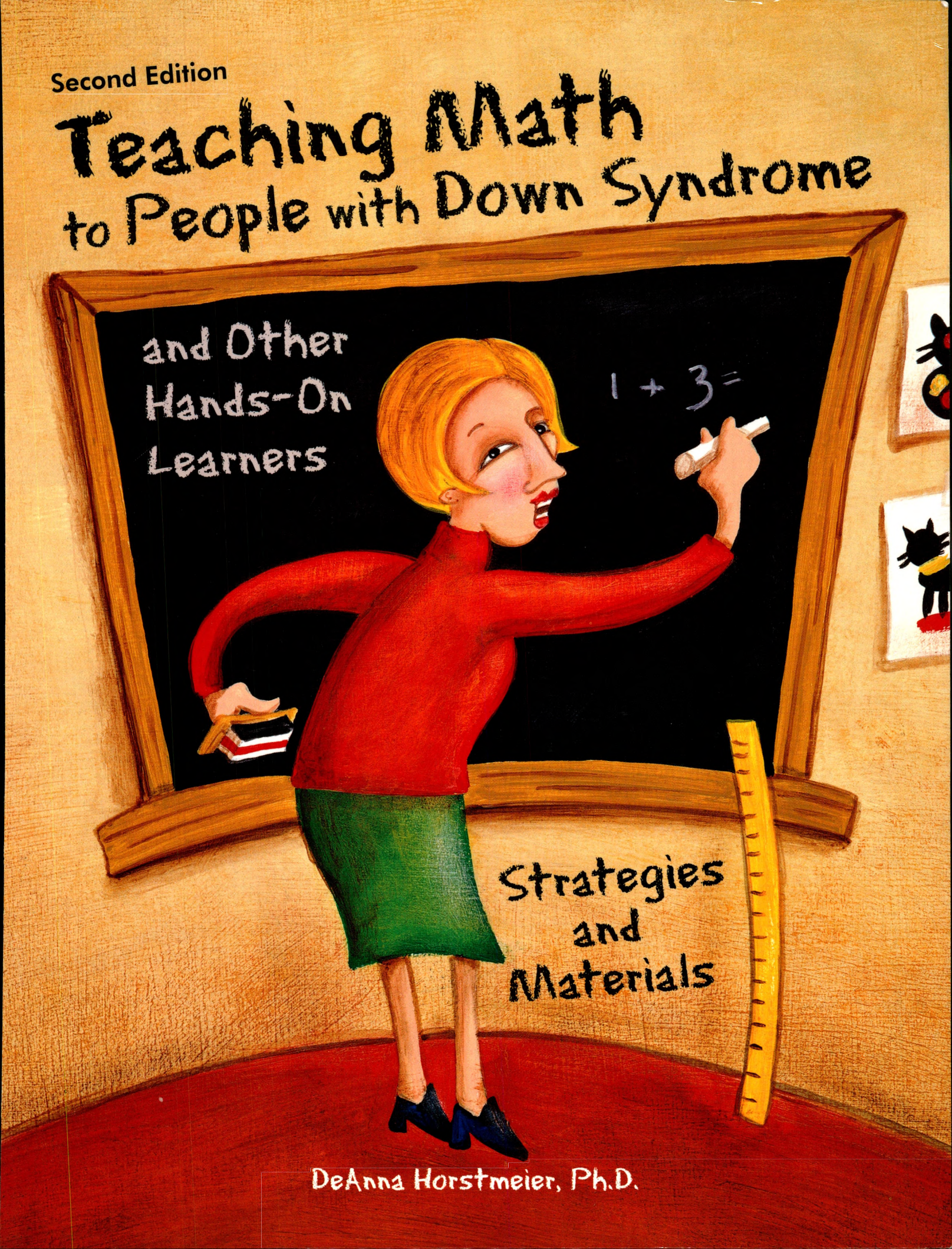
Second Edition

Teaching Math to People with Down Syndrome and Other Hands-On Learners

Strategies and Materials



DeAnna Horstmeier, Ph.D.

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TEACHING MATH to People with Down Syndrome and Other Hands-On Learners

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■ Strategies and Materials ■

DeAnna Horstmeier, Ph.D.

WOODBINE HOUSE ◆ 2016

DeAnna Horstmeier, Ph.D. (July 27, 1938–July 17, 2022) was an instructor of special education and speech, language, and communication at Ohio State University. Her interest in teaching people with Down syndrome arose in part from her experience as the mother of a son with Down syndrome. The publisher of several of her books, Woodbine House, ceased business in 2022. Her husband, Heinz Horstmeier, who now holds the copyright to the books, has kindly consented to make them available online for free. The books are:

Teaching Math to People with Down Syndrome and Other Hands-On Learners: Basic Survival Skills (2004) (408 pages)
Teaching Math to People with Down Syndrome and Other Hands-On Learners: Advanced Survival Skills (2008) (488 pages)
Teaching Math to People with Down Syndrome and Other Hands-On Learners: Strategies and Materials (2016) (a condensation of the two earlier books) (381 pages)
Try Reading Again: How to Motivate and Teach Older Beginners, Age 10 and Up (2012) (320 pages plus 88 pages, also included, from CD-ROM)

The books were scanned in November 2022 by Bound Book Scanning of Monsey, New York and reviewed before posting by Kurt Schuler. If you spot errors, please notify Kurt Schuler, kschuler@the-cfs.org.

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Second edition

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Many thanks to my husband, Heinz, the doctors, the medical staff,
and friends who helped me through my kidney implant;
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CHAPTER 1

Introduction to Teaching Basic Math Survival Skills

Scott Allen Horstmeier, my fourth child, was born with Down syndrome in 1970. At that time, most people had very low expectations for people with Down syndrome. I was an elementary school teacher, however, and eager to look into the early "infant stimulation" research that was just starting to paint a more hopeful picture for children like my son. Using what I learned, I worked with Scott regularly. He also went to both a typical and a special education preschool for some time.

In the meantime, I entered the Ohio State University to get further education so I could help Scott and other children like him. My early work concentrated on speech and language. As part of my graduate work, I visited the University of Washington, where preschool children with Down syndrome were being successfully taught to read. Encouraged, I applied what I learned to Scott's academic instruction. During his school years, I continued to teach him at home and worked hard to have him included in appropriate general education classes, especially in reading. Scott learned to read quickly and uses his reading skills daily—especially to read the ads for pizza places, the TV schedule, and the current movie listings in the newspaper.

Although I worked hard with Scott on his reading, I didn't pay very much attention to math. Teachers never indicated that math was a problem, and they seldom sent home math homework that taxed his abilities. Looking back on those school years, I realize that Scott had very little need to learn practical math skills. We freely gave him any items that he needed or desired, and he did not need to save money or use it in stores. Then, as a young adult, he moved into a supported living situation and worked in the community. To my consternation, I found that he was handicapped more by his deficits in math than by any other factor. He knew some single-digit addition and subtraction facts, but his budget was a disaster! We had given him a calculator, but he really didn't know how to set up the problems to use it. He spent any money that he had, regardless of the need to pay bills.

The four young men who inspired me to write this book: Scott Hays, Steve Langhirt, Scott Horstmeier, and Bryan Hiveley. Photo reprinted courtesy of the ***Columbus Dispatch****.*

By the time I discovered how inadequate Scott's practical math skills were, I was working as a consultant at a special education regional resource center in Ohio. I started to teach Scott and his roommates (who also had Down syndrome) how to use the calculator. I quickly discovered that the young adults needed much more than simple instruction in the use of the calculator. Some of the basic understandings that underlie the use of mathematics had flown—if they ever really understood them. I searched the literature for current research on teaching early number concepts and computation. Since the young adults in Scott's group were visual, concrete learners, I found out that a hands-on approach was essential. In addition, I learned that using games was the most effective way to have them learn and practice. Since they were paying for rent, utilities, food, and other necessities, I used their own experiences in working with addition, subtraction, and budgeting as the basis for my instruction.

I recorded the progress and activities that I used with Scott's group so that I could share them with other educators at the professional development classes that I taught. I discovered that many of the concepts and activities I used with the young adults could help students who were still in school, and could possibly give them a better chance at a fulfilling life. I then began giving regular workshops to teachers on teaching math to students who are concrete or hands-on learners.

Eventually, parents and classroom teachers convinced me that there was a need to gather my strategies and learning materials into a book so that many more students with Down syndrome and other hands-on learners could benefit from them. The result was the first edition of this book, *Teaching Math to People with Down Syndrome and Other Hands-On Learners, Volume One,* followed by a second volume that covered more advanced math concepts. Over time, it became apparent that some of the topics covered in Volume Two were not useful for users (either because their students never progressed to that stage or because their students learned the concepts better through other methods). So, Volume Two has been phased out as a stand-alone book, but some of the essential topics it covered (including multiplication, division, and measurement) have been added to this new expanded edition of *Teaching Math.*

Now parents, teachers, and tutors can use this *one* book for teaching math to any child or adult who needs a foundation in basic survival math and can benefit from its hands-on approach.

Who Will This Book Help?

There has not been a great deal of research into the math abilities of people with Down syndrome, but there is growing recognition that many of them have a great deal of difficulty acquiring math skills.

In the book *A Parent's Guide to Down Syndrome: Toward a Brighter Future* (Pueschel, 2001), H. D. Bud Fredericks cites a study of the academic skills of a sample of students with Down syndrome. In reading, the older students (ages 10–13) had achieved more than a third-grade level, which gave them access to many science and social studies materials, as well as works of fiction. In math, however, the same students (with the exception of one) had not achieved even a second-grade level. Similarly, a study of school-aged children (ages 6–14) with Down syndrome in the United Kingdom found that their math skills were an average of two years behind their reading abilities (Buckley, 2007). While my personal experience has been that some students with Down syndrome can achieve at higher levels, most authorities agree that students with Down syndrome have real difficulty with the abstract concepts of mathematics (Klein & Arieli, 1997; Nye & Bird, 1996; Broadley, 1991; Buckley & Sacks, 1987). A poor understanding of simple math concepts and money skills is also hindering many adults with Down syndrome in their quest for independent living. More explanation for these difficulties will be given in Chapter 2.

In addition, many students who do not have Down syndrome struggle with math. In standard achievement tests, US students always seem to do poorer in math than in reading, science, or other subjects. In the 2011 TIMSS (Trends in International Mathematics and Science Study), US students ranked in eleventh place in fourth-grade math and ninth place in eighth-grade math out of the fifty-six countries that administered the tests. Students in Asian and some European countries outdistanced those in the United States in average math scores. Why? Although some part of the differences may be due to the extreme diversity of ethnic groups tested here, the cause of the differences has not been researched widely.

Since much of math is built on previously learned concepts, low achieving students often disengage or drop out. The end result is that about 1 in 5 adults in the United States lack the math competence expected of a middle schooler. When older students and adults have such a poor understanding of simple math concepts and money skills, it hinders their quests for independent living.

Almost everyone needs math in their everyday living. Handling money is essential for most people, as are using numbers for quantities (for example, 6 oranges or a ½ pound of ground beef), calculating a tip, or checking your weight. The survival math taught in this book is just that—strategies to teach students enough math to survive in daily life.

After the early elementary grades, much of the teaching of math concepts in school is done at the abstract level. This book, however, is designed for concrete thinkers who need hands-on, practical activities to learn. This includes individuals (including some on the autism spectrum) who have difficulties with using abstract math skills in a meaningful way. The book's focus on the functional uses of math in daily living can motivate those individuals to learn and practice useful skills.

In short, this book is helpful for:

1. students with Down syndrome from early preschool through the elementary grades, as well as those in secondary school who have not mastered computation and basic number concepts and need teaching materials that are not "babyish"
2. students who have focus problems stemming from attention-deficit disorder

3. students with learning disabilities in math or a combination of math and reading difficulties
4. students who have difficulties with abstract learning, including those with fragile X syndrome, intellectual disabilities, or other developmental disorders
5. some students with autism spectrum disorder, who may be able to memorize basic number facts but have difficulty applying them to real situations.
6. any student who is consistently a low achiever in math
7. parents, teachers, or tutors who want *one* book that has clear instructions for teaching the basic arithmetic processes in a concrete, hands-on way
8. preschool teachers who want a developmental sequence to teach young students with activities beginning concepts using manipulatives

Who Is a Hands-on Learner?

In this book, the term **hands-on learner** is used to a describe student who is a concrete learner; that is, he or she learns best by actually handling and manipulating objects. Some hands-on learners may **only** be able to learn by handling objects. This is in contrast to many older children and adults who learn well by doing things concretely, but who are able to picture and manipulate objects in their minds or use language to solve problems. Concrete learners can be learning to picture things visually and later work toward representing things symbolically, but essentially their knowledge is on a concrete, hands-on level.

How Is This Book Different from Other Math Books?

1. It calls for frequent and early use of the calculator.
2. It emphasizes survival skills necessary for daily living.
3. It includes objectives for each area that can be made into instructional objectives by adding evaluation criteria.
4. Games and hands-on activities that can make practice fun are an essential part of the learning process.
5. Activities are suitable for older as well as younger learners.
6. Each operation (addition, subtraction, etc.) is taught to completion. (Most current math books mix addition with subtraction and multiplication with division.)

Who Can Teach Survival Math?

Anyone who can read the instructions can do the lessons: a parent, a teacher, a tutor, or an older student under supervision.

What Areas of Math Are Covered in This Book?

1. Prenumber concepts
    - Matching
    - Simple classification
2. Number sense
    - Counting
    - Visual spotting
3. Numeral recognition
4. Using a number line
5. Skip counting and counting-on
6. Place value
7. Writing numbers
8. Ordinal numbers and sequencing numbers
9. Addition
    - Basic principles
    - Facts (calculator)
    - Word problems
10. Subtraction
    - Basic principles
    - Types of subtraction
    - Word problems
11. Measurement and shapes
12. Money
13. Multiplication
    - Basic principles
    - Word problems
    - Larger numbers (calculator)
14. Division
    - Basic principles
    - Uses
    - Word problems
15. Simple fractions

CHAPTER 2

Characteristics of Concrete (Hands-On) Learners

Especially Those with Down Syndrome

Chapter 2 / Questions to be answered:

1. What characteristics of individuals with Down syndrome can pose challenges to math learning?
2. What strengths of individuals with Down syndrome can support their math learning?
3. What other students are hands-on (concrete) learners?
4. What about students who are on the autism spectrum?

Students with Down Syndrome

For years, many educators believed that individuals with Down syndrome were not really capable of learning to read and do math computation. And if our expectation was that they were not capable, we didn't give them much of an opportunity to learn these academic skills. In the United States, individuals with Down syndrome were often labeled "trainable" by schools and were excluded from the more academically oriented classes for "educable" students. Of course, there were infrequent examples of exceptional individuals who could read, write, and do some math (Hunt, 1967; Seagoe, 1964). However, until the mid-1970s, individuals with Down syndrome were mostly taught self-help and functional living skills.

Since the advent of what is now called the Individuals with Disabilities Education Act (IDEA), the special education law that opened the doors of schooling to every child in 1975, students with Down syndrome have been making strides in academic as well

as social learning. With higher expectations and more consistent education, achievements for most individuals with Down syndrome have been improving. These academic achievements are often higher when the students with Down syndrome are educated with general education students (Bird & Buckley, 2001).

Now that it has become commonplace to teach academics to students with Down syndrome, we are beginning to learn more about how Down syndrome affects learning. Importantly, almost all the research in the United States and Great Britain has shown that their achievements in reading are significantly greater than their achievements in math and number sense (Fredericks, 2001; Irwin, 1989; Buckley, 1985; Shepperdson, 1994). For example, three British researchers studied the academic achievements of twenty-four children with Down syndrome, ranging from six to fourteen years, who had been included in general education classrooms with good support (Buckley, Bird & Byrnes, 2001). After two years, their math skills were approximately four years behind the students' chronological age, while reading and spelling were only two years behind.

Similarly, John Rynders and his associate (1997) looked at data from three different studies involving the academic achievement of forty-six school-aged children with Down syndrome. The average reading level was fourth grade; however, the average math achievement was under second grade level. Although some of the authors have theories, no one actually knows why this discrepancy between math and reading achievement occurs.

If you are a parent who is wondering how Down syndrome might affect your own child's math abilities, it is worth remembering that research on the progress of students with Down syndrome in math is very limited. It is also quite probable that present-day adults with Down syndrome have had little systematic instruction in math and thus have not progressed up to their potential. Gillian Bird and Sue Buckley—educators and researchers at Down Syndrome Education International—have made it their life's work to help children with DS learn, and they have stated that in their clinical experience, "There is a wide variation in number ability among individuals with Down syndrome" (2001). This variation makes it very difficult to estimate what an individual person with Down syndrome is capable of doing in the math area.

What Characteristics of Individuals with Down Syndrome Pose Challenges?

In the absence of research pinpointing exactly why math is difficult for individuals with Down syndrome, this section offers some of the most common theories. Not all people with Down syndrome have all of these challenges to the same degree, and it is

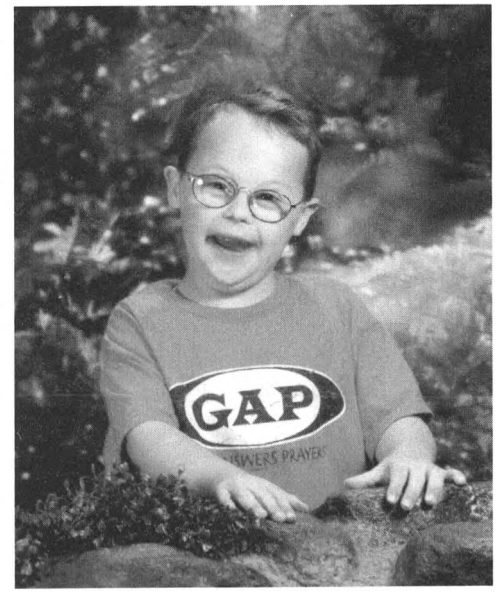
important for anyone working with a particular student to find out exactly which of these problems are stumbling blocks for that individual. In general, however, many children with Down syndrome have at least some of these challenges, and some children may have all of them.

1. Students with Down syndrome frequently have **problems with short-term and working memory.** Short-term memory helps us to keep specific facts or items that we have just heard or observed in mind as we decide what to do with them. For example, you keep a telephone number in memory for a short time until you have used it or have decided to memorize it. Working memory usually contains general processes that you must remember as you work with short-term items. For example, a student needs to keep the steps in division in mind as she plugs in the specific data from the problem she has been given.

   In math, we often use short-term memory in learning numerals, computation facts, and specific details about the current problem that is to be solved. Working memory underpins the processes of math—addition, subtraction, multiplication, division, and other multistep processes. Students with Down syndrome need assistance in learning to overcome these possible deficits in memory.

   Of course, if students have difficulties with short-term and working memory, they will not be adept at putting facts into long-term memory storage. When information (such as math facts) is not of interest to them or has little emotional impact, they may seem to remember it one day and forget it the next. This is not willful forgetting on the child's part—instead, it just means that the information was never properly stored in his or her memory to begin with. However, when items are securely in long-term memory, children with Down syndrome can remember them for a long time—sometimes even longer than their parents.

2. Children with Down syndrome may also have **fine motor delays in their hands and problems with eye-hand coordination.** These delays make it difficult to manipulate objects and make writing numerals difficult and slow. My son says, "I just have fat fingers. I want them to be fast fingers." It may be that they do not get as much experience with exploring objects in their world if they have difficulty manipulating them. Certainly they will have less energy to devote to understanding number relationships if they are struggling with just writing and lining up the numbers in a math problem.



3. Individuals with Down syndrome usually have **significant delays in speech and vocal expressive language**—that is, with communicating their thoughts in words. Problems with the oral motor mechanisms of the mouth such as sluggish tongue control, a narrow, high roof of the mouth, and difficulties with lip control can often make

them hard to understand. Research has shown that children with Down syndrome usually can understand more that they can speak clearly. Because of their difficulties with expressive speech, others tend to underestimate what they can understand (Miller, Leddy & Leavitt, 1999; Horstmeier, 1985). This underestimation of their abilities could make them lose opportunities for more advanced math learning. They are usually able to learn the meaning of math vocabulary words, though they often have to be taught them directly.

4. Although individuals with Down syndrome usually understand more than they can express verbally, they can also have **difficulties with receptive language**—that is, with understanding other people's spoken messages. Difficulties with receptive language are especially likely to complicate math learning if a concept can be expressed in several different ways in English. Speech-language pathologist Libby Kumin provided a clear example in her book *Classroom Language Skills for Children with Down Syndrome* (2001):

   "Synonyms can...be a problem when children are learning to sequence numbers. The simplest question is what number comes before or what number comes after, but worksheets to practice sequencing use all of the following:

   - Write the missing numbers.
   - Write the number that is one less.
   - Write the number that comes between the numbers.
   - Write the numbers in the correct order."

   In addition, students with Down syndrome may need instructions broken down into smaller steps. This is especially evident when a word problem requires two steps. For example:

   > Jan has 1 candy bar, Al has 7 candy bars, and Heinz has 4 candy bars. How many candy bars would they each have if they shared them so that each had the same number of candy bars?

   If the problem were broken down into parts, the student would be told to count up how many candy bars there were in total. The next step would be to divide the candy bars equally between the three individuals, yielding four each (by physical manipulation or by division).

5. **Processing information given *orally*** can sometimes present another problem. Studies indicate that about 40 percent of individuals with Down syndrome have a mild hearing loss. Ten to 15 percent have a more severe hearing loss (Fowler, 1995). Certainly, if children can't hear verbal instructions, they will have difficulties understanding words and concepts and distinguishing between similar sounding words, such as *forty* and *fourteen*. However, hearing loss does not tell the whole story, since even individuals with Down syndrome who do not have hearing losses usually have receptive language problems.

   Although children with Down syndrome can understand much of what they hear, they seem to process speech slowly and tend to miss details and information that is given sequentially, as in directions. Most children with Down syndrome have enough delay in auditory processing of speech that they learn much better through

their visual senses or through a combination of the various senses such as tactile (touch) and kinesthetic (movement).

Teachers who teach primarily by talking to the class can be very frustrating to students with Down syndrome. Students may be unable to keep up with processing the speech that they hear and may miss the important points the teacher is trying to make. If the teacher would just show the students *real* dollars and cents rather than numbers on a page, they could make picture in their minds. Sometimes students with DS get overwhelmed by the flood of speech and just tune out. They may think, "Someone else in the class will just have to show me how to add those dollars and cents later."

6. Children with Down syndrome may get **insufficient experiences using math** in the real world, unlike typically developing children. My son's behavior in a large crowded grocery store was so difficult to handle that I eventually left him home with his dad and did grocery shopping at the midnight hours. It was so much easier! But he was missing all those naturally occurring experiences of counting the oranges, seeing which breakfast cereal was cheapest, or finding aisle 4 so he could get the ketchup. I didn't always let him explore in the neighborhood and find the third house down where his friend Ray lived, or allow him to experience other number and location concepts. I got more adventurous as he got older, but I still ran along the backyards on our street, watching him, when he first walked down the sidewalk on his own.

   At home, parents or siblings may take over for their children in the interest of speeding things up. For instance, when playing a board game, your other children might shout out the number on the dice before your child with Down syndrome gets a chance to count it up, or you might program the phone number of your child's best friend into the phone rather than letting him or her try to dial the number independently.

7. Children with Down syndrome usually **take longer to progress through the stages of development** when learning how to think. Swiss psychologist Jean Piaget found that typical children go through these stages at particular ages. For instance, at the earliest stage of sensorimotor learning (birth to two years for typical children), children learn through their senses and motor activity. At the preoperational level (two to seven years), children learn to imitate and to play with toys and others and can use language to describe what they are doing. Schools generally gear their curricula to the developmental ages that they expect typically developing children to have reached at any given grade level. This means that academic tasks are often at a level that the student with Down syndrome has yet to reach.

   Remember, many students with Down syndrome are concrete learners. At the concrete operational stage (seven to twelve years for typical children), students are far more effective and capable at problem solving than before. However, their problem solving suffers from one important limitation. They can solve problems in an organized way only when dealing with concrete information that they can see and perceive directly.

   Concrete operational individuals deal poorly with abstract ideas (Berk, 2001). For example, a child who is at the concrete operational stage may be able to look at a pie that has been divided into sixths and tell you how much of the pie is left when

two pieces are taken away. But when confronted with a worksheet that asks him or her to subtract 2/6 from 1, the problem may be meaningless to the student. In other words, children who are concrete (operational) learners can compute with real things that they see, but have little chance of learning when given situations that require picturing the problem in their minds and doing mental math to solve it.

Several authors maintain that many individuals with Down syndrome do not progress much beyond the concrete operational level of cognition (Wolpert, 1996; Batshaw & Perret, 1993; Horstmeier, 1985). If so, this would explain why math skills that are highly abstract (such as using the letters *x* and *y* to stand for two unknown quantities in an algebraic equation) would be difficult or impossible for most people with Down syndrome to master. They have not reached the formal operations stage that would enable them to think abstractly about a situation and figure out possible consequences based on experiences and formal learning.

The concept of concrete learners helps to explain why people with Down syndrome do much better when using manipulatives in math and are motivated by real-life situations even when they are in middle or high school. The real problem solving goes on with concrete materials that they can see.

8. Sometimes students with Down syndrome have had **so many experiences with frustration that they are not motivated to try** and give up easily or refuse to participate. They also may not see how the math activity could be important to them. Perhaps one reason that people with Down syndrome are often more motivated to read than to do math is that reading tends to make the world more understandable to them, by allowing them to see words that flit by too quickly for them to process, but math often doesn't make anything clearer to them (at least the way that it is usually taught).

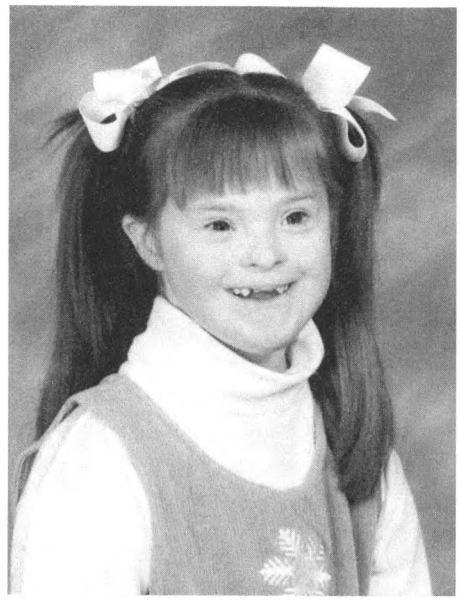
   Fortunately, children with Down syndrome will usually respond to interest-catching activities and games and to real-life situations that require the use of math. An enthusiastic parent or teacher is halfway there in the learning process. It is also very important to start by making sure the student experiences success when learning a new concept and to make sure those successes come frequently during the learning.

What Strengths Do Individuals with Down Syndrome Have?

Students with Down syndrome often have strengths that can assist in their math learning. Again, it is difficult to generalize, so it is possible that your student with Down syndrome may not have the strengths listed below. However, many individuals with Down syndrome have many, if not most of these characteristics. The key is to get to know your student as an individual and use teaching methods and materials that are suited to his or her strengths.

1. Many individuals with Down syndrome are **eager to please,** especially when young. Establishing rapport is crucial for motivating these students—and also for

the enjoyment of their instructors. Children with Down syndrome are often a joy to work with because of their enthusiasm for learning and appreciation of the person doing the teaching.



Of course, like any children, if they have been affected by frustrating experiences and inept teaching, they may create behavior problems. However, a study by Stephanny F. N. Freeman and Robert Hodapp (2000) indicates that children with Down syndrome have fewer problems with inappropriate behavior compared to children with intellectual disabilities due to other conditions.

2. Many individuals with Down syndrome **learn very well through their visual senses.** Often teachers will find that the student with Down syndrome can learn the concept if they just supplement their verbal explanations with a picture or graphic. Pictures, symbols, and written words are all helpful to understanding for the student with Down syndrome.

3. Even when individuals with Down syndrome have speech that is difficult to understand, they are often **able to communicate** with gestures and can show understanding by pointing or other visual ways of choosing an answer. Teachers who spend time with them can often tell by their expressive faces and imitations of actions what they want to communicate. They may be able to demonstrate the answer even if they cannot write it.

4. Children with Down syndrome seem to **go through the same developmental stages** in learning about math as typical children do. They just go through them more slowly. They *do* need to practice and practice important concepts so that they become automatic. They do *not* need endless drills on facts and worksheets of problems to do over and over again. Repetition should be creative and used only on important concepts, not to keep them busy during reading groups.

   Often individuals with Down syndrome enjoy working with "gadgets" such as calculators, smartphones, and computers, and these devices can be used to give the practice that they need to get their math skills securely into long-term memory.

5. Children with Down syndrome usually **enjoy interacting with their peers and often model peer behavior** both in and out of the classroom. Parents and teachers should take advantage of this characteristic and include typical children in play and academic settings. A study of parent and teacher attitudes after inclusion experiences for students with Down syndrome rated peer tutoring one of the most positive factors in the student's success. Teachers of sixth grade and higher reported that peer tutoring often worked better than teacher assistance (Wolpert, 1996).

Other Students Who Are Concrete Thinkers

Piaget's stages of cognitive (intelligence) development do not just apply to children with Down syndrome. Many other children struggle with basic number skills. Some of these students have cognitive disabilities, and some are typically developing in other areas besides math. If they are at the concrete operational stage or below and can work math solutions out with hands-on materials and games, then they will learn best with the "survival math" approach in this book. Certainly if a child's mental age is seven or below, you would want to try step-by-step procedures and concrete materials such as those recommended in this book.

Other students who are concrete thinkers may have specific deficits in learning strategies similar to those experienced by students with Down syndrome. These students will probably have similar experiences with frustration when they are unable to learn as easily as others are. They will also benefit from hands-on, visual instruction, motivational strategies, and rapport building. Some students with specific learning disabilities in reading may also have difficulties with math and learn more quickly with the hands-on, game-like approach to teaching math emphasized in this book. If, however, the student is primarily an auditory learner who pays little attention to visual instruction, he or she will not show as much progress using the activities in this book as the student who learns well from visual supports.

Students with attention-deficit disorders often need their learning sessions to be of shorter duration. However, it is quite possible that students with ADHD could focus long enough to complete the *games* in this book. Russell Barkley, a leading authority on attention-deficit disorders, has noted that many children with ADHD can play video games for long periods of time, probably because the games give immediate feedback that keeps them involved. Barkley concludes that the most important factor in ADHD is not a deficit in attention but the children's impulsivity and the fact that they are easily bored. The many interesting games and activities in this book could therefore be an addition to traditional math teaching for students with ADHD.

Students on the autism spectrum are also visual learners, perhaps even more so than most children with Down syndrome. The concrete activities and games in this book could hold interest and involvement for many children with autism. Some students with autism can easily memorize the basic facts and may be able to compute worksheets of number problems with ease. However, they often have difficulty learning how to use the facts appropriately. Teachers of students on the autism spectrum have asked for ways to help their students generalize their computation skills to real situations. The sections in this book on problem solving might prove very valuable to their overall functioning, especially if teachers or parents frequently incorporate math skills into various settings and situations in the student's daily activities.

CHAPTER 3

Teaching Strategies

If you are systematic, creative, and tuned in to the learning styles of your student or students, you will probably have the skills that you need to work with students who are concrete thinkers. Of course, it helps to be a "ham" and a good storyteller and to be enthusiastic about learning, too. There are also specific strategies and procedures that seem to work best with students who are concrete learners. These are summarized in the box below and discussed in more detail within the chapter.

Effective Teaching Strategies

- Emphasize visual learning.
- Use hands-on activities.
- Provide structured learning with some flexibility.
- Make learning relevant to the real world.
- Focus the student's attention and speak so he or she understands you.
- Give simple, clear homework and minimize distractions.
- Expect and encourage appropriate behavior.
- Ensure early success in the lesson.
- Consider the use of peer tutors.
- Facilitate short- and long-term memory.
- Use the calculator early and frequently.
- Make your interactions enjoyable.
- Use the computer, smart phones, and tablets.
- Break down the task into small steps.

Effective Teaching Strategies for Hands-on Learners

Emphasize Visual Learning

Students who struggle with math are very often visual learners—that is, they learn better by seeing actual objects or pictures of concepts than by hearing someone talking about those concepts. Time after time, I have seen students' eyes light up when I show a picture of what I am explaining. Most elementary teachers often use visual aids (commonly called visuals) in their teaching. However, when a student goes to middle school or high school, people seem to assume that reading a text should be enough. Visual learning through pictures and graphs are important to all of us as we are learning, regardless of age. In addition, because today's students spend so much time looking at television, playing video games, and using computers and smartphones, many of them have come to expect visual explanations for learning.

To make learning visual, it is not sufficient to just write numbers on the blackboard or on an overhead projector as you are talking. For younger children, using objects or pictorial representations of activities will be the most effective way of teaching. Once a child is able to read fluently, written numbers or words can supply a lasting explanation that is useful. (Visuals can be looked at repeatedly to help with working memory.) Even older students may be able to learn more quickly when pictures are included with words or numbers.

Educators working with students with autism spectrum disorders (ASD) have developed good innovations in the use of visuals for academic learning, direction following, and social skills. For example, Carol Gray (1991) has come up with the concept of "Social Stories"—illustrated stories written by parents, teachers, or speech-language therapists tailored to specific social problems a student is having that can be read and reread to help her understanding. And many students with ASD learn new skills through activity schedules—sequences of photographs or drawings that illustrate the steps involved in the skill. Many of the materials designed for students with ASD can be used as-is or modified for use with other students who are concrete learners.

Use Hands-On Activities

By definition, students who are concrete learners learn by manipulating objects and working out solutions with hands-on activities. Sometimes materials are left out for the students to discover their properties, but more commonly the parent or teacher demonstrates the actions and then the student reproduces the activity with her own materials. These hands-on activities may be more difficult for the parent or teacher to orchestrate, but the student's learning is usually greater and longer lasting.

PROVIDE STRUCTURED LEARNING WITH SOME FLEXIBILITY

Many students who are concrete learners are more comfortable with having a schedule and following it faithfully. They would like to have things stay just as they usually do it. As teachers, we need to address their need for structure and also build some flexibility into their learning.

For younger children, we can warn them that in five minutes we will be switching activities. Sometimes this warning is not enough, and we will have to think up a leadership-type activity to help them transition into the next subject. "Tom, I need you to collect the scissors for your group."

Making a classroom schedule with every activity on a separate card (illustrated, of course) and posting it in the front of the room may be useful. I put magnets on the back of large cards so they can be used on the refrigerator or a metal-backed chalkboard. Then, if a shift in sequence is necessary, I can make the change as the child watches. For some students, it may be helpful to have their own individual schedules for each day of the week. These schedules can be put in a small pocket chart holder so they can be changed. Parents working at home on math should also have a visual schedule or routine that helps their child know what to expect. Routines are especially important for students on the autism spectrum, who often have much difficulty with change.

Although schedules are useful, it is also important for children to learn to deal with unexpected changes to the schedule or routine because in real life, things don't always go as planned. One teacher in a workshop told me that she had one card with a question mark on it that she used when plans had to change suddenly. Sometimes the card signaled a nice surprise or a treat so the students had a pleasant experience with change.

MAKE LEARNING RELEVANT TO THE REAL WORLD

Just because the visual symbol for math appears in the schedule at a particular time, it doesn't mean that is the only time you work on math with the student. It is very important that you weave the math skills you are teaching into other parts of the academic day and into real-life settings. Involving students in activities such as measuring sugar for a cake or buying a video with their own money helps them understand why they need to learn something and motivates them to learn.

Parents often have the most opportunities to make math teaching come alive. Grocery shopping, mall shopping, counting calories, using the phone, punching in numbers on the TV remote, going to a restaurant, and many other activities can provide opportunities for math learning and practice and motivate your child to use math.

Be sure, however, that the real-world activities are not too complex for your student. Ads and signs often include numbers that can be confusing to students who are learning about money. For example, prices that are given per pound can give the student the idea

that the entire piece of meat is only $2.99. Or a grocery newspaper ad may offer 3 oranges for $1.00 when the student has not yet learned the process of division.

Focus the Student's Attention

Some parents and teachers have said, "I could teach her if I could only get her to sit still long enough." Getting the student's attention may be difficult for several reasons. First, if she is young, she is at the stage where she *should* be exploring and going from activity to activity. How long does a typical child sit to do activities at the same age? Are you expecting too much?

Another factor may be that you are not matching your teaching to your child's interest level. If the activity is dull or your voice is whiny or monotone, you may have lost her at the first turn of the race. Are your words at her level? Are your sentences fairly short? Are you stating the information clearly? Are you enthusiastic? Is the information concrete enough? Do you have pictures or models that are hands-on?

In addition, you may need attention-getters just to awaken a student's interest. When beginning to teach a new idea, you can do something unexpected such as coming in the room with a ruler balanced on your head but acting as if you don't know it is there. Most students have a good sense of humor—if the joke is simple and the joke is not on them.

When working one-on-one with a student, I sometimes got a frown, a head down on the table, and a refusal to do any more. I might set a puppet up on a pop bottle and give the instructions to the puppet and laugh and tease it just as if the puppet were a student. Often that is enough to get the student to look up and try. Sometimes I have to say something that is absolutely wrong, and the student can't resist correcting me. My son and his buddies (all young adults) can be kidded about being grouchy. I imitate what they look like (exaggerating, of course), and they usually start giggling and get back to work.

Many students on the autism spectrum and those with learning and attention disabilities process oral language quite slowly. This may be due to a mild hearing problem, but it is more likely to be a problem with auditory processing of oral language. Teachers who teach primarily by talking to the class can be very frustrating to student with delays in receptive understanding. A student may be unable to keep up with processing the speech that she hears and may miss the important points the teacher is trying to make. Speaking slower may help the situation, but many teachers have trouble maintaining slower speech. It is usually easier to insert regular pauses between phrases and short sentences. If you periodically write on the board, you will not only furnish a pause to help a child catch up, but also give her a visual, lasting cue. You may also have another student repeat directions using informal peer language that all the class understands.

Expect and Encourage Appropriate Behavior

Most typically developing older students can motivate themselves to do assigned work. Older students who are concrete learners, however, may not make the connection that doing work now will help them get good grades later on. You may have to use a system such as putting a sticker on the student's paper if she starts work within three minutes. The reward must be immediate or part of a token system until students have enough experience to want to do well just for themselves (internal rewards).

Token systems based on positive actions have been successful for many students with learning difficulties, but they may work best if the whole class is on the system.

Token systems may also work in a family, where the children earn tokens such as stickers for desired behaviors and receive some reward at designated times (such as after 10 stickers). Privileges can often be used more easily as rewards in families than in classrooms because of school principles or rules.

PROVIDE NONDISTRACTING, SIMPLE, CLEAR WRITTEN WORK

The written material that is given to the student should not distract from the purpose of the activity. Sometimes creators of school materials want to make them more aesthetically pleasing, so they use a special color or design to highlight items. These additions may be distracting to concrete learners.

Sometimes there are just too many pictures on a page. Watch the student's eyes. If they dart back and forth, or if she is unable to locate a prominent feature on the paper, take a look at the materials and see if they have too many distracting elements. One way to minimize distractions is to cut a hole in a sheet of paper, cover the picture, and show the student only one part of it at a time.

Don't give too many directions at the same time when explaining how to do the written work. Processing multistep directions can defeat a student at the very start. Write the instructions on the worksheet, especially if it is going home. Parents will often help at home, but not if they can't understand what has to be done.

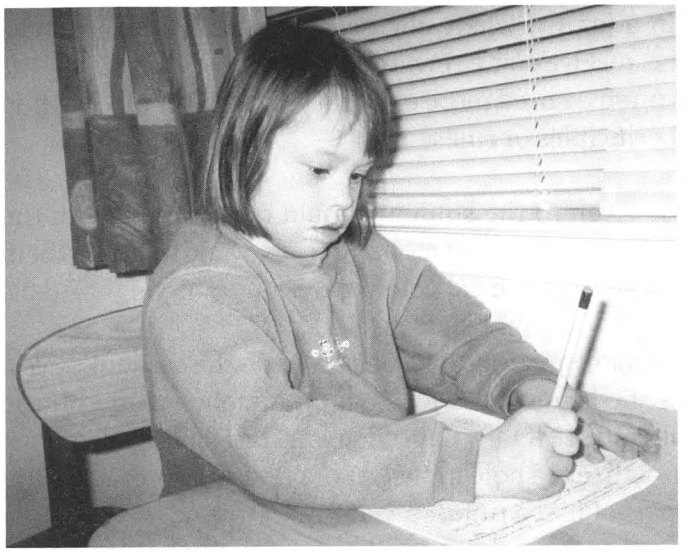
If you are preparing written work for the student, use your computer or copy machine to make things larger, even if it takes two sheets instead of one. It is better to have fewer math problems on a page and leave more space for the student to write in the answers. Make sure that the print quality is dark, crisp, and legible so that the student does not have to guess whether a number is a seven or a one, or whether she is looking at a picture of a nickel or a quarter.

A study done in the United Kingdom (Wolpert,1996) with ninety teachers who had students with Down syndrome in general classrooms found that workbooks were not useful at all. The authors felt that the workbooks were either heavily dependent on language comprehension or had too many distractions or problems on a page, which made it confusing for students with Down syndrome. The teachers felt that the most effective materials for instruction were concrete objects or manipulatives that the student had to use to perform an activity.

GIVE SIMPLE, CLEAR HOMEWORK

Homework sometimes becomes a source of friction between school and home. The work sent home for math homework should always be something that the student has already learned. Parents should be reinforcing what the student has learned at the school, not teaching her what she didn't pick up in school that day. If the concept has really been learned in class, lower-achieving math students should be responsible for doing some of each type of problem, but not necessarily the same amount of work given to typical students. Students with fine motor difficulties who have to spend excessive amounts of time writing recognizable numbers should be allowed to do only a representative number of problems, fewer than the rest of the class.

If a student has really worked hard at a homework paper for twenty or thirty minutes (depending on her age) and doesn't seem to understand the assignment, Mom or Dad should have an agreement with the teacher that she can stop work. They should then send a note explaining that the student has worked hard for twenty or thirty minutes straight and still does not understand. Ask the teacher to explain and demonstrate again to the student what is expected. This will lower the frustration



level at home—for student and parents. Parents have to resist the temptation to do the problems for their child. The teacher will not know what areas need to be retaught if the student brings in work she hasn't done herself. (However, it may be acceptable for you to write down your child's answers for her if her hand gets too fatigued.)

Parents can help by having a consistent time for homework that doesn't compete with favorite TV shows or outdoor play. Homework should not be a constant time for conflict between student and parents. If homework is creating such trouble at home, parents, teacher(s), and the student need to talk together and work out some solutions.

Rushing out the door, Mrs. Landon said, "I have to go home now. It is Heidi's homework time, and nothing happens if I am not there. I am just getting to hate the time after supper at my house. I don't know how much help to give. Sometimes she says, 'I can do it myself,' and sometimes she says, 'You never help me.'"

It is possible that your child just will not accept you as a teacher. That is all right. Your most important role must be as a parent. You may be able to find a tutor or an interested neighbor who can teach your child using this book. You also might be able to arrange some individual time with a teacher to use the one-on-one activities given in the book. I usually put on a funny-looking hat when I worked with my son Scott as a teacher. I told him to call me Mrs. Horstmeier, not Mom, when I was being his teacher.

Ensure Early Success in the Lesson

Success is very important at every level of teaching for low-achieving students. The steps in teaching should be very small, and each one should be praised for effort, if not correctness. As the students get older, they often meet with frustration as they try to do what others do. They may give up quickly because it is better *not* to do the activity than to do the activity and fail. Sometimes students who are "higher functioning" are more aware of those discrepancies than those who are "lower functioning," and, consequently, feel more stress about the situation.

I have tried to include a "success step" before each major learning activity in this book. That is, I recommend a relatively easy activity related to the new skill for the student to complete at the beginning of the lesson. It will only be a success step if the student is able to do it easily. Do not hesitate to change the step to something you know that the student can do well. You may find that some of the activities jump too quickly from one step to another. By observing the student, you can tell whether an intermediate step is needed and can act to preserve the feeling of success. Don't hesitate to try your own intermediate step.

As we get older, we are able to see our own accomplishments and get satisfaction from them. We learn that in most work environments, "No news is good news." But

even adults need occasional, quiet praise for their self-esteem. So, we should sincerely praise our students but also teach them to feel good about their own success.

CONSIDER USING PEER TUTORS

Both parents and teachers in the U.K. study mentioned above felt positive about peer tutors. In fact, the teachers of sixth grade and higher reported that peer intervention worked better than teacher assistance. In the upper grades, students are striving for more independence, and being helped by a friend was perceived as better than being helped by a teacher or aide.

However, peer tutoring should be organized with care. When the students are in first grade or below, they should be just helping each other, not having a peer-tutor relationship. When a peer is actually doing tutoring in an academic subject, the teacher should carefully select the tutor and teach him or her some teaching techniques. See if you can enlist a "laid-back" student with a sense of humor and the ability to follow directions. I have the tutor observe a session when I am working directly with the child to model the teaching strategies.

After the first one or two tutoring sessions, the teacher should talk to both students individually about the relationship and the skills learned. The tutor may be puzzled about problems he or she has encountered, and the tutee needs to know that she is expected to study and work with her peer tutor. It is important to look at what goes on in the tutoring session. I have seen sessions where the tutor makes fun of the student she is supposed to be helping, which is exactly the opposite of the effect we are hoping to create.

FACILITATE SHORT- AND LONG-TERM MEMORY

Assist the student to facilitate short- and long-term memory storage by using musical or rhythmic associations with the concepts, concrete visualizations, creative practice, and mnemonics.

Music is an excellent way to help students memorize numbers and important concepts. Many children (and adults) can remember the tune and the lyrics to a song that teaches a concept. Think of counting songs like "One Little, Two Little, Three Little Tigers," "Ten Little Monkeys Jumping on the Bed," and "This Old Man." Parents and teachers can make up teaching verses to many common childhood songs. It is important to remember that the tune should be simple and easy to sing so that the emphasis is on learning through the words. See Chapter 8 for suggestions of counting songs.

Concrete visualizations of objects and pictures can help students understand abstract concepts. We have found that many students need to see the passage of time on a round analog clock even though they can read the time on a digital clock. Likewise, many people picture a cut-up pie or pizza when they think about fractions. Actual manipulation of the materials in problem solving adds the tactile sensation of touching to the visual learning.

Mnemonics—making picture associations with the numbers and concepts—is also a helpful strategy. For example, we easily remember the shape of Italy because we have learned to associate its looks with a boot. Some of my older school classes have shared mnemonic strategies that helped them remember important items. For example, when reading a word problem that included the word *together,* one student observed, "*Together* starts with *t* and that looks like a + (plus) sign for addition."

Try some of the memory strategies suggested in this book and create ideas tailored to the student's own interests. The student can then choose the strategies that work best for her.

Short reviews at the beginning of each class period will help students make important facts automatic. Practice also should be introduced in a fun way with varied and creative activities. Ideally, each student should learn the addition and subtraction facts from 1 to 20 by heart, although this is not an essential skill for survival. That way, simple calculations can be done without paper, pencil, or calculator. Most individuals will need to practice and practice the simple addition and subtraction facts if they are to be useful in this way.

USE THE CALCULATOR EARLY AND FREQUENTLY

When pocket calculators first became widely available, typically developing students weren't usually allowed to use them much before middle school, on the grounds that early calculator use might interfere with learning math facts. However, an analysis of many research studies (Hembree & Dessart, 1986) concluded that use of a calculator, along with traditional math instruction, improves the average student's ability to do pencil-and-paper calculations and to problem solve. The National Council of Teachers of Mathematics (1989, 2001) has suggested putting less emphasis on paper-and-pencil calculations and using more technology.

I have noticed that many adults with Down syndrome and other disabilities do not use much mental math in everyday situations, perhaps because of working memory problems. They can, however, be taught to use calculators well. If some individuals with cognitive disabilities are not able to use mental math in a practical way, why not teach them to use the calculator from a young age?

After Chapter 7, the numeral recognition chapter, you should be having your student(s) use the calculator to check what you are doing with the concrete materials and using concrete materials to check the calculator answers.

MAKE YOUR INTERACTIONS ENJOYABLE

Make your teaching fun and show your pleasure in working with your students. As they approach school age, they often can be motivated by mild competition. If there are no other children present, challenge the student to compete with you. Of course, you will not always win, and you are *so* surprised that she has learned so much. Use humor as much as you can. Students often have a broad sense of humor, and you may be able to dispel their grouchy moods with gentle teasing.

USE THE COMPUTER, SMARTPHONES, AND TABLETS

Most computer programs and math apps are not designed to teach math but instead provide practice for concepts that have already been learned. The color and graphics in these programs usually get the student's attention immediately. Then the immediate feedback for right or wrong answers can keep her on the learning track. Practice-type programs can usually be taught to the student in one session, and she can work indepen-

dently from then on. Working with devices such as calculators, cell phones, and computers seems to be intrinsically interesting to children who are concrete learners. They see color and graphics in front of them, and they get immediate feedback on their answers.

Other computer programs do some teaching; for instance, when the student is told the correct answer after one or two tries. The computer is definitely not a substitute for a teacher, and it takes a teacher to set up the situation so that the computer session is productive. However, allowing the computer to do the teaching now and then can be a welcome change for both the teacher and the student, as well as being good motivation for needed practice. Computer software and apps are continually changing, but a few good apps and websites with helpful games, activities, and links are listed in the Resources section.

BREAK THE TASK DOWN INTO SMALL STEPS

I have tried to break the goals and tasks in this book into small steps that can be mastered easily. However, every student will learn in her own way. I'm sure that you will find times when a particular student just can't make the next step in the math goals. You will have to find a way to make the task shorter or less complex or try a completely new activity. Most parents and teachers who know how a student learns best can find a simpler or more effective way to teach the concept. Try it. Breaking down or modifying a task is a lot easier than most people think.

Adaptations and Modifications to General Classroom Work

In the early elementary years, it is possible that some students with Down syndrome and other concrete learners may be able to do exactly the same work in math class as the other students. By third or fourth grade, however, almost all students who are concrete thinkers will need *adaptations* in the way math is taught or in how the student shows her learning. There will probably need to be some *modifications* in the content of what the student will be learning as she gets to the higher grade levels.

It is beyond the scope of this book to go into great detail about ways to make math work more appropriate for students with disabilities. I do want to stress the importance of determining the most appropriate adaptations for your child and spelling them out in her individualized education program (IEP). With increasing numbers of children with learning problems being included in general education, you cannot count on your child's teacher having the special education training to know how to modify classroom demands for your child. There are books dedicated to helping teachers modify the curriculum in the general classroom (Blenk, 1995; Hammeken, 1995; Beninghof, 1998; and Stainback & Stainback, 1992). (See the References.) One of the most frequently mentioned programs is explained in the book *Adapting Curriculum and Instruction in Inclusive Classrooms: A Teacher's Desk Reference* (Ebeling, D.G., Deschenes, C. & Sprague, J., 1994) from the Institute for the Study of Developmental Disabilities, Indiana University. This combination book and staff development kit categorizes nine types of adaptations:

1. **Size** - reduce the number of items
2. **Time** - extend amount of time for test or assignments

3. **Level of support** - provide more assistance
4. **Input** - modify the way the instruction is given to the student (for example, read the problems aloud to her, or provide manipulatives)
5. **Difficulty** - make the problems easier (for example, by using a calculator or simplifying the rules of a math game)
6. **Output** - adapt how the student reports her learning (for example, using stamps or labels with numbers printed on them, rather than writing them, or having an aide write down the student's answers)
7. **Participation** - the student participates in only part of the task (for example, the student could gather data about favorite ice cream flavors with the other students, but then not figure out what percentage like vanilla best)
8. **Alternate goals** (modifications of classroom goals) - have less complex goals than the rest of the class (for instance, learning single-digit subtraction instead of three-digit subtraction)
9. **Substitute curriculum and goals** - student has different instruction and activities for her specific goals

Because I have found that I often need reminders of each student's special needs, adaptations, and modifications, I have compiled a checklist that can be filled out at the beginning of the year for each student and given to all other teachers involved. I have modified the checklist to be more specific to mathematics education.

Parents can send the checklist to their child's teacher or bring it to the IEP meeting and discuss the most appropriate adaptations to help their child. You have the most accurate, complete knowledge of your child and can be of real assistance to the school.

Student Name: _____ **Date:** _____

Checklist for Adaptations and Modifications to the General Curriculum

The following adaptations are appropriate and necessary for this student. Check all that apply.

Pacing

____ Extend time requirements
____ Vary activity often
____ Allow more breaks for student
____ Omit timed assignments
____ Work on vocabulary before lesson
____ Pick out only major concepts for learning

Environment

____ Reduce/minimize distractions
____ Provide extra paper and pencils close to student

Presentation of subject matter

____ Teach to student's learning style
  ___ Visual
  ___ Auditory
  ___ Tactile-kinesthetic
  ___ Experiential

Such as:

- Use visual whenever possible
- Use visually colorful computer programs
- Use pictures and mnemonics for memory
- Use chants or songs
- Use sand or Kool Aid in a pan for writing
- Use manipulatives and hands-on activities
- Write with finger on desk when learning
- Do wet writing on chalkboard
- Use dry erase marker on whiteboard
- Practice with board games
- Reduce copying from the board/book

Type of instruction

____ Individual and small group instruction
____ Functional application of academic skills
____ More review
____ Move around the room to gather information
____ Errorless learning

Materials

____ Large print
____ Arrangement of nondistracting material on page
____ Calculator for all math
____ Graph paper
____ Computer (not just as reward)

Assignments

____ Visual daily schedule
____ Calendars and assignment books
____ Use written back-up for oral directions
____ Request parent reinforcement
____ Reduce paper and pencil tasks
____ Shorten assignment
____ Lower difficulty level

Testing and proof of learning

____ Provide thorough reviews before tests
____ Oral testing
____ Correct missed problems for extra credit
____ Test administered by aide or special ed person

Continued on next page . . .

Social interaction support

____ Peer advocacy
____ Shared experiences in school
____ Extracurricular activities
____ Structure activities to foster social interaction
____ Train peer tutors
____ Debrief peer tutors

Motivation and positive climate

____ Offer choice
____ Planned motivating sequence of activities
____ Mostly positive reinforcement
____ Verbal praise
____ Concrete reinforcement, if needed
____ Set up token system
____ Use strengths/interests often
____ Cultivate a general positive attitude

Individual hints for working with this student:

CHAPTER 4

Assessing Basic Math Skills & Beginning the Lessons

Chapter 4 / Questions to be answered:

1. How do you decide where to start a student in this book?
2. What is the Pre- /Post Checklist used for?
3. Why use an informal screening assessment?
4. Do you have to score the Informal Screening Assessment?
5. What skills can be assessed in game or hands-on activity format?
6. Why does this book advocate early and frequent calculator use?
7. Why should students who are concrete learners use calculators?
8. What type of calculators can be used?
9. What are the basic principles for using calculators?

How Do You Decide Where to Start a Student in This Book?

Determining what math activities to give to a student you are just beginning to work with can be done in several ways:

1. Start at the beginning of the book.
2. Use the Pre- /Post Checklists at the beginning of each chapter.
3. Do the Informal Screening Assessment found in this chapter.

START AT THE BEGINNING

If your child is young and has not had much training in math concepts, start at Chapter 5, try out the activities and games, and enjoy yourself. You may be able to try these activities for the first time when your child is about two or three, or once he can indicate a choice or answer by pointing or another nonverbal method. If you find that your child already understands one of the skills you are doing, drop it and go on to the next activity. The chapters are mostly hierarchical, with one chapter building on the previous chapter. However, many activities in Chapters 5, 6, and 7 can be done together because their skills are interwoven. You will want to be working on calculator skills along with all the chapters from Chapters 5–15.

Some students in the upper elementary grades and middle and high schools may have had so little exposure to math concepts or have had so much frustration with doing math that you might want to just start from the beginning of the book and do each chapter. If you find that your student understands a concept, just skip ahead.

USE THE PRE-/POST CHECKLISTS

If you know the student's skills quite well, just look at the Table of Contents to see the skills that are taught in the book. Estimate where the student needs to start and turn to that chapter. At the beginning of each teaching chapter, there is a list of tasks that the student should be able to do once he has mastered the skills taught in that chapter.

Each item in the checklist describes an objective that will be taught in that chapter. Suppose that the student easily names the ordinal positions in a line of ten people. You would check off that item and not teach the activities in that section. However, you may have noticed that if he opens a book in the middle, he does not know what direction to turn the pages to get to a higher number. He does not know how to use sequence to find a specific number, so you would teach him the activities and games about using sequence.

If you have worked with a given student for some time, you may be able to easily place him in the activities. However, if you are not sure of his skills in a specific area, you can assess him on that one area. Look at the teaching section of the appropriate chapter for ideas on how to determine whether he has the skill. Then teach the skills he has not mastered. In general, the chapters get more advanced as they get higher in number.

GIVE THE INFORMAL ASSESSMENT

The informal assessment is very useful if you are not sure what the student's skills are. It is called an *informal assessment* because it is given for diagnostic purposes only and does not sample all the math skills covered in the book. Therefore, no age norms or grade levels will be given. The assessments are given in mostly a game format. Some students who have had difficulties with math respond poorly to written tests. Most will enjoy the game format and will be more motivated to succeed without feeling pressure. Some of the young adults I have worked with would *not* take another paper-and-pencil test because, they said, "I'm out of school now." Younger children can probably attend longer with a game format.

Most of the assessment information will come from observing the student. Each section is quite flexible so that the person doing the assessing does not have to use specific words or follow a specific order of items. Not every area covered in the book is tested in this informal screening. Only the key areas are assessed.

Informal Screening Assessment

ADMINISTRATION

The assessment may be administered over the course of two or three days. The board games should be played to completion so the students feel that they are playing real games. The Earn and Pay board game gives the most assessment information of all the games, so if you have limited time, play that game first. Only give those parts of the assessment that are appropriate for the individual student.

A scoring chart is provided after the procedure for each game. This chart also lists the chapters that cover the skills tested in the game.

SCORING

You really do not have to score this screening if you have only one student. If you note the areas where the student has difficulties, it may not be necessary to record any scores. If you work with several students, or if you need numerical scores to show growth, the scoring system is very simple.

TASK	POINTS
Student attempts task but does it incorrectly	1 point
Student does the skill inconsistently or prompted	2 points
Student does the task correctly after training* *Teacher demonstrates skill, gives the same item again (slightly modified by changing a number), and student succeeds. Usually the areas that the student can do with training are the first areas that you target for teaching.	2T points
Student does the task correctly without assistance	3 points

Check the student's learned skills on the Pre- /Post Checklists and begin the teaching.

EARLY GAME SCREENING FOR YOUNG STUDENTS

MATERIALS:

- Commercial board game Chutes and Ladders (Hasbro) or any simple game that has squares that need to be counted and dice to be rolled
- 20 pennies
- At least 3 dimes and 3 quarters
- Die

PROCEDURE:

1. If the child is young (grade 2 or younger) have him play a game of Chutes and Ladders or another game with you and/or another child. Observe whether he understands the concept of one-to-one correspondence when moving his token around the board.

2. Then see if he can count 1–20 pennies.
3. Give the student 5 pennies, and then give him 1–3 more and ask how many he has. Then take away some quantities and ask how many he has.
4. See if he can sort 3 pennies, 3 dimes, and 3 quarters. He doesn't have to know the value of the coins.

EARLY Board Game Scoring	Chapters to Be Taught	Points
1. One-to-one correspondence	Chapters 4 and 6	
2. Counting to 20	Chapter 6	
3. Sorting coins	Chapter 5	
4. Use of simple subtraction or addition	Chapters 9 and 10	

EARN AND PAY BOARD GAME

MATERIALS:

- Earn and Pay game (Appendix A-1), glued to the center of a file folder
- Earn and Pay cards for older or younger players (Appendix A-2 and A-3). If possible, copy them on cardstock or heavier paper
- Currency: $1, $5, $10, $20 bills ($85 in play money for each player)
- One die
- Game markers (small buttons, tokens, pennies, etc.)
- Money Total slip (A-4)
- Calculator

PROCEDURE:

1. Copy the game board from A-1 and paste the pages on the inside of a file folder. If you are going to use the board several times, it would be wise to laminate it.
2. Cut out the Earn and Pay cards from A-2 or A-3. You should be able to tell the Pay cards from the Earn cards when they are placed face down. Put the cards face down on the game board as indicated.
3. Give each player: 5 $1's, 2 $5's, 3 $10's, 2 $20's = $85. The adult must play if there is only one student. The adult also serves as the banker.

4. Have the players throw the die, and the highest number starts first.
5. The first person rolls the die and moves his marker that number of spaces. The space landed on will be either a Pay space or an Earn space. The player picks up the top card from either the Pay pile or the Earn pile. The player receives money (from the banker) or pays out that amount and is given the Pay or Earn card.
6. Sometimes the mixture of the amount of money needed and the description of what is to happen will be funny—e.g., buy ice cream for $40. Comment on the amounts if they are quite improbable. The corner squares involve some kind of

direction such as lose a turn or go back 2 spaces. If the player moves up or back following one of those directions, she does not pay or earn what is on the square he lands on.

7. The adult serving as the bank should have the student try to figure out the amount of change he gets back by subtraction.
8. The first person to reach the finish line is Winner I. He must throw the exact number needed to land on the finish square.
9. Winner II is the person who has the most money left. Have the players sort their currency in piles by denomination. Have each one separately try to count the number of bills in each pile. See if they can count the ones with one-to-one correspondence.
10. If the students are able to sort **and** count accurately, have them use the Money Total Slip found in Appendix A-4 to figure the total. See if they can count the $5's by skip counting (5-10-15, etc.) See if they can skip count the $10's (10, 20, 30, etc.). See if they can get the total amount of money for each pile by using the calculator. Help them with adding up the entire amount, either by calculator or regular addition. You will be learning much about their skills as they figure out how much money they have.
11. Each player parades his playing piece around the board in order (according to their finishes) and counts the number of squares on the board up to 20.

Earn and Pay Game Board Scoring	**Chapters to Be Taught**	**Points**
1. One-to-one correspondence (one number for one space)	Chapters 4 and 6	
2. Counting to 20	Chapter 6	
3. Recognizing numbers to 25	Chapter 7	
4. Skip counting by 5s and 10s (when counting the currency)	Chapters 8 and 15	
5. Sorting (currency into piles of 1, 5, 10, and 20 dollar bills etc.)	Chapter 15	
6. Simple money concepts (paying and receiving)	Chapter 15	
7. Use of the next-highest-dollar strategy (when buying a card)	Chapter 15	
8. Use of simple subtraction for figuring out change from bills (calculator)	Chapter 10	
9. Use of addition in figuring out total money left (calculator can be used)	Chapter 9	
Note here any multiplication or division 10. Use of multiplication or division in scoring	Chapters 11 and 13	

A student who does well on the Earn and Pay board game may also be able to do multiplication, division, or word problems using any of the math operations. You may want to give him the problems below to know where to start your teaching.

WRITTEN SCREENING (FOR MULTIPLICATION, DIVISION, AND WORD PROBLEMS)

- Calculator may be used.

MULTIPLICATION (TWO PROBLEMS CORRECT)

$$\begin{array}{r} 44 \\ \times\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 67 \\ \times\ 12 \\ \hline \end{array}$$

DIVISION (TWO PROBLEMS CORRECT)

$44 \div 2 =$ $24 \div 6 =$

WORD PROBLEMS

A. Mark and his 2 friends each bought 3 bananas. How many bananas did they buy in all? _____

B. Allison dealt out 52 cards to her 4 friends to play a game. How many cards did each of her friends get? (Do not count Allison as a player.) _____

TOP THIS GAME

All students should try the Top This Game. It shows recognition of numerals and comparison of their values.

MATERIALS:

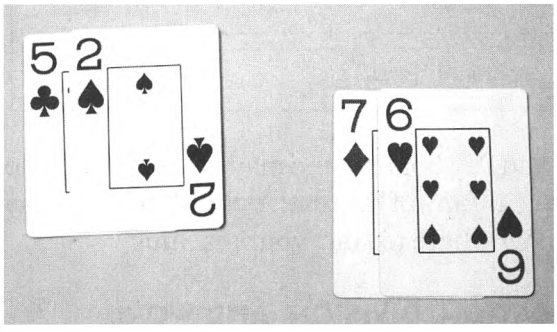
- Playing cards (omitting face cards—Joker, Queen, and King) and changing the Ace to a 1 with a marker)

OR

- Flashcards (2 sets) of the numerals 1–9. (If the flashcards are not in two different colors, you will not be able to do the sorting task.)

PROCEDURE:

1. Divide the deck into the red cards and the black cards. (If the student does the separating, it may give insight on his or her sorting abilities.) Deal 6 cards (if possible, of one color) to each player. The game is easiest with 2 players. If there are 1 or 3 players, the tester will need to play.



2. The purpose of the game is to assess numeral naming and the concept of big and small numbers. We will use the word *largest* to refer to numerals with the highest value.
3. To begin, each player's cards should be in a pile turned face-down in front of her. The first player turns over the first card in her pile and says the numeral. The second person turns over her first card and decides which number is largest. The person with the largest number takes both cards.
4. Then the action is reversed: the second player shows her next card and the first player decides which is the largest number. If the two numbers are the same, each player keeps her own card.
5. The winner is the one with the most cards after all cards are used or after about 15 minutes.
6. If the players successfully named the numerals and determined which number was larger using single-digit numbers, proceed to assessing knowledge of two-digit numbers. For two-digit numbers, the first player turns over one card that will be in the *tens* place (e.g., 4 tens). He then turns over a second card to be in the *ones* place

(e.g., 7 ones). Lay the two numerals next to each other to form a two-digit number (e.g., 47). The first player should name the two-digit numeral formed.

7. The other player does the same. The second person says which number is larger and the game continues as with the single digits. Only the person laying out the second set of cards should say which is the larger! Keep refilling the players' cards from the pile of red or black cards not used at the beginning of the game.

Top This Game Scoring	Chapters to Be Taught	Points
1. Single digits named correctly.	Chapter 7	
2. Two-digit numbers named correctly.	Chapter 7	
3. Knows which single-digit number is the largest.	Chapters 6 and 8	
4. Knows which double-digit number is the smallest.	Chapters 6 and 8	

Younger students should try the single-digit naming and determining which is larger. They do not have to try the double digits if they can't do the single digits.

STICK MANIPULATION

This game assesses single- and double-digit addition and subtraction, as well as counting on.

MATERIALS:

- Rubber bands
- Popsicle sticks (craft sticks), tongue depressors, plastic tableware, or unsharpened pencils, etc. Give each student 1 bundle of ten, and 8 single sticks.
- Construction paper of 3 colors to use as mats. One mat should always be the answer mat.
- Teacher-made handout with written problems (space left for students work)

PROCEDURE:

1. **Single-digit addition:** Put 2 sticks on one mat and 5 sticks on the other.

$$2 + 5 _____$$

then

$$4 + 2 = _____$$

2. **Double-digit addition:** Put 11 sticks on one mat and 5 on the other and ask the student the total.

$$\begin{array}{r} 11 \\ +5 \\ \hline \end{array}$$

3. **Single-digit subtraction:** Give the student 7 single sticks and ask him to give you 4 of them. Ask him how many are left.

   $7 - 4 = _____$

4. **Subtraction with regrouping/place value:** Give the student one bundle of 10 and 4 single sticks. Give him permission to take the rubber band off. Ask him to give you 7 sticks. He should place 3 sticks on the answer mat. (Needs to regroup the ten.)

5. Do the written problems at the end of this section if the student does the stick manipulation accurately.

Stick Manipulation Scoring	Chapters to Be Taught	Points
1. _____ **Single and double digit addition** done correctly.	Chapter 9	
2. _____ **Single and double digit subtraction** done correctly.	Chapter 10	
3. _____ Knows which single-digit number is the **largest**.	Chapter 8	
4. _____ Knows which double-digit number is the **smallest**.	Chapter 8	
5. _____ Multiplication (can use the calculator)	Chapter 12	

D. WRITTEN PROBLEMS

45 x 4	23 x 36	74 ÷ 2	18 ÷ 6

A. Paul and his 2 friends each bought 8 bananas. How many bananas did they buy in all?

B. Allison dealt out 52 cards to her 4 friends to play a game. How many cards would each of her friends get? (Do not count Allison as a player.)

Use of the Calculator

WHY DOES THIS BOOK ADVOCATE EARLY AND FREQUENT CALCULATOR USE?

This book strongly suggests using a calculator as soon as the student is introduced to the numerals that are the symbols of our number system. Early and frequent use of the calculator can be very helpful for children who have difficulty with short-term memory and rote learning.

I further recommend that children who are concrete learners be systematically *taught* to use the calculator. A student cannot simply be given a calculator and be told, "Just use this if you are having trouble learning your multiplication facts." He needs to be familiar with the keyboard of the calculator, including the position of the numbers and the operation keys, have experience in entering numbers accurately, and learn what keys *not* to push. But even more importantly, he needs to be taught how to set up problems that can be solved with a calculator.

Parents and teachers are sometimes worried that students will get so dependent on the calculator that they will not be able to compute without it. For students who are concrete learners, I would not be upset if they understood how to set up a problem but used the calculator instead of pencil and paper to solve it—and then knew whether their calculations were reasonable. Many adults use the calculator in exactly this manner. You should also know that an analysis of many research studies (Hembree & Dessart, 1986) concluded that use of a calculator, along with traditional math instruction, improved the average student's ability to do pencil-and-paper calculations and to problem solve.

The National Council of Teachers of Mathematics (1989, 2001) suggested putting less emphasis on paper-and-pencil calculations for *all* math learners, especially with numbers having more than two digits. The NCTM also thinks that math teachers should help students develop the skills necessary to use appropriate technology and then apply the results to problem situations. The NCTM wrote in the first *Curriculum and Evaluation Standards* (1989), "Appropriate calculators should be available to *all* students at *all* times." The analogy is given that calculators simplify work, much like word processing programs help writers (Smith, 1997). Many adults today, including the author, use calculators when adding several items or when working with numerals with more than two digits.

WHY SHOULD STUDENTS WHO ARE CONCRETE LEARNERS USE CALCULATORS?

Students with Down syndrome have difficulties with short-term and working memory, especially if the occasion is not very meaningful to them. Tests show that they have short digit-span memories, meaning that they can't remember as long a string of numbers as other children the same age. In addition, many of these students have difficulty processing the things they hear. When I talk through a problem situation with concrete materials, some of the students say, "Please put those numbers on the board; I just can't keep them in my head." They are visual and tactile learners who derive meaning from seeing the numerals and from touching them on the calculator. Using the calculator capitalizes on their sensory strengths in learning. It is easier for most people to press a button than to shape numerals on paper.

Students who are concrete learners, by definition, do not do math "in their heads." They need many activities and much practice to learn important math concepts such as numeral order and the sequence of numbers from 20 on. These fundamental skills need to be "overlearned" so that they come automatically and the student only has to think how to apply them. Therefore, it often takes concrete learners more time to learn the reduced number of important things that they have to memorize. Summer after summer, I worked with my son using flashcards of addition and multiplication facts so he would not forget them before the fall. Since these drills didn't work, I was essentially wasting time when I could have been helping him learn to solve real math situations.

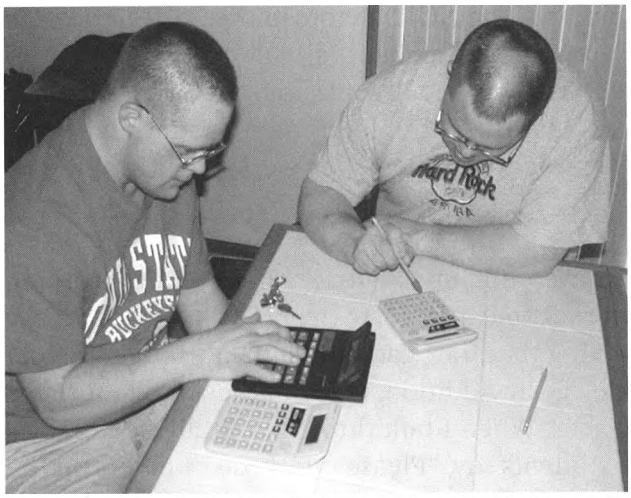
Lastly, there is a certain intangible feeling of joy to seeing someone who has previously shied away from numbers start to work on a calculator confidently. When I gave a group of young adults with Down syndrome a written assessment in simple computation, they complained and fussed and found

all sorts of reasons to dawdle. After a short while, I said, "Would you like to use your calculators?" "Of course!" they said. They immediately got to work and did the problems like the young adults they are. The difference in their confidence when they used the calculator was amazing!

WHAT TYPE OF CALCULATORS CAN BE USED?

Students who are hands-on learners need explicit instruction on the use of the calculator. Unfortunately, the keyboard is not set up in the same way as a telephone dial. Even if students recognize the difference, they may have motor and visual difficulty in pressing the correct numbers.

When I first began teaching students with Down syndrome, I started with the small calculators (3" x 4½") that have an identical keyboard to the overhead calculator that I use (*Calc-U-Vue*). Most of the students had difficulty accurately entering numbers with those calculators. They were more successful with the large (8" x 6"), inexpensive calculators that are often given away by companies. The numerals are easy to read and widely separated. The answer window shows large numerals and can be tilted so the teacher can observe the answers of several students at a time.



Using a large-keyed calculator (left) and a talking calculator (right).

Of course, after students are proficient on a large calculator, we want them to get a small calculator that can fit in the pocket or purse. Make sure you get a calculator with the numbers from 0–9 arranged in four lines of three buttons each. An on-off button, the operation signs for addition, subtraction, multiplication, and division, and the *clear* button and *equals* sign are essential.

Either solar- or battery-powered calculators are fine. I do not recommend using a membrane calculator because the students seem to need the definite edges of a button and the slight soft click to make sure they have entered the number. If the student has a smartphone, check to see if the default calculator or a calculator app will work for him. For example, the calculator on the iPhone makes clicks when numbers are entered, so it may be a good option for some older students. You should teach the students on their individual cell phones because steps and keyboards may differ.

Another calculator that has proven useful is the talking calculator (PCI Educational Publishing). It repeats the numbers entered and tells the total. Some students seem to need the reinforcement of hearing that they have plugged in the correct number.

For a student who has difficulty writing, you may use a calculator that has a printer. These calculators print the whole problem out on a roll of white paper, something like an old-time cash register would do.

Many personal computers include a calculator in one of their basic programs. You will have to teach the students how to go through several steps to access the calculator on either a PC or an Apple computer. (For example, on a PC, you need to go to "all programs" and then "accessories.") With the adaptations that can be made on computers for fine motor and visual problems (adaptable keyboards, touch screens, etc.), the student may be able to do the work on a computer when other solutions do not work.

Using a printing calculator.

A special consideration to bear in mind when you are working on math problems involving prices is that most calculators are not set up to calculate dollars and cents as we visualize them. If your answer is $10.80, the calculator will show 10.8 in the window. Some students have real difficulty understanding this concept—which is really just a function of the calculator, not of the math itself. PCI Educational Publishing has a brightly colored calculator called Money Calc that has the option of always showing the money amount correctly (to two decimal places). It also has a button for calculating the tax that can be set for your state percentage. Older students may want to use the Texas Instruments TI-15 calculator, which can be set to always give two decimal places. This calculator has quite a few other buttons and functions that could be distracting, however.

Left: Money Calc (PCI Educational Publishing); Right: Texas Instruments 15 calculator

What Are the Basic Principles for Using Calculators?

1. **Make it simple.** Cover unnecessary buttons on the calculator. If necessary, put black tape over all but the number buttons, on/off, clear, and the operations buttons (+ and –, etc.).

2. **Give the student some time to fool around with the calculator** before you have him do work with it. All of us like to fiddle around with a new toy. If you give some unstructured time for playing, the students will be more apt to use it for work later.

3. **Practice frequently using the keyboard of the calculator** before using it to solve problems. Students may find it easier to punch in a number when you show it visually than when you just say it. Teach students to enter numbers *both* ways. Have the students punch in the numerals and then hold the answer up for you to check. It is important to give the students time to train their motor skills in using the calculator as well as their understanding of how to use it.

4. **Use concrete materials for understanding.** It is very important that the student be given concrete materials to work out the concepts that are being taught. Concrete teaching materials should be used often for better understanding. Tally marks can also be used when appropriate.

5. **Use the calculator to verify.** After the initial concept has been taught, have your student use his calculator to verify the concept that has been worked out in a concrete manner. For example, if you are using manipulatives to teach addition, after the student has counted 5 green and 5 red tokens to reach the answer of 10, have him punch in 5 + 5 on the calculator to verify the answer. Sometimes using manipulatives, making concrete drawings, or drawing tally marks to verify the answer on the calculator will be useful.

6. **Use the calculator as soon as possible.** Students must be able to recognize the numerals 1–9 and be able to press one key at a time before learning to use the calculator. If you have a student who is starting to work on the book at a later chapter, introduce him to the calculator right away if he has never used one before.

7. **Get in a rhythm when using the calculator.** You can even chant the sequence:

   - **Three** (or whatever the number is), as you push the number 3.
   - **Look** (check the number in the window)
   - **Plus** (or whatever the operation is)
   - **Five** (or whatever the second number is)
   - **Look** (check the number in the window)
   - **Equals** (as you press the = sign) to get the answer

   It is very important that you have the students enter the data slowly and check at the window every time. You are not looking for speed, just accuracy. It will take them more time if they have to go back and find where they made a mistake because they hurried.

   The rhythm in the chant will also help the students put in the operation sign after each number and end with the equals sign.

8. **Ask whether the answer is reasonable.** Students should know whether the calculated answer is reasonable. This concept takes a good deal of time to learn. Students often don't even think about the answer they give. I tease a little to make them think. "Stephen, do you really want to spend $80.00 for lunch? What are you going to eat for that amount of money? Doesn't $8.00 sound more reasonable for a lunch?"

CHAPTER 5

Prenumber Concepts

Pre- /Post Checklist

Can the student:

1. _____ Find the bigger item in comparisons of two items?
2. _____ Identify same and different using objects and pictures?
3. _____ Match identical objects and pictures?
4. _____ Sort items by basic colors (red vs. yellow)?
5. _____ Sort items by shapes and sizes?

Many people think of math skills as only involving numbers and quantities. However, before you can count, add, measure, or do anything else with numbers, you have to learn certain "prenumber" concepts. For instance, you can't begin to be able to count how many red circles there are until you can recognize both *red* and *circles.* Or you can't tell someone how tall your block tower is unless you know what the word *tall* means.

Children begin to develop some prenumber concepts even before they start counting. They know that some things are heavier or bigger than others. Some children learn some math comparison words such as *big* and *little, up* and *down, fast* and *slow, hot* and *cold,* and other simple word pairs from others around them. Others need these concepts to be deliberately demonstrated and taught to them. Sometimes it may be difficult to see these early experiences as forming a base for math, but they do form the cognitive foundation skills that are important for learning math concepts.

Another essential concept is that of sorting or simple classification. If we want to be able to count how many girls are in the family or in the class, we first have to classify the individuals as either girls or boys. Classification is needed to tell us *what*

to count. To sort or classify, we also need to pay attention to the details of items and determine whether they are the same or different from each other. Therefore, it is important to learn to distinguish same and different items, and progress to matching, sorting, and classifying them.

The beginning math concepts that will be covered in this chapter are:

- **Simple comparisons**
  - Big (a lot) or little (tiny)
  - Same or different
- **Matching identical items**
- **Simple classification (sorting)**
  - By color
  - By size and other characteristics

Simple Comparison

In the beginning, you do not need to teach your child many comparison words. You just want her to look at a few things and develop an early understanding of quantity. Which one has more? Which one is bigger? The most basic comparison is seeing that one item is bigger than the other is. Just talking about *big* things and *little* things with your child in everyday experiences probably teaches the concept best.

You can try some of these simple comparisons when your child is between one and two years of age. If she is not interested, don't force the issue, just continue pointing out comparisons in daily conversation. Or watch a television show such as *Sesame Street,* which shows a lot of colorful comparisons, if your child focuses in on that program.

BIG OR LITTLE

OBJECTIVE: The child will compare two items where one is roughly twice the size of the other and be able to identify which one is bigger.

MATERIALS:

- A big box and a small box (if possible, the same color)
- A big book and a little book
- Dad's shoe and child's shoe (if the child can identify them both as shoes)

SUCCESS STEP: Have the child identify both boxes by pointing or naming. (Say, "What is this?" or "Show me a box.") Praise her by saying something like, "Right. You did great!"

PROCEDURE:

1. Put the two boxes in front of the child. Ask her to point to the *big* one. If she is correct, praise her. If not, hold the big box closer to her and the smaller box behind it and repeat.
2. Try the same procedure with the big and little books and big and little shoes.

Emily looks at her Dad's big shoe and her own little shoe that are placed before her. Her mother asks her which is the ***big*** *shoe. Emily shakes her head and says, "My shoe." She picks it up and tries to put it on her foot. Her mother realizes that Emily has not responded to the word* ***big*** *but is focused on the possession of her own shoe. Her mother changes the items to a complete Sunday newspaper vs. a half page of the newspaper. Emily then responds to the word* ***big*** *as applied to the newspaper.*

GENERALIZATION INTO DAILY ACTIVITIES: Now it is important to show your child big and little objects in her own world. The items should be quite similar except for size. You might try to show *big* and *little* in helpings of food. At first make comments about things that are *big* or *little*. For instance, "Wow, Dad took a *big* serving of mashed potatoes. Mine is little!" Later you may have your child participate by asking, "Can you point to the *big* helping of mashed potatoes?" You can also give her opportunities to make a choice between big and little. "Do you want a *big* piece of paper or a *little* one? A *big* crayon or a *little* one?"

MORE

OBJECTIVE: The child will compare two groups of items where one group has roughly twice as many items as the other and identify which one has *more*.

Note: *When children first learn to use the word* more*—especially if they are learning to sign—they use it to request more of something they already have. For example, requesting more food or toys implies a desire for an additional quantity of what the child already has (more banana). She can't have more bananas until she has had some banana. The activity in this section is not focused on this meaning of more, but instead on the concept of comparing two groups to see which group has more items or a larger quantity than the other.*

MATERIALS:

- Blocks of the same size
- Poker chips of one color or pennies
- Dry macaroni (or other small items your child will not try to eat)

PROCEDURE:

1. Build two towers with blocks or with poker chips. One should be much taller than the other. Count the chips or blocks aloud as you make the towers. Ask your child which tower has *more* chips or blocks. If she is correct, let her crash down the bigger tower. If she guesses wrong, say something like, "No, *this* one has more. Look how *big* it is. The other tower is *little*." Repeat as needed. If your child enjoys building, challenge her to make a tower with *more* blocks than yours.
2. Put a few pieces of dry macaroni in one of her hands and put much *more* macaroni in the other hand. Ask her which hand has *more*. Repeat several times. Then see if

she can put more macaroni in one of your hands than in the other. This is harder than looking at the two amounts and judging which has more. If she is not able to do it, try it again in a month or two.

COMMERCIAL GAME TO BUY OR TRY

If your child is beyond the stage of putting everything into her mouth, you may want to try playing the commercial game Don't Spill the Beans by Milton Bradley (about $7-9). As each person places a plastic bean on the bean cover, she can say, "Now I have more." The next player says, "No, I have more," until the beans spill over.

GENERALIZATION INTO DAILY ACTIVITIES: Your child will learn far more about the concept of more in the course of your regular daily living than can be gained from the structured activities given here.

When serving foods to the class or the family, purposefully give a lot more pretzels or other snacks to one child. If your child does not protest that someone else has more, ask her who has more. If she can't recognize this yet, say, "Whoops! I gave Jason more than you!" Equalize the quantity of snacks and say that everyone has to have the same. One child can't have more in our class or family. Likewise, when you are handing out craft materials, blocks, dominos, or other items, see if your child notices that another child has more than she has.

Parents, help your child to give Dad or an older child more of some food and then to tell Dad or brother that he did. (For example, "Let's give Dad more broccoli than Anna. Dad really loves broccoli!") Talk about who needs more in your family and who needs less. Talk about who has more socks than others when you are putting laundry away or who has more leaves in their pile when you are raking leaves or other similar activities.

Same and Different

For children to start sorting or classifying, they have to be able to look at the details of each object, compare them, and decide if they are the same or different from each other. In teaching about sorting, start with items that are exactly the same (such as three identical shoes) and add another item that is markedly different from those items (such as an orange). Then you can progress to using items that are in the same category but not identical (such as three nonmatching shoes) and again add an item that is markedly different (an orange). This task is more difficult than just finding one different item among other identical items.

INTRODUCING THE CONCEPTS OF SAME AND DIFFERENT

OBJECTIVE: The child will be able to indicate which item is different in a group of four items in which three are the same. Use both objects and pictures.

MATERIALS:

- Several sets of 2 (or 3) identical items and 1 markedly different item small enough to spread on the table in front of the child. Start with two pencils and a piece of paper crushed into a ball.

  Other examples are:
  - Cars and a book
  - Bananas and a salt shaker
  - Forks and a spoon
  - Pieces of paper and a piece of chalk
  - Socks and a shoe
  - Pieces of red paper and a white piece
  - Same size blocks and an apple
  - Cereal pieces and a marshmallow
  - Pictures of 3 identical items and one that is very different
  - Three coats and one boot

SUCCESS STEP: Show the child one familiar item such as a pencil and tell her to touch or point to the pencil. Praise her for success. If she doesn't respond, touch the pencil to her hand and say, "You touched the pencil. Great!"

PROCEDURE:

1. Show the child the two identical items one by one and ask her to touch or point to each one. Hold them all in front of her and tell her that they are the *same.* Repeat at least once. Then show her the crushed paper ball and say, "Is this a pencil? No! No! This is a ball (or a paper ball). It is *not* the same as the pencils" (showing the pencils).
2. Play catch with the ball with her for a few minutes. Then show her the pencil. "Is this the *same* as the ball? No! No! It is *not* the same. (It is different.)"
3. Put the ball behind your back and hold out the two pencils. "Are they the same?" Repeat the activity, if necessary.
4. Put one pencil on the table. Hold out the ball and the pencil in one hand. Ask, "Which one is the same?" using your other hand to point to the pencil on the table. If necessary, hold the pencil up toward her face and the paper ball back farther. "Good, you found the same." Repeat the above activity several times. Later, switch to a different set of materials.
5. At this stage, you are teaching the concept of *same* and *not same*. You may also say the word *different*, but you are looking for the concept of *same* and *not same* at first.

   Note: *If the student has autism or Asperger syndrome, you should probably use the word* different *from the start. Students with autism sometimes have problems processing the word not and its connection with the following word.*

6. You may need to put a different set of materials out and model touching the different item. Then help the child do the same thing, using up to 4 items.
7. It is better to start with tangible objects and then progress to pictures. However, sometimes when you use tangible objects, children want to play with one of the objects and will persist in playing with it without learning the concept of same and

different. If so, you should go on to pictures. Since all four pictures may have about the same play value for the child, she may be able to concentrate on the activity. If the student has difficulty with 4 items, try using 2 that are the same and 1 that is not. (If you are working with a child who has autism, she may have notable difficulty understanding that a picture can stand for an object, and may need to learn what are called "picture-object correspondence skills."

8. When your child is able to do same and different with pictures, you can use preschool activity and coloring books that have pictures that teach that concept. If she has trouble holding a pencil or crayon, though, don't expect her to circle or cross out the different picture.

Teaching with Music

I like to use the song that was used in *Sesame Street* when I am asking a child to show me which thing is different.

One of These Things
Words and Music by Joe Raposo and Jon Stone

One of these things is not like the others.
One of these things just doesn't belong.
Can you guess which thing is not like the others?
By the time I finish my song?

Did you guess which thing was not like the others?
Did you guess which thing just doesn't belong?
If you guessed this one is not like the others,
Then you're absolutely...right.

GENERALIZATION INTO DAILY LIFE: Parents have the best chance to help their child learn to use the concepts of *same* and *different.* Your food cupboards are full of items that are the same and different. You just have to point them out. When you go food shopping, you can pick out produce that is the same and different. You can have your child find the different dark sock in the basket of white ones, or one yellow apple among the red ones. You can save fast food wrappers or boxes and have her point out the different one, or, when you are eating at McDonald's, look around the restaurant and see which children got the same Happy Meal toy as she did. You can have her pick out dress pants from a group of jeans. There are many ways to bring this concept into daily living. Just be sure that the *same* items are a lot alike and the *different* item is very distinctly different.

Educators can use things in the school environment such as pencils, pens, colored paper, juice boxes or milk cartons, and books to emphasize *same* and *different.* You may try comparing students' clothes, but most pieces of clothing are not exactly alike.

Matching

By working on *same* and *different* above, you have cued the child to look at the details of the items. She should now be ready to match simple identical items. Matching is the concept of one-to-one correspondence using *like* things. If your child has a visual problem, you will want to start with concrete objects that can be easily felt. Even if you are using pictures, you can add some different textures such as sand glued to one pictured item as well as to its match. When teaching your child to match, have her move the item to a position *beside* its identical match, or she can point to the matching item and you can move it for her.

MATCHING REAL OBJECTS

OBJECTIVE: The child will be able to match identical objects.

MATERIALS:

- One roll of unopened toilet paper, a cup, and a brush (or similar items) on the table
- An open box containing a roll of unopened toilet paper, a brush, a cup, and a spoon

Note: *You may also use some of the items that are mentioned in the materials section of the same or different activity above. However, it would be wise not to use the same items you used when teaching about same and different, because you may confuse the student.*

SUCCESS STEP: Have the child name or point to the items in the box. Praise her for correct answers.

PROCEDURE:

1. Lay the toilet paper roll and the brush on a table in front of the student, just out of her reach. Put the box on a chair beside her. Take the roll of toilet paper from the box and put it beside the roll on the table. "See, this roll is the same as that roll. They match. Can you do the same thing (or match them too)?" Give her the roll and prompt her to do the same thing. Praise her and repeat, if needed.
2. Then put the one roll of toilet paper back in the box, and see if she can match the items on the table with the items in the box. When she is successful, add the cup to the items on the table and see if she can match three items. If the objects are distracting her from learning, go on to using pictures, which seem to have less distracting value (next step).

MATCHING PICTURES

OBJECTIVE: The child will be able to match identical pictures.

MATERIALS:

- Pictures of two dogs, cows, horses, and rabbits found in Appendix B-1.
- A mat so that the pictures won't slip around on the table. The latex mats used for shelf lining work quite well.

SUCCESS STEP: Have the child name or point to several pictures of familiar things. Praise her for correct answers.

PROCEDURE:

1. On a mat on the table, place three different animal pictures in a row, spaced about four inches apart from each other, directly in front of the student. Show the picture cards to her one at a time.
2. See if she can match the pictures in front of her by putting the correct picture next to its match. Say, "Find the one that's the same," pointing to the animal picture you want her to match. If she puts one on top of the other as she is learning, she can't self-correct as easily as if they are side by side. Repeat, mixing up the pictures.

   Note: *If animals are not motivating to the student, try using pictures of toys or cartoon characters that she really likes. You can use black-and-white line drawings, colored drawings, or computer clipart, if that helps motivate the student.*

3. Try again, using three and then four pictures to match at the same time.
4. You probably will need to repeat this activity many times with different pictures to make sure the student can really match.

MATCH A BATCH GAME

OBJECTIVE: Same as above.

MATERIALS:

- Pairs of pictures from Appendix B-1
- *Match a Batch* game (constructed as described below)
- A piece of cardboard or poster board that is the same size as nine of the picture cards when they are placed in three rows of three

PROCEDURE:

1. Cut a piece of cardboard or poster board so it is the same size as nine of the picture cards arranged in three rows of three.
2. Make two copies of each page of pictures so you have nine pairs of pictures.
3. Separate the picture cards so you have two identical sets. One set will be the playing cards and the other set will go on the game board.
4. Begin by placing nine picture cards face up on the game board (in three rows of three). Shuffle the duplicate cards and place them face down in the draw pile in front of the student.
5. Have the children in turn pick a card from the draw pile and try to make a match on the game board. If they are incorrect, they must keep the card. If they are suc-

cessful in the match, they put the draw card on top of or to the side of its match on the game board. The one who makes the most matches wins.

6. *For a more challenging game,* turn the cards on the game board face down. The first child turns over the top card on the draw pile. She guesses which card on the game board will match and turns it over. If the card is the same as the card she drew, she puts it on top of its match on the game board. If the card does not match, she turns the game-board card over again and keeps the draw-pile card. The next player draws a card, turns over a card on the game board and looks for a match. The idea of the game is to remember where the game cards are located so you can choose the right match. The winner is the one who makes the most matches.

COMMERCIAL GAMES TO BUY OR TRY

The commercial game called ***Memory*** by Milton Bradley uses a little plastic form to hold the cards for the game board. If the student has fine motor problems, she can put her finger in the holes under the frame and push out the cards, which are made of thick cardboard. The commercial game has 12 items to match, but it is a game that families and students can play together with success. At the present time, there are also Winnie the Pooh and Bob the Builder ***Memory*** games and probably games with other themes.

There are also many simple lotto games that can be bought at discount stores, teacher's stores, and on the Internet. (Lotto games generally include game boards that are preprinted with an array of pictures or photographs of objects, plus individual cards with single pictures of those same objects. Players draw cards and are allowed to place them on their board if they have a match.) For younger children, there are apps with matching tasks that give immediate feedback.

Simple Classification

Classification, the grouping of items that have a property or properties in common, is an important skill for math, reading, and language. Much of the instruction in all content areas consists of making more subtle classifications and giving them names or grouping together items that have characteristics in common and giving the grouping a name. For example, a large body of water may be further classified as a lake, sea, ocean, or river. Dogs, cats, cows, and horses are grouped together as animals. These skills will be taught briefly here, but activities in classifying are expected to be part of reading readiness, cognitive development, and language activities.

Children usually start to classify by size, color, number, or appearance. Jean Piaget describes the development of classification as children learning to form groups of items based on a fixed rule (Copeland, 1984). They may not be able to tell you the rule, but they are using one.

First, children learn to sort basic items for one characteristic such as color. For example, the child sorts poker chips into a white pile and a red pile. Later, two or more

characteristics may be used, such as square red items/round red items and square white items/round white items. Then more difficult distinctions may be required, such as determining the various shades of blues that fit into the category of blue.

SIMPLE SORTING

OBJECTIVE: The child will be able to sort items by different colors, shapes, sizes, and types of items.

MATERIALS:

- Knives, forks, spoons
- A divided silverware tray or 3 separate boxes
- 12 pieces of construction paper (4 pieces each of 3 different colors)
- 3 or 4 different types of small food items that differ in color or size (for example, colored miniature marshmallows, raisins and peanuts, Lucky Charms cereal pieces, gummy bears, jellybeans, chocolate and regular Teddy Grahams, and any other small items of food that the student enjoys)

  OR
- If using food is not wise, use 3 types of small inedible objects such as plastic teddy bear counters that come in 3 or 4 colors, blocks that come in 3 or 4 colors, paper cups differing by size or by color, hair accessories ("scrunchies") of different sizes and colors, socks, plastic utensils in different colors, or cans of food (only 3 kinds)

SUCCESS STEP: Have the child point to a knife, a spoon, and a fork as they are named (when they are laid in front of her).

PROCEDURE:

Sorting by Color

1. Show the child how to sort by color. Say, "Let's put the papers that are the same color together." Then start sorting the papers into piles by color, commenting when you find one that looks "the same."
2. Have the child try sorting colored construction paper into 3 different piles by color. If she makes a mistake, show her that the colors are "not the same" by holding the two mismatched pieces of paper next to each other and saying, "These two are not the same."
3. Later, or if the child is not interested in sorting colored paper, have her sort colored play money, small cars, Lego blocks, or things she collects by color.

   Note: *The object of this activity is to sort the different colors, not name them, but you can talk about the names of the colors at the same time. Do not mix shades of colors such as light blue, navy, and royal blue at this time. These distinctions are very difficult to make. Expect some students to have difficulty distinguishing black, purple, and navy blue.*

Sorting by Shape and Size

1. After the dishes have been washed or before a meal, show your child how to sort the knives, forks, and spoons into a divided silverware container. You may want to talk yourself through this to show her what to do by picking up a fork and saying, "OK, I'm putting the forks here. Here's another fork, so I'll put it in the same place...."
2. Next, have her sort the knives, forks, and spoons and place them in separate boxes or the divided silverware container. At first, do not present the smaller salad forks or large dishing spoons. When she is confident about sorting, you can introduce the different-sized forks and spoons.
3. As an alternate activity for younger children, have them sort their toys into different small bins as part of cleanup (blocks in one bin, dolls in another) or separate different-sized balls for storage.

Difficulties Sorting by Category

There are many different ways that similar items can differ. If your child is able to do some tasks and not others, help her make a rule to use when making her choices and have her repeat it when choosing. For example, if she is having trouble sorting cans from glasses, you could tell her the rule, "Cans have a closed top; glasses have an open top you can put your finger into"(putting finger into top of the glass). Or you may give her an easier sorting task until you figure out what the problem is.

The underlying problem may be with language as well as cognitive skills. Libby Kumin (2003) explains that as children learn language, they may use ***underextension*** and ***overextension:***

"Underextension occurs when a child uses a word such as ***jacket*** only to describe the specific jacket in her experience. So, if there is a picture of a blue jacket, and her jacket is red, she will not use the word ***jacket*** for the blue jacket....

"Often, when children are learning new words and concepts, they use ***overextension*** or ***overgeneralization***.... For instance, a child may learn that a dog has four legs and use the word "dog" for any animal with four legs."

So, for instance, if your child calls pretzel sticks and chips "snacks," you may need to teach her the specific words for each type of snack to facilitate more distinction in her classification, but she still can use the word ***snacks*** for whatever she eats when she gets home from school. You could tell her that ***snacks*** is the big word that takes in all different kinds of great foods.

COIN ACTIVITY

OBJECTIVE: The child will be able to sort coins functionally by color, size, weight, or appearance.

MATERIALS:

- Loose change (pennies, nickels, dimes, and quarters)
- 4 clear bottles or containers

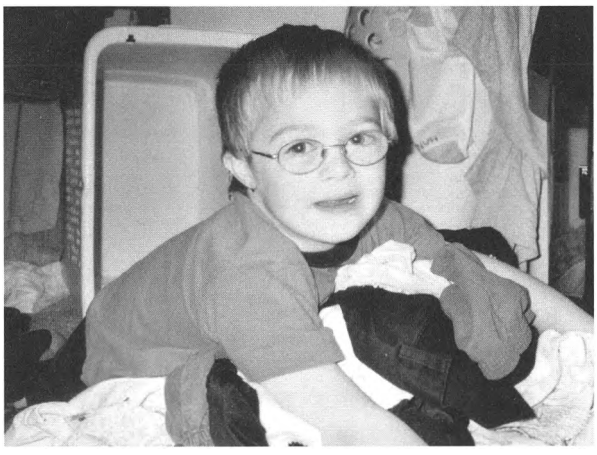
SUCCESS STEP: Have your child hand you a penny from two types of coins on a table in front of her. Praise correct choice or help her to make the correct choice.

> ***Note:*** *if you have a student who is ready for more advanced coin sorting activities or is ready to learn the names and values of coins, see Chapter 15.*

PROCEDURE:

1. Once your child can successfully sort three colors and the flatware, you can try this very functional activity. Many homes have a big jar or drawer for small change. Sometimes that change will have to be sorted to put in wrappers (especially pennies). Bring out a clear glass or jar for each type of coin you have accumulated. Have the child put the pennies in one jar, the nickels in another jar, etc. She does *not* have to know the values of each coin.
2. Schools may find a time when change has to be sorted from a food sale, a field trip, or other money activity.
3. After your child has sorted the change, show her how you put the coins in wrappers to take to the bank, so she knows the function of her coin sorting task, but don't require her to count the coins. As you are wrapping the coins, show her any that may have ended up in the wrong jar and point out why they are not the same as the others in the jar. ("Whoops. This one is too little and thin to go in this jar. It's not the same as these big silver coins.") If possible, take her to the bank when you exchange the wrapped coins for dollar bills.

GENERALIZATION INTO DAILY ACTIVITIES: There are many ways you can work on sorting during everyday activities, but it will take some planning on your part. Everyday activities usually call for more complex sorting than the simple tasks described in this chapter. Sometimes you will find it difficult to explain the differences between common categories, such as jackets vs. coats. It is also possible for children to make different categories out of the same group of objects than you would, depending on the characteristics they choose. For example, fruits can be classified by similar colors, types of fruit (apples vs. oranges), or similar sizes or weights.



Daily experiences both at home and school present numerous opportunities for sorting and classifying. The most common characteristics that are used for classification are:

1. color (white or red, etc.)
2. size (big and little)

3. shape (round or square)
4. weight (heavy or light)
5. appearance (light or dark, bumpy or smooth)
6. temperature (hot or cold)
7. sound (loud or soft)
8. location (up or down) (Smith, 1997)

Lanell's family used a laundry sorter that had three pockets. Lanell's mother told her to sort a pile of dirty clothes into white clothes, colored clothes, and dark-colored clothes. About ten minutes later, Lanell called for her mother in frustration. "What do I do with the clothes that have both white and colors in them? Is red a dark color? Where do I put the pink nightgown?"

Her mother had not established the rules for sorting or classifying in enough detail for Lanell to be able to sort the clothes easily. Her mother might simplify the task by having her sort the clothes in two piles of white or non-white. She could be asked to chant the rule, "White or not white." Her mother might also simplify the task by saying, "White or mostly white clothes go in the first pocket. All other colors of clothes go in the second packet except blue jeans and black clothes." Again the rule could be chanted. "White here, colored there, except for blue jeans and black."

To master classification skills, your child will require much repetition and variation in the materials used. Use of mass-market workbooks or early childhood materials will probably be needed. There are also apps and computer games for sorting and classification (and other math concepts) that even older children like to play. Only the adult working with the child can decide when she has worked long enough on classification.

Learning about matching, sorting, and comparing should be fun for both student and teacher. For young children, you can just incorporate learning into daily life. Here are a few ideas:

- If you're playing together with blocks, ask your child to give you all the *big* ones so you can make a really *big* garage.
- If you are decorating cookies, ask her to help you find a bunch of *yellow* M&Ms to put on Dad's cookie.
- If you are folding laundry, ask her to find all the *socks* in the basket of clean clothes for you (and if she can, put all of Dad's socks in one pile, her sister's in another, and hers in another).
- If you're doing a jigsaw puzzle together, search for all the blue sky pieces together or all the brown pieces that are part of the dog.

For older children or students at school:

- Organize the pantry together. Put all the canned goods on one shelf, the cereal on another, etc.

- Ask her for help organizing the supply closet at school. Put the scissors in one container, the markers in another, etc.
- Separate glass, metal, and paper items to be recycled.

This section on sorting and classifying is only intended to scratch the surface of classifying experiences. Eventually, your children(s) will need more experiences in classifying by two or more characteristics and in making more difficult distinctions such as white and nonwhite.

(Note: I have referred to ***learners*** *as* ***children*** *in this chapter. Starting with Chapter 6, I will use the term* ***student*** *to clarify that activities are appropriate for older students as well as children.)*

CHAPTER 6

Number Sense

Pre- /Post Checklist

Can the student:

1. _____ Rote count the numerals from 1–10 in correct sequence. Later from 1–30. (He does not have the items to count.)

2. _____ Say the correct numeral for the corresponding item (1–5) in one-to-one correspondence.

3. _____ Count with meaning up to 10 items.

4. _____ Give the last numeral counted when asked for the total number of counted items (cardinality).

5. _____ Use the word *zero* to show that nothing is there in that place.

6. _____ Count from 11–19 objects, using the correct numeral names.

7. _____ Demonstrate knowledge of the pattern for counting larger numbers by counting from 20–29 correctly.

General number sense is defined as the understanding of number concepts and relationships, the ability to compute accurately, and the ability to effectively use numbers in daily life (Reys, 1995). Number sense is not something that a person either has or does not have. Development of number sense is a lifelong process The National Council of Teachers of Mathematics has emphasized the importance of development of number sense, especially in elementary school.

Essential for the development of math understanding are the following:

- **Counting/Early number concepts**
- **Rote counting the correct sequence**
- **Early math principles/beginning counting with meaning (1–5)**
  - Conservation of number
  - One-to-one correspondence
  - Last number counted is the total (cardinality)
  - The value of *zero*
- **Counting larger numbers**
- **Patterns in Counting**
  - Teen numbers
  - 20 and beyond

Katri counts rapidly, "1, 2, 3, 4, 5" up to 10. However, when asked to count out 10 chocolate candies, she pulls out only 5 candies as she counts to the numeral 10.

Counting/Early Number Concepts

Most children learn how to rote count (say the names of the numerals in sequence) from people in their environment. Family and peers usually count objects for them starting at a very early age. At first we will expect the child to count to 3, then 5, later 10, then to 20 and 30, and up to 100. (It may take some time—months or years—for students who are concrete learners to learn the names of the numbers, so don't get discouraged if this skill does not come quickly.) In addition to the names of the numerals, students need to learn the *patterns* to follow when counting the teen numbers and the numbers from 21 to 100. Although we cover these skills in this chapter, if your student is not ready for them yet, come back to this chapter later and try working on other skills in other chapters (such as recognizing written numerals) for now.

ROTE COUNTING (NAMING THE NUMERALS)

OBJECTIVE: The student will rote count to 10 (say the numerals from 1 to 10 in correct sequence, but without connecting the number words to actual quantities). Later, he will say the numerals from 1 to 30. (He does not have to have items to count.)

MATERIALS:

- For younger children, counting books that go up to 10 or 20 (see Resources)
- Short songs or cheers that feature counting

SUCCESS STEP: Have the student touch and count his own hands and feet (counting 1, 2) and then praise him. (You say, "I am going to count your hands— 1…2. Can you count your hands for me?") Just do body parts that he can see directly.

PROCEDURE:

1. Teach a song or rhyme with counting words to 10 (Children under 10 or 11).

For example,	1 - 2 - 3 - 4 - 5	*Put up fingers in order.*
	I caught a bee alive,	*Put pointing finger inside a hive made of the other fist.*
	6 - 7 - 8 - 9 - 10	*Put up fingers as named.*
	I let it go again.	*Open both hands; open again fast.*
	OW!	*Act as if your hand was stung.*

Another variation on the above song is used on the *Barney* show as:

1, 2, 3, 4, 5
Once I caught a fish alive.

6, 7, 8, 9, 10
Then I let it go again.

2. Sing the song that used to be called *10 Little Indians* (traditional), with an animal replacing the Indian. For example:

One little, two little, three little elephants,
Four little, five little, six little elephants,
Seven little, eight little, nine little elephants,
Ten little elephant babies.

ADAPTATION FOR OLDER LEARNERS:

Use a football cheer instead of a counting song.

1, 2, 3, 4	Who are we rooting for?
5, 6, 7, 8	Who do we appreciate?
(Name of favorite team)	
9, 10	Do it over again.

Note: *If the student is not speaking at this point in the instruction, you might go on to recognition of numbers (Chapter7), so that he can point to the numerals or indicate them on an augmentative communication device. Or you may teach the signs for the numerals so that he can use gestures to indicate numbers in the songs.*

GENERALIZATION INTO DAILY ACTIVITIES: The purpose of the above activities is more for familiarizing students with the names and sequences of the numbers from 1 to 10. They still need to be able to say the numbers as a memorized sequence, not just in a song. For this reason, it is especially important to help your student(s) generalize rote counting into daily activities.

Children do much of their learning of rote counting during regular daily living. Most children need to hear their parents and teachers count out loud many, many times before they begin to memorize the sequence of numbers. Don't expect a student to memorize and be able to repeat the whole sequence from 1 to 10 in one fell swoop. He may need to practice counting from 1 to 3 for quite a while before he is ready to try counting to 5. Here are some ways you can naturally expose students to counting throughout the day, and encourage them to rote count themselves when they are ready:

- Throw a ball in the air and see how far you or the student can count before it hits the ground.
- Play Hide and Seek and have the student count to ten (or as high as he can) while others hide.
- You count to 10 when transitioning to a new activity. For example, "I am starting with number 1. When I get to 10, it is time for you to go to speech class (or to get ready for your bath)."
- Some teachers use a rhythmic chant to get their students' attention: "One, two, three...eyes on me!" (clapping on every syllable). Students are expected to reply, "One, two...eyes on you!" (also clapping on every syllable).

Early Math Learning Principles

As you are teaching the names of the numbers in rote counting, you should also be teaching counting with meaning for the numbers from 1–5. Certain principles underlie the process of counting with meaning. Your child does not have to be able to state those principles, but he needs to show understanding of conservation of number, one-to-one correspondence, cardinality, and the meaning of zero. Although I have listed these principles in a sequence, your child might learn them in a different order or learn them almost at the same time. He also may use them inconsistently at first. So we start with counting with meaning for the smaller numbers of 1–5 as we give experiences with these principles of early math learning.

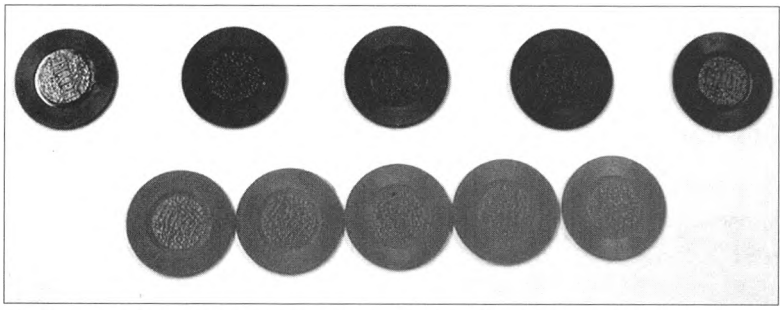
Conservation of Number

Jean Piaget described the phenomenon of the conservation of number as the realization that a given number of items does not vary, no matter how items are arranged (Thornton, Tucker, Dossey & Bazik, 1983). For example, five red blocks can be arranged in a straight line exactly across from five blue blocks, and the child will say the number of red and blue blocks is the same. If, however, the red blocks are arranged in a much longer line than the blue blocks, a child who has not yet mastered conservation of number might say that there are more red blocks than blue blocks. Typically developing children usually learn this concept by age 4 or 5.

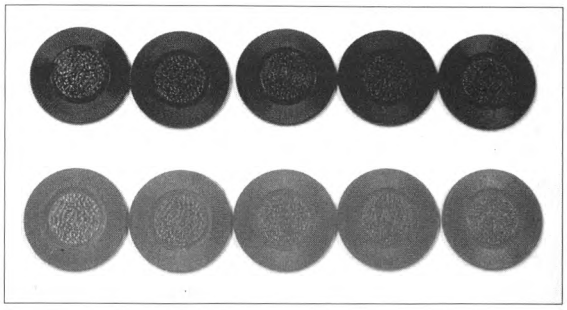
When a child thinks that a number (or substance) can change without anything being added or subtracted, he effectively believes in "magic," not the orderly structure necessary for true understanding of mathematics. Children seem to learn conservation of number slowly, and may sometimes seem as if they understand it one day, but not the next. Continue working on prenumber and number concepts with your student while he is acquiring conservation of number.

INTRODUCING THE CONCEPT OF CONSERVATION

OBJECTIVE: The student will demonstrate by comparison that the amount of items stays the same, even if they are arranged differently.



Children who don't understand conservation will say that there are more chips in the top row as lined up here...



...than here.

MATERIALS:

- 5 blue poker chips or other identical items
- 5 red poker chips or items that are a different color than the above items

SUCCESS STEP: Ask the student to touch a blue chip (making sure he knows the color blue). Praise the correct answer or model touching the chip. Then ask him to touch another blue chip.

PROCEDURE:

1. Put 5 red chips in a straight line. Place the 5 blue chips across from the red chips. Ask the child if there are more red chips than blue chips. (They have the same number.) Move both sets of chips closer so that the red chips are lined up exactly even with the blue chips to show that there are the same number of each color.
2. Then spread the blue chips much farther apart than the red chips and ask the child if there are more blue chips now. (No, they are still the same.) If necessary, show him by lining up the chips across from each other.
3. Spread out the red chips and repeat the question.
4. If necessary, use different objects and repeat, pointing out the conservation of number. A good class demonstration would be to line up students in two lines, facing each other, with the same number of students (4 or 5) in each line. Then spread out one of the lines and ask which line has the most people. Then have each person in the first line find a partner in the second line to show that the number is the same.

Note: *Jean Piaget felt that children develop conservation of number as they mature, but other researchers believe that exposure to guided experiences with conservation can speed up the process. If your student(s) can't seem to get this concept right now, go ahead and teach some of the other early math principles such as counting with meaning for small numbers. Return to the activity later.*

One-to-One Correspondence/Beginning Counting

An extension of the principle of conservation of number is the principle of one-to-one correspondence in which one counting word is paired with each item. For example, in counting five apples, we say one number word per apple. A child who does not understand one-to-one correspondence may throw the dice when playing a

board game and say, "I get to go four spaces." However, as he counts, "1, 2, 3, 4," he skips over 6 spaces on the board. He doesn't understand one-to-one correspondence. Although other children may say that he is cheating, he is just not able to match the counting words with each item.

INTRODUCING THE CONCEPT OF ONE-TO-ONE CORRESPONDENCE

OBJECTIVE: The student will be able to say the correct numeral for the corresponding item in one-to-one correspondence for up to 5 items.

MATERIALS:

- Poker chips or other counters
- Items that need to be distributed such as cups, pencils, napkins, etc.

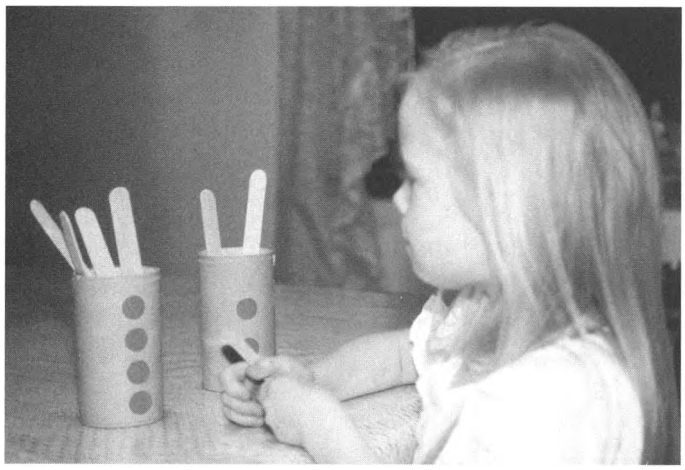
SUCCESS STEP: Have the student hand out cups, pencils, or plates, giving one item to each person present. Say, "Please give a cup to everybody." Tell the student that he has been fair. He has given an item to each person present (one-to-one correspondence without counting). If the student is not successful, model the task yourself and then have him repeat it.

PROCEDURE:

1. Have the student hand out some other items (napkins, etc.) one to each person present. Praise his fairness in giving one napkin to each person. (If you are working with your child alone at home, he can pass out items to several stuffed animals or dolls.)
2. Then go on to one-to-one correspondence that includes simple counting (1–5). Count out 5 blue chips, 1, 2, 3, 4, 5, and put them on the table one by one in front of the student as you count them. Give the chips to the student and ask him to do the same thing. If he is not able to count to 5, repeat the activity with 3 chips. He will be saying a numeral and putting the chip on the table in one-to-one correspondence as well as counting with meaning for small numbers.
3. If the student is not successful, draw 3 circles on a piece of paper and tell him to put a red chip on each circle. Have him repeat the task as you count out the numbers in one-to-one correspondence. See if he can do the activity independently.
4. Count other objects (1–5) in the same manner. Do not count higher even if the child knows the names of higher numbers. The emphasis should be on the one-to-one rule while the student is doing a simple counting task with meaning.

MATCH MY DOTS

OBJECTIVE: Given empty cans labeled with 1 to 5 dots, the student will match the correct number of sticks with the dots. (You can also use the game for counting with meaning for small numbers if you have the student count the dots on the cans and also count the sticks as he puts them in the cans.)



MATERIALS:

- Empty soup cans or frozen juice containers (5–9). (They can be painted or wrapped in construction or Contact paper.)
- Round adhesive dots or permanent marker for making dots on the cans
- Tongue depressors or craft sticks (15+)

PROCEDURE:

1. Label one can with 1 dot, another with 2 dots, another with 3 dots, etc.
2. Challenge the student to put one stick in for each dot on the cans. Check when he is done.
3. If he seems secure with the matching, you can have him count the dots and the sticks to check his answer. If you are working with several students, let them take turns filling the cans.

GENERALIZATION INTO DAILY ACTIVITIES: Many daily living tasks can be used for generalizing one-to-one correspondence:

- When making muffins or cupcakes, have the student put the liners in each cup. You may have to separate the liners for him if he has fine motor difficulties.
- Have the student pass out construction paper in school, straws in the fast food place, or rolls at mealtime. Any occasion where one item has to be given out per person will be good practice because the person who does not get the item will immediately let the student know that he or she does not have one.

Count with Meaning for Amounts Up to 10

The student has already counted small amounts (1 to 5) with meaning when he was learning one-to-one correspondence. When he can rote count to a number higher than 5, you should progress to counting with meaning to that number. This activity is designed to make counting with meaning automatic for numbers through 10 through repeated practice.

INTRODUCING THE CONCEPT OF COUNTING WITH MEANING

OBJECTIVE: The student will count up to 10 items showing one-to-one correspondence.

MATERIALS:

- Varied items such as blocks, pencils, etc. that can be counted, especially objects that the student especially likes or collects
- Container for items

PROCEDURE:

1. Have the student move and count some items and put them in a container. The student says "One," picks up one pencil, and puts it into the container. It sometimes

helps if he can hear the object hit the bottom of the container. Then he continues saying the numerals as he picks up other pencils and puts them in the container. By dropping the object into the container, he is less likely to count it twice. (If the student drops more than one item into the container at a time, try handing him one item at a time to count.)

2. Repeat with varied motivating items around home and school. For example, if your student is interested in Pokemon DVDs, have him count them as he puts them back in the storage cabinet. Any item that he collects or has great interest in will help motivate him to count. My son collects Power Rangers, which he faithfully counts. Likewise, I worked with a student with an autism spectrum disorder who would count the cars on the trains that he loved.

Last Number Counted Is the Total (Cardinality)

A student must be able to see that the last number he says when counting items tells how many items there are. A child can count six forks, one by one, as he has seen adults do, and still not understand that his counting answers the question "how many?" and determines the total number of items. This concept (known as the principle of cardinality) tends to be learned all at once in sort of an "ah ha" moment. Usually, the child can then generalize it easily.

COVER UP GAME

OBJECTIVE: The student will give the last numeral counted when asked for the total number of counted items.

MATERIALS:

- Poker chips or other interesting items to count
- Handkerchief

SUCCESS STEP: Ask the student to name the colors of the chips (if he knows them). Praise the correct answer.

PROCEDURE:

1. Put 3 blue chips down in front of the student, counting 1, 2, 3. Say, "You now have 3 chips in front of you. How many chips do you have?" (3)
2. Then count out 5 chips in front of him, counting 1, 2, 3, 4, 5. Ask, "How many chips do you have now?" See if he understands that the last number counted tells how many chips there are.
3. If he is not successful, try using the numbers from 1 to 3.
4. Play the game *Cover Up* by covering different numbers of chips with the handkerchief after you have counted them. Then ask, "How many chips are hidden under the handkerchief?" After the correct answer is given, say, "You peeked."

WHAT'S IN MY BAG? GAME

MATERIALS:

- Poker chips or other small items to count
- Small paper bag (lunch bag size)

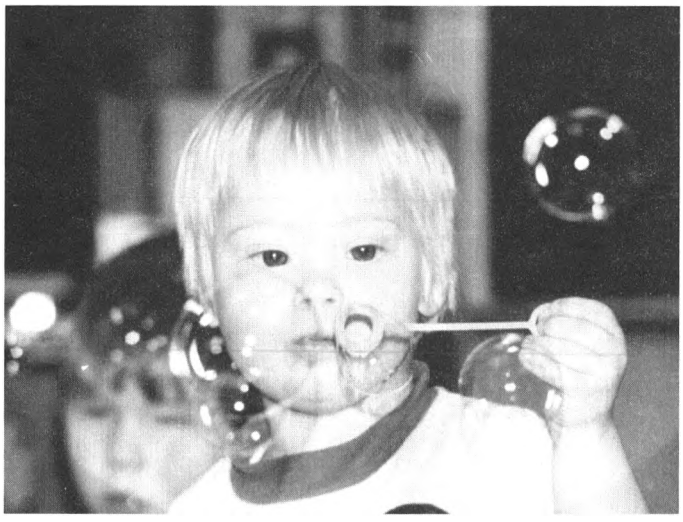
PROCEDURE:

1. Count chips or other motivating items in front of the student and drop them in the bag, one by one.
2. Hold the bag closed and say, "Do you know how many chips I have in this bag?" When the student answers with the last number counted, say, "You got me! You saw through the bag."
3. Then take the chips out and count them to be sure. Repeat with different numbers of chips. Each time the student says the right number, he is a winner...and you lose. If the student doesn't understand how to play the game, try again with only 1 or 2 chips.

ADAPTATIONS FOR OLDER LEARNERS: Don't use the counters that are used with young children (e.g., teddy bear counters). Use items that are used in daily living such as spoons, poker chips, keys, or coins. Instead of making games out of Cover Up and What's in the Bag, do the same activities and say firmly, "You are right!" each time the answer is correct.

Kyle correctly pairs a numeral with each of the six balls that he drops one by one into a bucket. However, when asked how many balls are now in the bucket, he stammers and guesses that there are four balls. Although he knew how to count each ball, he did not know that the last number he said would give the total number of balls in the bucket.

GENERALIZATION INTO DAILY ACTIVITIES: Once your student begins to get the hang of counting with meaning, it is important to weave this skill into his daily routine. At first have him count numbers from 1 to 9. As he learns the counting words for the teens, have him count items to 20 and higher.



- Count:
  - Plates on the dinner table
  - Dirty clothes that go in the hamper
  - Produce into a bag in the grocery store
  - Keys on a key ring
  - Soap bubbles you or your child blows
  - Small toys like Legos or cars as you tidy up
- Make a lot of paper airplanes and see how many the student can land in a drawn circle or beyond a certain line. Count them again as they are picked up.

- Have the student guess (estimate) how many Goldfish crackers, small blocks, or clothespins he can pick up and put in one hand. Then count the objects to see how close the estimate was.
- Count the number of ball tosses when playing catch. Begin again when someone misses.
- Play the *Simon Says* game and have the student take 3 steps forward, move back 2 steps, and other movements that involve small numbers.

Understanding Zero

The concept of *zero* is a basic part of our system of numbering. Remember, the Romans didn't have *zero*, and we are still struggling with quick recognition of Roman numerals today. *Zero* is most commonly used to express that *nothing* is there. In addition, the *zero* is essential as a placeholder for our base ten system. When we add 1 item to 9 items, we place a zero in the ones column and move to the *tens* column and write 1. The *zero* is used to show there are no 1s in the ones column, but one 10 in the *tens* column.

INTRODUCING THE CONCEPT OF ZERO

OBJECTIVE: The student will be able to use the word zero to show that nothing is there (at that place).

MATERIALS:

- Small items of food such as jellybeans, pretzels, gummy bears, or strawberries

OR

- Nonfood items such as Legos or Matchbox cars

SUCCESS STEP: Ask the student to name some of the items you are going to count. Praise correct answers.

PROCEDURE:

1. Give the student two pieces of food such as jellybeans or pretzels. Tell him that he has two items. Then take one item.
2. Ask him how many he has left.
3. Have him eat the last piece of food and ask, "How many do you have now?" When he answers that he has none, tell him that the name for none is *zero* (and show how the zero looks written).
4. If you want to use nonfood items, make sure that the items you choose have some interest value for the student. Use the same procedures as above—except take the last item away from him instead of having him eat it! (If you are working with a student with ASD, however, it may be better to use low-interest items if he gets upsets when you take away a favored item.)

GENERALIZATION INTO DAILY ACTIVITIES: Continue talking about zero with many different types of items.

- Make zero part of your daily counting daily activities by asking questions such as, "How many shoes do I have on? That's right. I have no shoes on. I have *zero* shoes on."
- If the student is young, you may want to say, "Zero...Mr. Zero...sad Mr. Zero...he has nothing." Make a zero with your thumb and forefinger and say, "Zero looks like this."
- If the student is interested in sports, point out when one team has zero points because they have not scored.

Counting Larger Numbers

Learning to count the teen numbers can be very difficult for students. There is a pattern from 14 to 19 involving saying the *ones* first and then the word "teen" (standing for 10). However, this pattern is just opposite from the pattern used in 21 and beyond, naming the *tens* first and then the *ones*. Some students have difficulties writing down the teen numbers because they hear the digit in the ones place (such as four) before they hear the indicator for the *tens* (teen). Thus, some students write 41 for 14. In addition, 11, 12, and 13 do not fit any pattern. (I have found that most young students do not hear the sound "thir-" in thirteen as indicating 3.) If the student has great difficulty with the teens, move on to recognition of numbers (Chapter 7) so the written numbers can reinforce the student's memorization of number names.

PATTERN FOR TEEN NUMBERS

OBJECTIVE: The student will first be able to count from 1 to 13, and then from 14 to 19 objects, using the correct numeral names for the teen numbers.

MATERIALS:

- Counters, pennies, or poker chips
- Egg carton, muffin pan
- Pegboard and 13 pegs (optional)

PROCEDURE:

1. Teach your student(s) the numerals 11, 12, and 13 as separate number names like 6 or 7. Line up 11 people or objects and have the student count them. When the student gets to the last object or person, say, "Whoops! What do we do with this one? We'll call it eleven. Eleven, eleven, eleven—that's a good name. Let's chant it—eleven, eleven, eleven." Set up a rhythmic chant.
2. When the student is successful at counting to 11, move on to introducing the numbers 12 and 13, as above.
3. Use some counting structures that require 12 items to be complete. Have the student put pennies, eggs, and other small items in egg cartons, one item per compartment, naming each one. Use muffin tins for the same purpose. Alternately, cover some of

the holes on a pegboard with tape so there are only 12 or 13 holes, and have the student put 12 or 13 pegs in the pegboard and count them.

4. Counting from 14 to 19 can be quite difficult, for the reasons discussed above. Once your student can reliably count to 13, explain briefly the pattern in 14, 15, 16, 17, 18, and 19. If the student knows 4, 5, 6, 7, 8, and 9, suggest that he can just make them *teenagers* by saying the word *teen* after those numbers.

GENERALIZATION INTO DAILY ACTIVITIES:

- Count to one of the *teen* numbers when you are warning your child that a play period is almost over or that it's dinner time. Give him plenty of chances to hear you count to the teen numbers.
- Talk about *teenagers* you know and give their ages frequently.
- If he has a collection of toys, CDs, or other items, have him count them, counting into the teens as far as he can.
- Practice counting with the teen numbers frequently with packs of soda pop, other groceries, etc.

Pattern for Numbers 20 and Larger

Learning to count to 20 may take a long time for students who are concrete thinkers. Grasping the pattern for counting and putting in the sheer work of memorization may be very difficult for them. You may find that if you go on and teach recognition of numbers (Chapter 7), the visualization will help them to learn the numerals and patterns. If the student has few fine motor difficulties, writing the numbers (Chapter 7) may give tactile feedback that will assist in memorization.

INTRODUCING THE CONCEPT

OBJECTIVE: The student will demonstrate knowledge of the pattern for counting larger numbers by counting from 20 to 29 correctly.

MATERIALS:

- 40 craft sticks or short pretzels
- paper plates

SUCCESS STEP: Ask the student to count to 19 as you put down one craft stick (or pretzel) for each number that he counts. Praise him.

PROCEDURE:

1. Say, "I am going to count some more: 20, 21, 22, 23, 24, 25, 26, 27, 28, 29."
   - Put the craft sticks or pretzels on paper plates, 10 to a plate.
   - "Let's do that again. Twenty...what? Right, 21."
   - Continue to ask him to supply the number following 20 all the way up to 29. You can help him by saying, "What number comes after 2? That's right, 3. We have 23."

2. You may write the numbers as you say them, but do not expect the student to recognize them until you have taught Chapter 7, Recognition of Numerals.
3. When you have thoroughly taught 20 to 29 by rote and with objects, see if the student has figured out the pattern. If he has, you may quickly go on to 30. Eventually you will want to have him count to 100 with meaning, but in my experience, it is easy for students to be overwhelmed by so many digits. If you have taught the pattern, you can continue to larger numbers at a later date.

COMMERCIAL GAMES TO BUY OR TRY

As your student is introduced to the various skills covered in this chapter, there are a variety of ways you can use commercially available games to make learning fun.

As students begin to understand one-to-one correspondence and counting with meaning, they can play simple board games where players have to move their marker one space for each dot on the die thrown. Good examples include the following games:

- *Chutes and Ladders* (Milton Bradley),
- *Trouble* (Milton Bradley),
- *Clue, Jr.* (Parker Bro.),
- *Snakes and Ladders* (Fundex),
- *Monopoly Jr.* (Hasbro), or
- *Hi! Ho! Cherry O* (Parker Brothers) for younger students

CHAPTER 7

Recognizing and Writing Numerals

Pre- /Post Checklist

Can the student:

1. _____ Match the written numerals 1 to 5 with another set of numerals 1 to 5 (later, 1 to 10).

2. _____ Identify written numerals with appropriate groups of items (1 to 10).

3. _____ Recognize written numerals from 11 to 19.

4. _____ Recognize written numbers from 20 to 29 and apply the pattern to the numbers 30 to 39.

5. _____ Recognize written numbers 39 to 100.

6. _____ Enter single and double digits correctly on the calculator.

7. _____ Draw straight lines and circles (or semicircles).

8. _____ Write the numerals 1 to 5 and 7 on a 4" x 6" card starting in the upper left corner.

9. _____ Write the numerals (6, 8, 9) that do not use the above pattern. (Students with fine motor problems can choose from numeral cards that are already written correctly.)

10. _____ Demonstrate understanding of number problems by choosing correct answers from an array of already written numerals.

Students who have learned the concepts in Chapter 6, Number Sense, will already understand how to count with meaning from 1 to 20 or more. This chapter will help students to relate the printed symbol of a numeral to the number of items that it represents.

For some students, it may be important to teach recognition of numerals along with number sense. If the student does not have understandable spoken speech, teaching recognition of numerals would allow her to point to written numerals to show understanding of number concepts. In addition, seeing the written numerals can add visual cues to the learning of number concepts.

For a student with good small muscle control, writing the numerals may add tactile and visual cues to aid long-term memory. Research, however, has documented that typical first graders frequently have difficulties with writing numbers, probably because of their immature fine motor control and their limited eye-hand coordination (Payne and Huinker, 1993, as quoted in Rey, 1994). Many children with Down syndrome or low muscle tone have additional difficulties with fine motor skills (Bruni, 2016). Therefore, they should have frequent experiences with numbers and recognition of numerals before being required to write them.

This chapter covers the following important numeral recognition concepts:

- matching numerals;
- identifying written numerals with groups of items (1–10);
- recognizing the written numbers from 11–19 (teen numbers);
- recognizing the written numbers from 20–39 and their pattern;
- recognizing the written numbers 39–100;
- entering single and double digits correctly on the calculator.

Matching Numerals

Before I expect a student to learn the names of numerals, I teach her to match numerals by shape. The student must attend to the important features of the numeral in order to match it with the same numeral that is a different size. Matching also gives the student a successful experience before more difficult tasks are given.

> ***Note:*** *I suggest writing numerals in the style recommended by Jan Olsen in the* ***Handwriting without Tears*** *method. That is, make a 1 with a simple straight line and an open 4, as well as a 6 in which the back is not curved. Using this method, you can start writing most of the numerals in the upper left-hand corner of a box, which makes it easier for many students to learn how to write the numerals.*

THE PARKING GAME

OBJECTIVE: The student will be able to match one set of the numerals 1 to 5 to another set of numerals 1 to 5.

MATERIALS:

- Five small cars (e.g., Matchbox) with a large numeral written on the top (1 to 5)

- Five individual milk cartons or other small boxes with tops cut off, making a cube with one open side as a garage. Label each garage with a numeral from 1 to 5

OR

- Paper airplanes labeled 1 to 5
- Five boxes labeled 1 to 5 large enough for the airplanes

SUCCESS STEP: Ask the student to push one of the cars across the table to you or to fly the airplane to you. Continue to take turns and praise her for her successful race or flight.

PROCEDURE:

1. Line up the milk carton garages (with the large numeral on the top) in front of the student. Tell her that the cars need to be put in their own garages. (If you have more than one student, let other students "play" at the same time with their own sets of garages and cars.)
2. Model putting the numeral 1 car in the numeral 1 garage. Pick up the car, point to the numeral on the top and say, "This number is a 1. Let's see if we can find a garage with a 1 on it like the number on the car. Let's find the match."
3. Guide the student to successfully put the car or plane in the appropriate garage. You may name the numeral as you are teaching, but do not expect her to know or repeat it. You are having her match at first.
4. Have the student put all the cars or planes in the correct garages. See how confidently she can put the cars away.
5. The first player to put all the cars in the right garages wins. You can have many winners.

Note: *You may want to give elementary-aged students a few minutes to play with the cars or have a race. Older students may want to make paper airplanes and fly them into labeled boxes.*

Identifying Numerals with Items

The next step in learning about numerals is recognizing what numerals stand for. That is, numerals are ways of symbolizing the numbers the students have previously used to rote count and to answer the question "How many?"

INTRODUCING THE CONCEPT

OBJECTIVE: The student will demonstrate that a number of objects can be symbolized by a numeral by matching up groups of dots (1 to 5) with the numerals 1 to 5. Later, the objective can be expanded to 1 to 10.

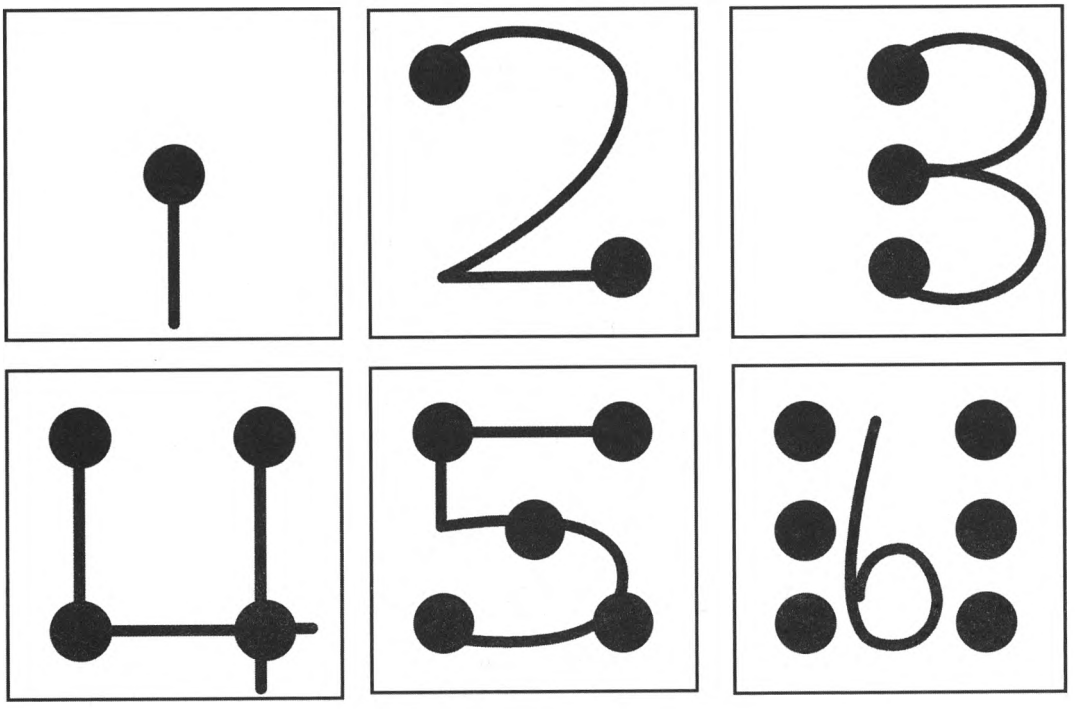
MATERIALS:

- Washable black and red marker pens
- Flashcards with dots 1–5 from Appendix B-2

SUCCESS STEP: Hold up the cards with 1 to 5 dots on them and ask the student how many dots are on each card. Praise correct answers.

PROCEDURE:

1. Using a new set of dot cards 1 to 5 or using the original set, which has been laminated, show the student the one-dot card and write the numeral 1 (in a different color than the dots) right below the dot. The dot should look like a round hat on the top of the 1. Say "One dot...its name is one" as you write the 1 on the card. Have the student repeat orally. Each time you introduce a numeral, point out the dot(s), and then have the student say the numeral and count the dot(s). Have her trace the numeral on the card with her finger, starting at the top.
2. Repeat with the number 2 card. Point out the 2 dots. Write the number 2 starting with the first dot and ending with the last dot, as shown in the illustration. Practice as above.
3. Repeat the above procedure with each of the five cards and have frequent, short practices.



CLIP IT, CLIP IT GAME

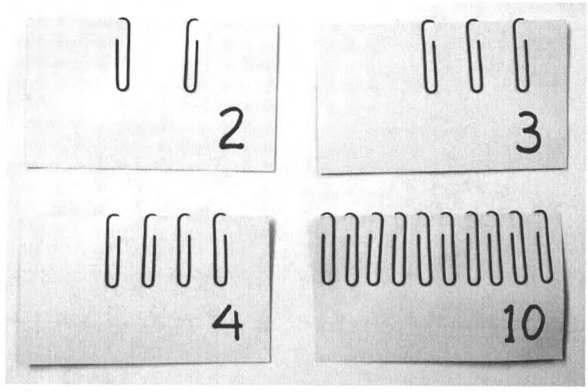
MATERIALS:

- Five 3" x 5" index cards, numbered from 1 to 5 in large, black numerals; prepare one set for each student playing the game. Use larger, thicker cards for students with fine motor difficulties.
- 15 paperclips or clothespins for each player

PROCEDURE:

1. Pick up a card. Model putting on the number of paperclips or clothespins to match the numeral on the card.

2. Let the student practice putting the clips on. If using clothespins (best option for students with fine motor difficulty), you may need to paste two index cards together or make the cards out of sturdier cardboard, as from a shipping box. If the student needs assistance, you or a helper can hold the card steady as the paperclips or clothespins are clipped on. If a student doesn't recognize the numerals, add dots to them, as in the previous activity. Fade (remove) the dots as quickly as possible so the student doesn't become dependent on them.
3. To play as a game, give each student one set of cards numbered from 1 to 5 and 15 paperclips or clothespins to match to the numerals. The student with the most correct cards wins. (They all can win.)
4. When the student does *Clip It, Clip It* confidently, the game can be given as independent work. Later, new larger cards can be made for 1 to 10, and students can compete to finish first accurately. Of course, do not have competitions if one student will be hampered by fine motor difficulties.



Remembering the Numerals

Students need to thoroughly learn to recognize the numerals. Many students need creative practice to transfer the short-term learning of numerals to long-term memory storage. Recognition of numerals needs to be automatic before students can concentrate on more advanced math concepts.

Memory experts have told us that we often remember better when we associate an item with a picture in our mind, especially if the image is funny or unusual. We call this process of using visual or other cues to jog memory *mnemonics.* Mnemonics also includes using chants, songs, and letters to help us remember. For example, some people think of Italy as a boot, or the colors of the rainbow as Roy G. Biv (red, orange, yellow, green, blue, indigo, violet). Many of us need to softly sing the ABC song when we are alphabetizing files. We need to use mnemonics more often with students who have difficulties with short-term and working memory.

If the student has trouble remembering what the numerals look like, use mnemonics where possible. You can make up your own that are meaningful to your particular student(s).

Some students may be able to write the numerals to help them remember how to form the numerals. Writing the numerals may help by involving their senses of movement (kinesthetic) and touch (tactile). You don't even need to have the student use a pencil and paper. She can just trace the numbers on the desktop or practice with chalk on a chalkboard.

Another aid to memory is creative practice. Although concrete learners do need much repetition to recognize numerals, workbooks and flashcards can be boring and ineffective. Use of games and activities can be fun as well as effective. An example of creative practice, Hopscotching the Numbers, follows.

HOPSCOTCHING THE NUMBERS

OBJECTIVE: The student will practice numeral recognition while actively involving her whole body.

MATERIALS:

- Chalk (for outdoor play)
- Masking tape (for indoor play)

PROCEDURE:

1. Use masking tape to make a hopscotch board with 12 squares on the floor inside, or use chalk to make the board outside on the sidewalk or playground. Place the numbers from 1 to 12 in order at first. Later put the numbers randomly on the squares.
2. The child hops on the numerals in order from 1 to 12, saying the numeral name. Since this is not intended to be a motor skill game, the player may just walk from one square to another as she calls out the numeral.
3. If the player calls out or steps on the wrong number, another person takes a turn, or if the player is alone, she starts over from the beginning.

ADAPTATION FOR OLDER LEARNERS: Instead of Hopscotching the Numbers, older students can use the floor mat of a Twister game (Milton Bradley) and tape numerals to the various colored circles. You will need to put the same numerals on the spinner or just use dice instead. You can use simplified rules or invent your own, depending on the age or skill of the players.

COMMERCIAL GAMES TO BUY OR TRY

Once the student can recognize numbers from 1 to 10, she can try many regular children's games such as the following:

- *Go Fish* (various makers)
- Standard 52 playing cards)
- *Uno* or *Winnie the Pooh Uno* for younger students (Mattel)
- *Monopoly Junior* (Parker Brothers)
- *Skip Bo* (Mattel)
- *Crazy Eights* (various makers or standard 52 cards)

Many of these games are also available as apps or online computer games.

For older learners, find a board or card game that is appropriate for their age that requires recognition of numerals. Examples include Uno, Sorry, or Racko. Then simplify the rules, if necessary, so they can play it. Use Post-It notes on the board if you need to make changes. For instance, cover any squares on the game board that have directions that are too complex or that prolong the game too much. Since games for older students can be very long, I often just say, "This game takes too long. We are going to make changes so we can play it in twenty minutes. These rules are classroom (family) rules."

Recognizing Numerals from 11 to 19 (Teen Numbers)

As discussed in the previous chapter, the *teen* words often confuse children who are learning about numerals. They hear the *ones* place first, so they may think the numeral starts with the number they hear in the *ones* place. For example, they may think that 19 is written as 91. Teach the numbers from 13 to 19 individually. Most students will find learning the pattern for 20 and beyond much easier than the teens.

1 TO 19 BINGO

MATERIALS:

- Bingo cards from Appendix B-3
- Metal-trimmed Bingo chips and magnetic wand (See Resources)

PROCEDURE:

1. Photocopy the Bingo cards from Appendix B-3 that only have the numerals from 1 to 19. Laminate the cards if you anticipate using them frequently.
2. Review the teen numbers from 13–19 with the student. I tell the students that the rowdy teenagers want to do something different from the other numerals, so they say their names backwards.
3. Randomly call out the letter above a column and then a numeral that fits in that column—e.g., M – 14. Have students cover up matching numerals on their game boards. Some numerals may be printed twice on a board, and the students may cover both numerals if they recognize that there are two. After the turn is over, they can't go back and cover the second numeral.
4. After each turn, check that the students have covered the correct numeral. If students have difficulty recognizing the numerals given orally, write the numbers you call out on a piece of paper or a whiteboard so the students have a matching cue. However, the point of the game is to make sure that they recognize the spoken names of numbers, so you must fade that assist.
5. The winner is the first one to fill in a complete row or a complete column. (You can discreetly look at the students' cards and call a number that will enable a frustrated student to win.)
6. The winner gets to pick up her chips with the magnetic wand first. Then, the others may do the same.

Recognizing Numbers from 20 to 39

Learning the pattern for writing the numbers twenty and above is usually easier for students than learning to write the teen numbers. The pattern rule for naming the twenties and beyond is consistent and names the digits in the order that most people say them (42 = forty-two). However, if you have thoroughly taught a student the teen number pattern, it may interfere with applying the pattern for naming the twenties and

beyond. If the student mistakenly applies the teen pattern to naming larger numbers, stop and explain both patterns both orally and visually.

MATH BINGO GAME PLUS

MATERIALS:

- MATH Bingo Plus cards (from 10 to 30) from Appendix B-4
- Bingo chips and magnetic wand (see Resources)

PROCEDURE:

1. Play MATH Bingo Plus as described above in *1 to 19 Bingo,* except use the cards with numerals from 10 to 30 (later use the cards up to 99). If you detect that the student is having trouble with some groups of numerals, call these numbers more often. Watch the "teen" numbers closely.
2. Bingo may be played frequently for fun without the students knowing that they are getting good math practice. You can change the numbers on the cards and modify the rules so they do not get bored with the cards provided.

GENERALIZATION INTO DAILY ACTIVITIES: Use everyday life experiences to teach numeral recognition and give practice. For example, have the student do the following:

- Push the numbers on the elevator so they light up.
- Choose TV channels with the remote.
- Recognize numbers on the calendar.
- Use a school schedule (room numbers, sequence of activities or classes).
- Identify page numbers when you are sharing a book.
- If appropriate, enter the cooking time on a microwave.
- Punch in her home number, Dad's work number, etc., on a phone.

Recognizing the Numbers on the Calculator

When students can recognize written numbers, they should be introduced to the numbers on the calculator.

INTRODUCING THE CALCULATOR

OBJECTIVE: The student will be able to enter single and double digits correctly on the calculator when numbers are given orally.

MATERIALS:

- Calculator

SUCCESS STEP: Have the student show where the number 1 and the number 9 are on the calculator. Praise correct answer.

PROCEDURE:

1. Call out single-digit numbers and check to see if the students can accurately enter them on their calculators. Have them say the number out loud as they enter it. Next, have them say, "Check" as they look at the window to check that they have entered the correct number. Be sure they know how to clear the window before entering the next number.
2. Slowly call out double-digit numbers and check the students' calculator windows. If necessary, hold up a visual of the number called. Fade the visual number when possible. Some students will always need a visual image to copy. This is okay, but in daily living, people frequently need to enter numbers that they hear given orally.
3. Pair off the students and have one student call out numbers from 1 to 20 while the other student enters the number in the calculator. Then reverse the roles.
4. Practice for 5 to 10 minutes each session until students can enter the digits smoothly and accurately (but not swiftly).

Writing Numerals

As was discussed in previous chapters, young children typically have difficulty writing numerals, even when they can recognize numbers and do simple addition. This is mainly due to the limited development of the muscles necessary to write and to immature eye-hand coordination

Many children with low muscle tone have additional difficulties with fine motor skills (Bruni, 2015). Insisting that they write numbers too early may bring frustration that has little to do with number concepts. This section will discuss different ways to teach the writing of numerals and will also explain ways that students can indicate the correct number without writing. For example, by using numeral cards, a student can show that she knows the answer to the problem even though she can't write it.

The areas covered are:

- drawing lines and circles;
- ways to teach numeral writing;
- cues for remembering;
- alternatives for recording numerals.

DRAWING LINES AND CIRCLES

Objective: To write numerals, the student must have a mental picture of the numeral and then must duplicate this picture with motor actions. Children who are just scribbling back and forth on paper are probably not ready to write numbers. First, they must be able to draw a straight line and stop where desired. They also must be able to draw controlled curved lines. From plain scribbling, children usually progress to making circles, perhaps in representation of a face. When a child has enough control to make the curved line of a circle, she may be ready to try very simple number writing.

MATERIALS:

- Large sheets of unlined paper
- Large crayons or markers
- Music (for encouraging drawing large strokes)

OBJECTIVE: The student will be able to draw straight lines and stop on command and draw controlled circles or semicircles.

SUCCESS STEP: Have the student pick out her favorite crayon or marker.

PROCEDURE:

1. Demonstrate making large circles on chalkboard, whiteboard, or large paper. Play some music or sing a song that has long phrases while saying a chant like "a-round, a-round, a-round" while you are drawing. Have the student do the same. You can try slow movements of classical music such as waltzes by Johann Strauss or children's songs such as "Found a Peanut" or "Ring around the Rosy."
2. Turn off the music. Tell students you are going to play a game. They are going to draw circles and when you yell "STOP," they have to stop drawing and put down their crayon or marker immediately. Practice this activity until they are doing semicircles with control. You might want to try playing or singing "Pop Goes the Weasel" during this activity, having the children stop drawing at the word *pop*.
3. Practice drawing lines and stopping them. For younger students you can tell them to draw railroad tracks and then pretend to have an engine (a line down the middle) race down the tracks until you say "STOP."
4. If the student is unable to control drawing lines and circles, wait a while before teaching numeral writing. You really can't hurry physical maturity very much.

Ways to Teach Numeral Writing

Some children with conditions that affect their motor, sensory, or cognitive skills can learn to write numerals and letters using the same method as the other children in the class use. However, many benefit from slower, more systematic approaches that give them the time and instructional support to both visualize and practice the necessary movements.

There are several programs that teach writing numbers along with the writing of letters. See the Resource Guide for publication/ordering information.

- *Handwriting without Tears* is an excellent program that moves developmentally and emphasizes learning in multisensory ways. The program teaches simplified but completely legible number and letter shapes (without any extra curlicues) and uses paper with fewer guidelines than usual for beginning writers. Jan Olsen, the author, has programs for each grade, and they can also be used by older students who still need help with printing or cursive writing.
- In *Semple Math,* Jan Semple explains how to progress from using the spotting dots (explained earlier in the chapter) to writing the numerals. Her program focuses only on teaching how to write the numerals, not the letters. She also uses the students' fists as cues to help them remem-

ber the direction of writing, and mnemonics to create visual pictures of the numerals and what they mean.

Most parents and teachers teach how to write numerals and letters at the same time. It is possible, however, that a student could learn to write the more limited amount of numerals *before* he tackles the entire alphabet, including words and sentences. The primary principles are the following:

- Make sure students are able to draw straight and curved lines before you begin to teach numeral writing.
- Make your models of numerals large. Then have the students write their numbers no smaller than their fist.
- Give students a starting point for the numerals. Visualize a 4" x 6" index card. For numerals 1 to 5 and 7, the starting point is the upper left-hand corner, 8 and 6 start in the middle, and 9 starts at the upper right-hand corner.
- Give students some way to visualize the numeral and to memorize the sequence of strokes necessary to write the numeral.
- Vary your materials and methods of teaching.
- Gradually decrease the size of the numerals.

EARLY WRITING ACTIVITIES

OBJECTIVE: The student will write the numerals 1 to 9 on a small slate or dry erase board or a 4" x 6" card.

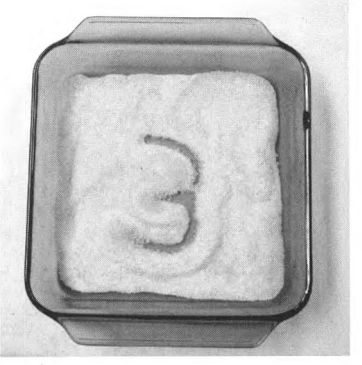
MATERIALS:

- Markers and paper
- Small slate and chalk
- A small wet sponge and a small dry sponge or paper towel. (The slate that is part of the *Handwriting without Tears* program has a wooden border that gives a tactile cue for keeping the numeral inside the slate. If your slate board has no border, use several layers of duct tape to make a tactile frame around the slate.)
- A dry erase board with a frame and markers can be used instead of the slate, chalk, and sponges
- Papers with one numeral per page for the student to trace (Appendix B-5)
- Play sand *or* bright-colored, sugar-sweetened Kool-Aid or Crystal Light drink mix in a 9" x 11" cake pan
- Cutout numerals that have a slightly rough surface (e.g., cut from sandpaper) to trace with the finger
- Cutout numerals for the overhead projector (for small groups)
- Yarn
- Small items such as macaroni, cereal, and popcorn to glue on paper to make numerals

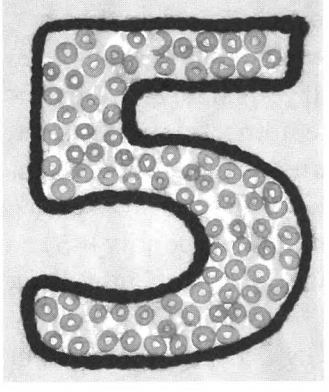
SUCCESS STEP: Have the student make a short vertical line with a marker on paper. Praise her and tell her she has just learned her first numeral—the numeral 1.

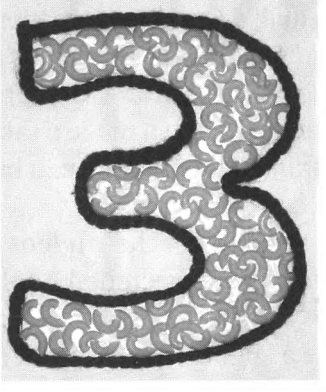
PROCEDURE:

1. Have the student repeat drawing the numeral 1 on paper. See if she can draw straight lines and stop them when told to.
2. Make some curved lines by demonstrating a large curve and two small curves. See if the student can imitate them. If she needs more practice, switch to the chalkboard and have her draw *big* curves and straight lines. Try having her do lines and curves in finger paint if she needs that tactile experience. Later, graduate to an individual slate.
3. Students often need experience with feeling the numerals to help them get a visual picture of their shapes. You may write the numeral on paper, put it on the bottom of a shoebox lid, and cover the numeral with fine play sand. The student then traces the numeral with her finger, pushing aside the sand. The texture of the sand makes the activity both a visual and a tactile (touch) experience.
4. You can try using sandpaper numerals and having the student trace them. My son wouldn't touch the sandpaper letters (due to tactile defensiveness). We found a substitute that worked every time. I spread sugar-sweetened red Kool-Aid on a rectangular cake pan. He had to trace the numeral several times, and if he did it, he could lick his finger. He loved this activity, but the red color stayed on his finger for a while. At first I wrote the numeral on a piece of cardboard and buried it in the Kool-Aid. Very quickly he could write the numeral independently without tracing.



5. If you are teaching a small group, you can take advantage of the overhead projector. Put cutout numerals on the overhead projector. One student can trace the numeral on the chalkboard or screen. The others trace the numeral in the air or on their desk. Students can also clap out the number (4 = clap, clap, clap, clap) to link the meaning with the numeral. Again, multiple senses come into play.





6. Another way to build on the tactile feeling of the numerals is to glue pieces of yarn in the shape of numerals on pieces of cardstock or cardboard. If you make numerals with double lines, the student can glue things such as popcorn, cereal, or macaroni on the outlines and have a variety of textures to feel.
7. Use songs, rhymes, or cues to help students remember how to write the numerals. One poem that has surfaced in many early childhood programs is *Writing Numbers*. I found this version in the book *Bucket Math* (no copyright date given). Younger students may like a picture for each numeral.

A straight line one is fun. **1**

Around and back on a railroad track - two-two two. **2**

Around a tree and around a tree is three. **3**

Down and over, then down some more. That's four. **4**

Fat old five goes down and around.
Put a flag on top and see what you've found. **5**

Down to a loop, a six rolls a hoop. **6**

Across the sky and down from heaven.
That is how we make a seven. **7**

We make an S, but do not wait. *(not useful if child doesn't know that letter)*
We climb back up to make an eight. **8**

A loop and a line, make nine. **9**

It's easy to make a one and a 0. **10**
Ten is all your fingers, you know.

8. Eventually, you should have your student progress from making large numerals to smaller numerals and from using a marker to using a crayon or fat pencil. Large graph paper can be used to help align the numbers correctly.

A variety of teaching strategies have been discussed here because the students need to be motivated to learn a difficult skill. You can think up other ideas that will be interesting to the specific students you teach. Be creative and find some way to keep their interest. For example, you may have a puppet that is learning to write numerals too.

GENERALIZATION INTO DAILY ACTIVITIES: Wait until the student is comfortable with writing numerals before asking her to generalize the skill into real life. She should be able to write the numerals somewhat automatically before you ask her to write numerals during daily activities. Both school and home have many activities where it is natural to write down numerals:

- writing down the number of students present/absent;
- lunch count: recording how many students are buying lunch;
- counting and recording the number in various groups (such as when taking a vote on several options);
- writing down channels of favorite TV shows;
- making temperature predictions or recording other data for science class (such as the number of tadpoles or butterflies that have hatched);emerged from the cocoons.
- numbering your paper (as for a spelling test).

Alternatives for Recording Numerals

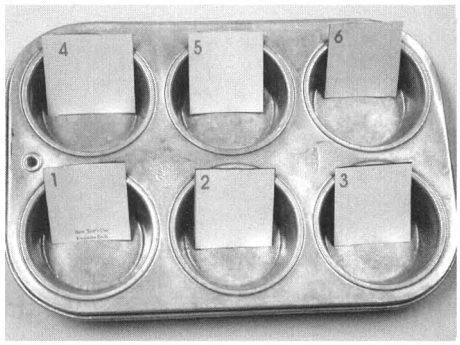
Students who do not yet have the fine motor skills to write numerals can still express mathematical understandings. They may be able to combine small items into sets

for addition or put numbered items in the correct sequence. They may be able to indicate small quantities by showing the appropriate number of fingers. In addition, they may be able to name the numerals orally. Still, there are times when you will want to have a permanent product to illustrate a student's understanding. Here are several alternatives that can be used:

- choosing answers from already formed numerals;
- using number stamps and other options;
- using assistive technology such as a computer, a printing calculator, or an app.

CHOOSING ANSWERS FROM ALREADY FORMED NUMERALS

Here are some options for students who can orally identify numerals but who are not yet able to write the numerals themselves. You can use these methods to enable everyone in a class to respond to a question at the same time or as a lead-in to the slightly more advanced ways of indicating responses described in the next section. Use only those materials that are appropriate for the individual student.



1. **Number Line.** Make a large number line with chalk or tape on the floor. (You could also write the numbers on white shelf paper or an old shower curtain.) Using the number line that you have drawn on the floor, have the student go to a number you name and stand on it. Choose the numerals at random from 1 to 10 or from 1 to 20, if you have room. Practice until the student goes to each number without hesitation. You are checking that she can recognize the numerals given orally. (For an older student, you can use the paper number line in Appendix B-6 and have her point to the number you name.)
2. **Numeral Cards.** Cut 3 to 4 squares of paper or cardboard for each numeral and write numerals on the squares with marker. Store the squares in separate compartments of an egg carton or muffin tin. (It is easy to make cards with the numerals up to 30 by cutting up an old calendar. Glue on some heavy backing to make it easier for students to pick them up.) Ask the student(s) to pick up a 2 card and give it to you. Then tell her to pick up the 7 card, etc.
3. **Magnetic Numerals.** Your student(s) can also use the magnetic numerals that are sometimes included with sets of magnetic letters. That way your refrigerator or most chalkboards can be a large math field. You can also give students cookie sheets or other small metal baking trays (not aluminum), use markers to draw rectangles on them, and ask students to indicate the correct answers by putting magnetic numerals in the rectangles.

USING NUMBER STAMPS AND OTHER OPTIONS

For students who were successful with the methods described above, have slightly better fine motor skills, or are ready to work more independently on math activities, here are some more ideas for indicating responses without physically writing numerals.

1. **Number Stamps.** You can have one stamp for each of the numbers from 0 to 9. They must be clearly labeled and easily accessible for the student to be able to use them accurately.
2. **Number Labels.** If the student has the ability to peel labels off a contact sheet, you could type numbers, one to a label, and let her use the stickers to complete math worksheets.
3. **Multiple Choice.** List the answers to math worksheets as multiple choice options. The student can then just mark the correct answer. Have her circle the correct answer so her decision can't be mistaken for a random pencil slash.

Assistive Technology

The Individuals with Disabilities Education Act (IDEA) defines *assistive technology* as "any tool or item that increases, maintains, or improves functional capabilities of individuals with disabilities." So, the adaptations described in the sections above would technically be considered low-tech assistive technology. However, often when people talk about assistive technology, they are thinking about computers, tablets, or other high-tech gadgets that can improve someone's functioning. In the United States, IEP teams are required to consider a student's need for assistive technology, both low-tech and high-tech options. Here are some high-tech options to consider writing into your student's IEP:

- **Printing Calculator.** A student may be able to use a calculator with a printer. If the numbers on the calculator are big enough for her to be accurate, the printer can produce a written product that can be corrected later by the teacher.
- **Computers.** The computer can be used to teach and for practice with students on basic number concepts such as counting and number recognition. These days, many activities and games are online, and many of them are free. Many of the computer games are self-checking so the student learns the correct answer. Computer keyboards can be adapted to the student's individual needs so that a regular keyboard with small keys is not a barrier to use. (Adapted keyboards such as IntelliKeys are available from Intellitools and others). A touch screen that registers the answer when the student touches the screen in the right place would also qualify as assistive technology. It is beyond the scope of this book to give more than a small sample of helpful websites, but here are some that may be appropriate for students at this level:
  - www.mathchimp.com
  - http://www.sheppardsoftware.com/math.htm
  - http://www.e-learningforkids.org/math/
  - http://pbskids.org/games/123/

- **Apps.** There are now a multitude of apps that can be downloaded to iPads and other tablet computers that can be helpful to students learning about number recognition and writing. Number tracing apps enable students to use their fingers to form number shapes, usually with some kind of visual and auditory feedback. Here is a small sample that you can find in the iTunes store:
  - Letter School – Learn to Write Letters and Numbers
  - Vehicle Number Trace
  - 123 Tracing

  And this website offers a short list of apps that are good for working on number recognition and counting:
  - http://technologyinearlychildhood.com/2013/03/08/5-best-apps-for-math-and-counting/

If you search for the specific type of app you are looking for (e.g., apps for number recognition), you will surely find at least one that fits the bill!

CHAPTER 8

More Counting Skills

Pre- /Post Checklist

Can the student:

1. ____ Rote count from 1 to 100.
2. ____ Use a number line.
3. ____ Count forward from any number.
4. ____ Find numbers one more and one less than a named number.
5. ____ Decide which number is larger than another (using a number line).
6. ____ Skip count by 2s (2, 4, 6, etc.).
7. ____ Skip count by 10s (10, 20, 30, etc.) to 100.
8. ____ Skip count by 5s (5, 10, 15, etc.).
9. ____ Count-on from the highest number when adding two numbers.
10. ____ Build numbers (under 40) using teacher's Clothespin Box.
11. ____ Read two- and three-digit numbers correctly.
12. ____ Use ordinal numbers to order objects

Counting is not just a simple skill that you learn in a short time. Counting strategies continue to be developed as you have more and more life experiences. Students with limited math skills may be able to count small numbers accurately but get confused by

numbers with several digits. Eventually they will learn the pattern of combining digits to make numerals and be able to use larger numbers in their daily living.

This chapter covers the following higher skills for counting:

- Counting to 100
- Using a number line
  - Visualizing the concepts of the number line
  - Counting forward on the number line starting with any number but 1
  - One more, one less using the number line
- Skip counting
  - By 2s
  - By 10s
  - By 5s
- Counting backward (optional)
- Demonstrating place value of two-digit numbers
- Reading two- to three-digit numbers
- Ordinal numbers

Counting to 100

The student should be able to count and use the numbers from 1–40 from previous lessons. Now the larger numbers are introduced so the student can work with the base 10 system for skip counting and place value.

ROTE COUNTING TO 100

OBJECTIVE: The student will be able to rote count to 100.

MATERIALS:

- A chalkboard or whiteboard
- Chalk or erasable markers

SUCCESS STEP: Have the student rote count to 40 orally. Praise his success. If he is unable to count correctly, you count to 40, having him repeat after every 2 or 3 numbers. Praise his participation.

PROCEDURE:

1. Review rote counting to 39, if you feel the student is not secure in the concept.
2. Write the number 39 on the board. Ask the student what number comes next. He should know that 40 comes next from the review. Write 40 on the board. Then write 41 on the board and say, "This is 41. What is the number?" and have the student repeat the number 41. Continue introducing 42, 43, and 44 in the same manner. Be sure to put commas and some space between the numbers as you write them.
3. Then ask the student, "What number comes next?" If he does not say 45, point out the pattern in the numbers on the board, pointing out the 0, 1, 2, 3, 4. Then ask again, "What number comes next?"

4. Continue the above procedure with the numbers up to 49.
5. Have the students chant the numbers from 40 to 49 as often as necessary.
6. Repeat the procedure for the numbers from 50 to 100. A good pace would be to work on 10 numbers per session. Eventually the student will be able to rote count to 100.
7. Praise the student's efforts. Have him rote count for brothers and sisters, parents, the school secretary, and anyone else you can rope into listening.

WHAT IS MISSING?

OBJECTIVE: The student will fill in the missing numbers on a chart with numbers 50 to 100.

MATERIALS:

- Missing Number Chart (Appendix B-7) for each student
- Small Post-It notes (2" x 1½")

SUCCESS STEP: Have the student point to the number 50 and the number 100 on the Missing Numbers Chart. Praise success.

PROCEDURE:

1. Cover 3 of the numbers on the Missing Numbers Chart with Post-It notes. (Cut the Post-It note in half on the longest side. Put the Post-It note over the number and then fold.) Have the student point to each Post-It note and say what the Missing Number hidden beneath it is. Lift up the notes so the student can see if he supplied the correct answer. He needs to *say* the written number, not write it.
2. Cover different numbers on the Missing Numbers Chart, gradually increasing the amount of numbers the student must supply.
3. Continue practicing rote counting to 100 at future sessions to help him maintain the skill.

GENERALIZATION INTO DAILY ACTIVITIES: Counting and recognizing these larger numbers will be more meaningful after students get more experiences with place value. At this stage, try to include numbers in your everyday activities so that the number names are almost overlearned.

- Play hide and seek and count to 100 (or as high as the student can count).
- Have the student count out small items for a snack or an activity (e.g., give everyone in the family or class 50 mini marshmallows, Goldfish crackers, or craft sticks).
- Count to make sure you still have 52 cards in your deck before playing a card game.
- When sharing a book, occasionally pause and ask the student to read the page number you are on.
- Play Chutes and Ladders by Milton Bradley (the squares on the board are numbered from 1 to 100) and comment on what number you land on, have to go back to, etc.

Using a Number Line

Using a number line is a skill that helps students to visualize the relationships between numbers. In school the number line is often taped to the desk for reference. Some students can count easily from 1 to 20, but if you ask them to start at any number besides 1, they are not able to do it. A number line can give them a visual picture of the numbers' relationships to each other and help them understand "one more" and "one less." Number lines can also be used to help students visualize what they are doing when adding or subtracting. To use a number line, students need to be able to recognize the numerals from 1 to 20.

VISUALIZING THE CONCEPTS OF THE NUMBER LINE

The student needs to have a visual picture of what a number line represents in his mind. That is, the further you go to the right, the "bigger" the numbers are; the further you go to the left, the "smaller" they are. The first step in using a number line is to use concrete materials to represent the numbers on the number line.

OBJECTIVE: The student will be able to represent the numbers on a number line (1 to 10) with concrete materials.

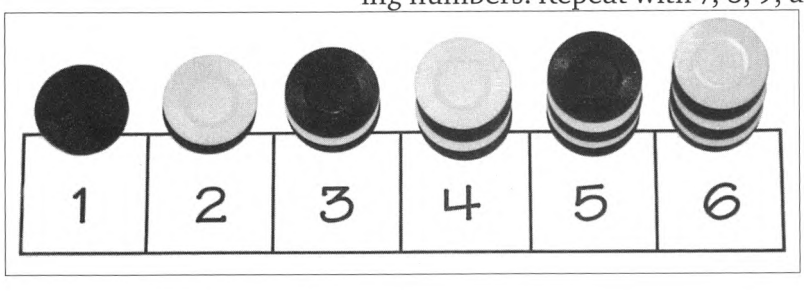
MATERIALS:

- Poker chips, Bingo chips, small blocks, or other similar items that can be easily stacked
- Number line with numbers 1 to 10. (Cut a strip of light cardboard or cardstock about 15 inches long and 1 inch wide—similar to a ruler. Write the numerals 1 to 10 clearly, about 1½ inches apart.)

SUCCESS STEP: Ask the student to make a stack of one, then two, then three chips or blocks. Praise him for counting out the correct number.

PROCEDURE:

1. Show the number line (1 to 10) to the student, point out the numbers, and have him identify them.
2. See if he can put the right number of chips or blocks on the numbers 1, 2, and 3. If he is not correct, model putting the chips on the numbers. Tell him that the chips represent the numbers on the number line.
3. Have the student make piles of 4, 5, and 6 chips and put them on the corresponding numbers. Repeat with 7, 8, 9, and 10 chips.



4. Make a game about pointing to a number on the number line and having the student make a pile of chips that represent the number pointed to.
5. For additional practice, have the student mark dots on the number line itself to show the number of items that the number represents.

COUNTING FORWARD FROM ANY NUMBER

Emily easily counts from 1 to 30, almost on one breath. If you stop her, however, she yells, "Now I have to start all over again from the beginning." If you say to her, "You were on number 11. Just start from 11," she says, "Oh, I can't! I have to go back to 1 again."

OBJECTIVE: The student will be able to count to 20 starting with any number other than 1, at first using the number line, and then without the number line.

MATERIALS:

- Number line with numbers 1 to 20. Use light cardboard or cardstock to make a 20-inch line about 1 inch wide (similar to a ruler) with the numerals 1-20 written clearly about 1 inches apart. You can copy the number line from Appendix B-6 (the 1 to 10 and the 11 to 20 segments) and glue them on to the cardstock.

PROCEDURE:

1. Using the number line marked from 1 to 20, point to a low number and ask the student to say that number and all the numbers that are higher up on the number line to 20. Say, "Tell me which numbers are larger (higher)." He can just read them off the number line. Practice as necessary. If the student is nonverbal, have him point to the correct number as you say it.
2. Tell him that you are going to play a game to see if he can really count. You know that he can say the numbers from 1 to 20, but can he begin at a different starting number each time? Start from numbers chosen at random, but let him use the number line if he chooses.
3. Then challenge him to count to 20 from any number between 2 and 19 without using the number line.
4. Have the students work in pairs by having one student name any number on the number line except 20. The other student has to count forward while the first student watches the numerals on the number line to make sure he does it correctly.

Note: *The skill of counting forward from any number is not easy. The student needs to have a picture in his mind of something similar to the number line in order to start at any number and count higher. Practice will make this skill come automatically if he has the concrete materials to help him visualize the sequence of numbers as he learns.*

ONE MORE, ONE LESS (USING A NUMBER LINE)

OBJECTIVE: The student will be able to name numbers that are one more or one less than called numbers using a number line (1 to 20). Later, he will name numbers from 1 to 100 on a hundreds chart, if possible.

MATERIALS:

- The number line used for previous activity
- A large index card or sheet of paper
- Teacher's number line, which can be written on the chalkboard or on an overhead transparency (or make a large number line from cardstock that can be seen by the students)

PROCEDURE:

1. Point to a number (5) on your number line and say, "What number is one more than 5?" (Or name any number from 1 to 9.) Move your finger over one number and say, "6."
2. Ask the students to do the same thing on their number lines. The students put their fingers first on the 5, and then move one more to the 6 on their number lines. Repeat with numbers chosen randomly.
3. When the students seem to get the pattern, quit modeling the answers immediately and observe the students.

PEEK GAME

4. When the students seem secure with the previous activity, they can play the game Peek. Cover up the numbers (on the teacher's number line) that are larger than the number you will call. For example, you want to call the number 4. Cover up 5 to 10 and point to the number 4. Then say, "What number is one more than 4?"
5. After the students respond, one student comes up to peek at the next number on the teacher's number line. He announces whether the class was correct and the class receives a point. If they were incorrect, the teacher gets a point, and the number is repeated. Of course, the class will win, but the teacher can bemoan the fact that the class is so smart.
6. After the students are secure in the concept of *one more*, change to another activity. In a week or two, use the same activities for *one less*. This activity is designed to prepare students for beginning addition and subtraction.

GENERALIZATION INTO DAILY ACTIVITIES: Deliberately set up situations that will prompt your child or student to ask for one more. For example, give him one too few items to hand out (cups, napkins, etc.) to the class or family. See if he will ask for one more. Or don't give him enough envelopes for his Valentines or buns for hot dogs and see if he will request one more.

WHICH NUMBER IS LARGER? (USING A NUMBER LINE)

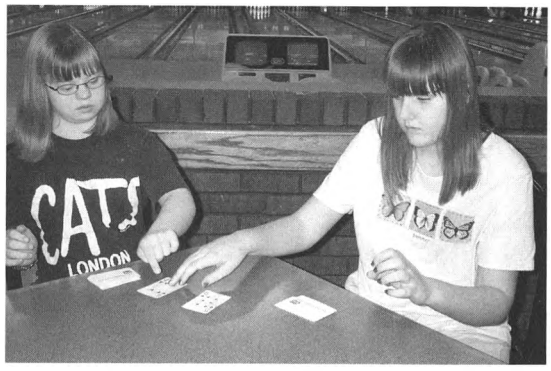
The game Top This (also called War) can be adapted so the number line is used to check each answer. This offers good practice in using a number line to determine which number is larger.

OBJECTIVE: The student will use a number line to determine which number is larger from pairs of numbers under 30.

MATERIALS:

- Deck of cards with face cards removed; aces representing one
- Number line numbered from 1 to 30

PROCEDURE:



1. Shuffle the cards and deal them to the players face down. (The game is usually played with two players, but more can play if they make teams.)
2. Each player turns over the top card or cards in his pile and compares cards to see who has the largest number. (Turn over two cards at once if you want to play with two-digit numbers.)
3. The student with the largest number must show that his number is the largest by pointing at the two numbers on the number line. He then gets to take all the cards in that round. If there is a tie, each player keeps his own cards.
4. The winner is the player with the most cards at the end of the game.

Skip Counting

Skip counting means naming only some of the numerals in a consistent pattern such as 10...20...30...etc. Many children come to school with the ability to skip count by 2s to 10, perhaps from playing childhood games. Counting by 2s helps a student distinguish between odd and even numbers.

Counting by 10s is an important skill that helps in understanding our base ten system and in determining place value. After counting by 10s, most texts teach counting by 5s. Skip counting by 5s is very useful in telling time and in adding nickels and five-dollar bills. If you have a student with significant memory problems, I would concentrate on teaching him to count by 10s first. Later, you can try introducing him to skip counting by 5s or 2s.

SKIP COUNTING BY TWOS

OBJECTIVE: The student will be able to count by 2s, first to 10, later to 20, understanding the terms even and odd numbers.

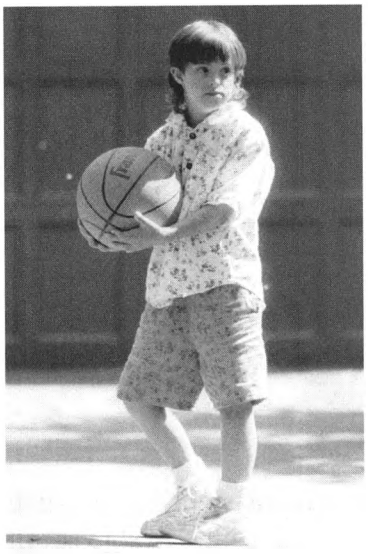
MATERIALS:

- A number line (1 to 20) for each student. You can use the one from the activity "Counting Forward from Any Number," above.
- One or more small game tokens or other counters small enough to sit completely on one square of the number line
- One die

SUCCESS STEP: Ask the student to repeat or chant after you, "2, 4, 6, 8, 10. Let's do it over again." If he is not successful, have him repeat fewer numbers. "2, 4, 6. Don't do any tricks." Praise him.

PROCEDURE:

1. Using the number line, repeat the chant from the success step and have the student point to each number on the number line as it is chanted. Repeat as needed.
2. Show the student how you can *hop* or *jump* your marker to every other number on the number line, starting with the number 2. Ask him if he can *hop* the numbers too. You say the correct number as he is making the hop.
3. If he can't get the concept of "every other," highlight the even numbers with a light pencil mark. After some practice with the marked number line, thoroughly erase the pencil marks and see if he can hop to every other number without cues. Have him name his jumps without instructor assistance.
4. Tell the student that the numbers he jumped to are called *even* numbers. When that concept is secure, teach the term *odd* numbers for the other numbers.



GENERALIZATION INTO DAILY ACTIVITIES:

- When cutting a pizza, brownies, etc., say that you want every person there to have two pieces. Then count the people by twos so you know how many pieces to cut.
- Shoot baskets and give yourself and any other players two points for each basket.
- Count how many shoes are in the house or class by counting each person as two.

SKIP COUNTING BY TENS

OBJECTIVE: The student will be able to skip count the numbers by 10, first from 10 to 50, then to 100.

MATERIALS:

- 5 bundles (with 10 each) of straws or sticks for each student
- Counting by 10s Chart (1 to 50) (Appendix B-8)
- Counting by 10s Chart (1 to 100) (Appendix B-9)

SUCCESS STEP: Have the student take the rubber band off of one bundle and count the straws so he knows that each bundle has 10. Praise correct counting.

PROCEDURE:

1. Give each student 5 bundles of straws (10 in each) and have them chant as they put down each bundle, "10...20...30...40...50." Repeat as necessary.
2. Open all 5 of your own bundles and have the student count the straws from 1 to 50 in rhythm with you. Then bundle up your 5 bundles of ten and compare them to the student's bundles. Then have the student chant "10...20...30...40...50" as he counts his bundles.

3. Introduce the Counting by 10s Chart (1 to 50). Have the student count by tens while looking at the chart. Continue until the student is secure with counting by 10s to 50.
4. Next, demonstrate counting by 10s with more bundles of ten (your own bundles) until 100 is reached. Repeat the demonstration with the students counting along with you.
5. Introduce the Counting by 10s to 100 Chart to the students. Sometimes they will be able to focus on the 10s in the hundreds chart if those numbers are larger. You may want to outline the 10s in red to help the students focus. Have them count by 10s, pointing to each one on the hundreds chart.
6. At another session, have the students work in pairs and put their 5 bundles with the other person's 5 bundles so they both can count to 100 by 10s. If the students have much difficulty with the numbers from 50 to 100, go back and review skip counting with only the numbers from 10 to 50. When the students are more familiar with the larger numbers, they may be able to pick up the skip count to 100 more quickly.

GENERALIZATION INTO DAILY ACTIVITIES:

- Play the game *Spud.* Assign each player one of the 10s (for example, "Josh, you're number 20; Lauren, you're number 30.") One player yells "Spud," throws up a ball, and calls out the number given to another player. The person whose number has been called must catch the ball before it bounces. The Spud thrower must know the 10s to make a good call.
- Count your pennies by making piles of 10. Count off the piles up to 50 so you can put the coins in penny roll wrappers.

SKIP COUNTING BY FIVES

OBJECTIVE: The student will be able to skip count by 5s from 5 to 50. Later to 100.

MATERIALS:

- Small paper cups
- Small items like raisins or macaroni
- 5 five-dollar bills (play money) for each student plus extra for the teacher
- Counting by Fives Chart (Appendix B-10)

SUCCESS STEP: Have the student show you how to put 5 items in the paper cup. Make sure he is successful and praise him.

PROCEDURE:

1. Place 5 paper cups on the table. Lay out a line of 25 small items in front of the student on the table.
2. Have the student count how many items are in the line.
3. Then have the student count out five items to put into each paper cup. (Younger students may enjoy the activity more if some of the cups are placed in front of stuffed animals.)
4. The teacher says, "I have a faster way to count all these small items." Chant, "5...10...15...20...25," touching each cup as you count, thus getting the total number

of items. "Is that the same number we had when we counted them one by one?" (It should be the same.) "See, that was faster! Would you like to learn the faster way?"

5. Then direct the students (together) to count the items in the same way, chanting or singing the 5s. Ask if that is faster than counting each item separately. Give each student a chance to skip count the items in the cups by himself.
6. Use the Counting by Fives Chart with the 5s magnified to help them learn counting by 5s. They can read off the 5s numbers until they have memorized the numbers. Point out that every other number ends in 5.
7. If young students are having a lot of trouble memorizing the sequence, you can make up a song to help them remember. For example, to the tune of the traditional "10 Little Indians," you could sing "5... 10... 15... 20/ 25...30...35...40/45... 50... 55... 60..., I can count by 5s."

Do not be concerned if the student's skip counting is not consistently correct. Repetition is the key to memorizing the numbers in skip counting. When the student works with money, counting by 5s and 10s will probably become a more functional skill.

Allen knew how to count his five- and ten-dollar bills so he could figure out how much money he had. He easily said, "5, 10, 15, 20." He could also skip count ten-dollar bills up to 100. However, when he was asked to skip count by 2s, he was unable to remember how. "I did that once," he said. He had many occasions to use his skip counting by 5s and 10s, but very little occasion to skip count by 2s.

GENERALIZATION INTO DAILY ACTIVITIES:

- Play games where you score 5 points at a time such as darts or another game where you can make up the scoring.
- Count how many fingers there are in your family or in a small group in your class. Touch each finger as it is counted. Then count the number of fingers by 5s. Point out how much faster counting by 5s is.
- Play hide and seek and have the seeker close his eyes and count by 5s to 100. You can use skip counting by 10s in the same way.
- Review skip counting until the students can count the 5s and the 10s without using the number lines or charts.

Counting On

When students learn the strategy of *counting on,* they become more efficient in counting two groups of items. When the student is combining two groups of items, and knows how many are in each group, he can start with the total of the largest group and *count on*. For example, when we combine 9 and 4, we do not have to count each one of the items starting with 1. We can start with the number of the largest group (9), then count 10, 11, 12, and 13 for the 4 additional items in the second group to get the total. Students must be able to *rote count forward* from any number to *count on* with meaning.

Research with typically developing children shows that they do not just "catch on" to *counting on* and use the strategy from then on. Sometimes they use it and sometimes they do not for a period of time. Eventually it becomes a consistent part of their math strategies. So, if you are working with a hands-on learner, just continue to remind him gently about *counting on* as he does other math problems and don't expect a quick change in strategies.

Jake was helping his older brother with his paper route. He had six Sunday newspapers at his feet. Then his brother handed him five more and asked, "How many papers do you have now?" Jake started counting the whole pile of newspapers at his feet. "1, 2, 3," up to 11. Jake got the correct answer, but it would have been much faster to ***count on****: "7, 8, 9, 10, and 11."*

WHAT'S IN THE BAG?

OBJECTIVE: When combining two groups, the student will count on from the total number of the largest group to get a final answer.

MATERIALS:

- Several varieties of items that are alike (pencils, poker chips, pennies etc.)
- Paper bag

SUCCESS STEP: Have the student count three of the items. Praise him.

PROCEDURE:

1. Put the three items already counted in a paper bag. Say, "I put in three."
2. Pick up one more item and say, "And this is four" and put it in the bag.
3. Pick up another item and ask, "What number is this?" (Five.) Put it in the bag.
4. Then dump all the items out of the bag and count them to be sure there were five there.
5. Point out how much faster it was to *count on* from the number that we knew was in the bag compared to counting each one. Repeat steps 1–4 with different items.
6. If the student is nonverbal, have him raise his fingers to show the number of items in the bag or point to the numbers on the number line.
7. Pair off students. Have one put the items into the bag, as done above, and the other check the total by counting all the items.
8. Repeat with a variety of items or counters.

GENERALIZATION INTO DAILY ACTIVITIES: Any time the student is counting two groups, emphasize how much faster it is to count on instead of starting all over again from the beginning. Playing games with two dice offers opportunities to point out the efficiency of counting on. Have the student look at one die and say the number of dots there. Then have the student count on with the dots on the second die. You can always have him check the answer by counting each of the individual

dots. (More advanced students can try Yahtzee by Milton Bradley and count up the total of five dice.)

When you are getting supplies that come in boxes or multipacks, you can have your child count on to get the total count. For example, have him figure out how many cans of juice, eggs, or any food item there are when the total of one group is already known. Then both of you can count on to get the total amount. For instance, if you have two six-packs of juice, point to the first six-pack and say "six," and then count on to find out how many cans you have in all.

Backward Counting from 10 (Optional)

Backward counting is much harder than forward counting. It requires reversing a well-known process. Counting backward is not an essential survival skill. But it can help students understand a number line and learn to do beginning subtraction. For example, "You have 9 pencils and I take away 3 of them. How many do you have now?" The student could count backwards, "8...7...6." If he knows how to use a number line, he can also count backward with the help of that aid, so it is not absolutely essential to remember the numbers in backward sequence.

BACKWARD COUNTING FROM 10 (OPTIONAL)

OBJECTIVE: The student will be able to count backward from ten, using a number line if necessary.

MATERIALS:

- Songs that incorporate counting backward (such as *Ten Little Monkeys)* where monkeys, frogs, or jumping children are eliminated one by one (best for elementary students)
- A picture or a model of a rocket
- A number line

SUCCESS STEP: Have the student clap and sing, "3, 2, 1; 3, 2, 1" to the tune of the first two lines of *Three Blind Mice*. Praise him for correct responses.

PROCEDURE:

1. Teach a backward counting rhyme such as *Ten Little Monkeys* (elementary students only).

   Ten little monkeys jumping on the bed.
   One fell off and bumped his head.
   Mama called the doctor and the doctor said,
   "No more monkeys jumping on the bed."

   Nine little monkeys jumping on the bed.
   One fell off and bumped his head.

Mama called the doctor and the doctor said,
"No more monkeys jumping on the bed," etc.

2. Imitate a rocket or shuttle takeoff by counting backwards from 10:
10...9... 8...7...6...5...4...3...2...1.

ADAPTATIONS FOR OLDER LEARNERS: Use items for counting that adults use such as poker chips, bingo markers, pennies, and pretzel sticks. Do not use the singing games or finger plays that are commonly used by younger children. If possible, use the number line to teach counting backward. You may teach the names of the numbers backward from 10 by making them into a chant or rap song.

If you can stand to hear the song, have the students sing the song *100 Bottles of Pop on the Wall* (to the tune of *100 Bottles of Beer on the Wall).* You can start with any number of bottles of pop on the wall, depending on what numbers you want them to be counting back from.

PLACE VALUE

Moira is reading out numbers for Rowan to type into the computer. Rowan does well with numerals up to 13. For 14, however, she types in 41.

This is an understandable error since she hears the words "four-teen" with the ***four*** *spoken first. However, if Rowan understands something about place value, she can be taught this irregularity in the English counting sequence.*

Our system of numeration evolved from a need to record quantities and a desire to convey the amounts of these quantities to others. Our system is a base-ten model with one to nine digits in the first column. When one more is added, there is an exchange of 10 *ones* for 1 *ten,* which is then placed in the second column. This regrouping allows us to record both small and very large numbers with a rule-driven system. The *value* of any digit is determined by the *place* that the digit holds in the given number. For example, in the number 582, the digit 2 is in the *ones* place, so there are 2 *ones.* The digit 8 is in the *tens* place, so there are 8 *tens.* The digit 5 is in the *hundreds* place, so there are 5 *hundreds.* You could not give the correct value for the digit 5 unless you knew that it was in the *hundreds* place.

This may seem so elementary to you that it goes without saying. However, young students have to be taught what place value means and have a mental picture of how the numbers are made. That learning is really not so elementary. The Romans struggled with a system that required letters and specific placements to record quantities. Every time I look at the production date of an older movie, I remember that those Roman numerals do not give me instant recognition as our Arabic numerals do...and that they string out much longer.

In teaching your student(s) about two-digit numbers, you have already worked with place value, even if the students were not aware of it. In the following lessons, you will help your students get a visual, concrete image of the place value of numbers.

Place value has its roots in counting with meaning. If your student can count to 30, understanding the patterns, he is probably ready to learn about place value. Essential steps in understanding place value are:

- understanding that single digits are *ones*
- grouping and counting items by *tens*
- making numbers showing place value
- reading two-digit numbers
- breaking numbers down into *ones, tens,* and *hundreds*
- reading three-digit numbers

Ones and Tens

If you have used sticks and straws to give concrete meaning to numerals, most students will have the mental image of the numeral 1 as being *one* concrete item. You may teach them the name *units* or *ones* for the single digits, depending on the name preferred in your school. The student will already understand that the sticks or straws can be bundled up in a group of ten, which is called *tens.*

Using the Clothespin Box described below is a good activity to help students understand place value. Some students are able to squeeze open clothespins and clamp them on a sturdy cardboard box edge. Others do better with the kind of clothespin that has no moving parts and slide it onto the box edge. I have found that students usually enjoy working with the brightly colored clothespins and shoebox.

When making Clothespin Boxes, you will find that they are easier to store if you cut the two short sides that are not labeled with ONE and TENS. Then you can fold the short sides in and fold the whole structure flat. If the cardboard is strong enough, the box will stand up well when you pull the sides out part way.

CLOTHESPIN BOX

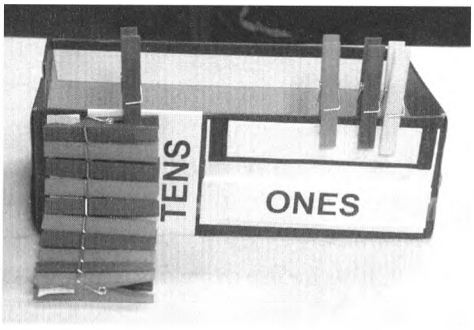
MATERIALS:

- 50 colored plastic clothespins per student (preferably spring-type)
- Several plain wooden clothespins or wooden clothespins painted black for each student
- 1 shoebox per student with **TENS** printed on one half of the long side of the box (on the left side) and **ONES** on the other half of the box
- The rectangular cardboard holders that the clothespins came on, cut to hold only 10 clothespins. (If no holders are available, cut cardboard strips about 1½" wide and long enough so that they will only hold 10 clothespins.)
- A larger box for the teacher to demonstrate on (optional)

SUCCESS STEP: Have the student put 2 clothespins on the ONES section of the box. Then ask him to put on one more. Praise him.

PROCEDURE:

1. Demonstrate to the student how you can clip 9 clothespins to the ONES section and no more. (Use marker to delineate the area where the clothespins can go if there is room for more than 9 clothespins.)
2. Then take one of the long strips of ten clothespins attached to a cardboard strip and hang it on the TENS side. I color one clothespin black or use a plain wooden clothespin and use it to clip the strip of 10 to the box. (Black color or plain wood keeps it from being counted as one of the strip.) Give the student a chance to try doing the same.
3. Have the student make all the numbers from 1 to 9 with the clothespins on the ONES side of the box.



4. Write the numerals from 21 to 29 on paper, one at a time, and have the student try to make each with the pins using two bundles of *tens* on the TENS part of the box. Repeat over a period of days. (To attach two bundles of ten, the teacher clips the second bundle on the first bundle. The students just lean the two groups against the box.)
5. Now have the student make the numerals from 11 to 20 on the clothespin box. The *teen* numerals are more difficult because the student does not hear the predictable pattern of *twenty-one, twenty-two, etc.* However, if you show him the written numeral, He will usually be able to reproduce the more predictable written numerals.

Clothespin Colors

The clothespins I use come in four bright colors. At first, I was afraid of confusing the students, so I only used one color (yellow) for the lesson. Later the students tried to replicate the lesson, and some of them used several colors of clothespins. I asked them to tell me which was the easiest way to work with the clothespins. One boy said, "If the pins are the same color, I can't count them from here. If you mix up the colors, they are easier to count." It seems that the yellow clothespins meld together and are hard to count when on the box. The group was able to disregard the difference in the colors in order to count more easily.

PLACE VALUE OF NUMBERS UP TO 100

If the student has learned to accurately make the numbers from 1 to 40 using the teacher's Clothespin Box, he should be ready to make the numbers from 41 to 100. However, make sure that he can count all the way from 41 to 100, understands the rules for making these higher numbers, and can recognize these higher numbers when they are written and given orally. It is helpful if he can count on from the highest number but not absolutely necessary. Then he should be introduced to the concept of *hundreds.*

HANGING NUMBERS

OBJECTIVE: The student will be able to place any two-digit numbers into ones and tens columns and read them correctly.

MATERIALS:

- Clothespins
- Clothespin Boxes for students used in previous activity
- Teacher's Clothespin Box marked with numerals 1 through 40

SUCCESS STEP: Have the student make a single-digit number on the Clothespin Box. Praise him if correct. Model making the number for him and have him repeat, if he is not successful.

PROCEDURE:

1. Using the Clothespin Boxes, show the students the numbers 27 and 19. Then write the numerals on paper or a board so the student can relate the written number to concrete items.
2. Using clothespins, make some of the numbers from 21 to 40 on the large teacher's Clothespin Box and see if the students can figure out what the numbers are. They do not have to read all these numbers correctly. If you need more space to hang ten strips, glue or staple a strip of cardboard to the left side of the box.
3. If you feel that the students are still shaky on recognizing the larger two-digit numbers, replay the MATH bingo game (in Chapter 7) using the double-digit numbers.

ANOTHER WAY TO VISUALIZE PLACE VALUE

For students who are having difficulty with the concept of place value, you may want to introduce them to a simplification of a mnemonic developed by Jan Semple (1986). In this illustration called *Whole Number Street,* the numbers in the ones place are visualized as Kids, those in the tens place are thought of as Teenagers, and those in the hundreds (and higher) place are Grown-Ups.

KIDS, TEENS, AND GROWN-UPS

OBJECTIVE: The student will be able to visualize and explain the relationship between ones/units, tens, and hundreds (place value) by comparing it with the relationship between kids, teenagers, and grown-ups and their allowances.

MATERIALS:

- Place-Value-People chart showing *ones, tens,* and *hundreds* from Appendix B-11. (Before teaching this lesson, copy this chart on to the chalkboard or make a laminated poster that can be referred to later)
- Teens and Kids Worksheet from Appendix B-12
- Pencil, colored pencils, or crayons

PROCEDURE:

1. Ask your student(s) if they would like an easy way to remember place value. Show them the positions of the *kids, the teenagers,* and the *grown-ups* on the chalkboard or on the teacher's Place-Value-People poster. The *kids* get allowances in one-dollar bills. Explain that the *kids* are closest to home, which is shown by the wall (and which represents the decimal point, used later on.) The *tens* column is for *teenag-*

ers, who get allowances in ten-dollar bills. The *hundreds* column is for *grown-ups,* who get their money in hundred-dollar bills.

2. Talk about various one- and two-digit numbers and write them in the proper places on the teacher's Place-Value-People chart. Be sure to talk about the *kids* and *teenagers.* Remind students that *kids* get dollar bills for allowance and *teenagers* get ten-dollar bills for allowance. If students don't understand what an allowance is, just say they "get paid" in one-dollar or ten-dollar bills.
3. Have students do the Teens and Kids Worksheet from Appendix B-12. Have them circle or scribble with ***red*** crayon or marker on the tens (teens) of the two-digit numbers shown on the page. Also have them circle or scribble with ***blue*** crayon or marker on the ones (kids).

ORDERING NUMBERS

Up to this point, students have been learning about regular or *cardinal* numbers that answer the question "How many?" Another important use of numbers is telling the location of the item or person in relationship to others. This use of numbers emphasizes the sequence or order that the numbers indicate, so numbers that indicate order are called *ordinal* numbers. Many children know the meaning of a few ordinal numbers such as *first* or *second* when they come to school.

Ordinal numbers can be used to give the sequence of items in space or in time (first you do *x,* second you do *y,* and third you do *z*). The following activity focuses on helping students understand how an ordinal number can give the *location* of each item in space by using the words *first, second, third, fourth, fifth, sixth, seventh, eighth, ninth, tenth,* and more.

INTRODUCING THE CONCEPT

OBJECTIVE: The student will be able to use ordinal numbers to order objects (first, from 1 to 5, and later from 1 to 10).

MATERIALS:

- Other students or family members willing to stand in line
- Dolls, stuffed animals, action figures, etc. that can sit or stand in a line

SUCCESS STEP: Ask the student what the objects are. (Or ask him to point to the named objects.) Praise him.

PROCEDURE:

1. Set up four objects in a straight line in front of the student so that he can see each object. Or, ask four students or family members to line up in front of him.
2. Starting at the left, ask the student to touch the object or person that is *first* in line. If he does not know, model touching the object or put his hand on the first object. Praise him. Repeat the step several times.
3. Repeat step one, and then ask the student to touch the *second* object. Model or physically prompt if necessary. Repeat with *second* several times. Then ask for the first object and then the second object. When teaching ordinal numbers, it is important to show the second location in relation to the other objects in the line. Practice *first*

and *second* several times. If possible, set up a second line of different objects and repeat the questions to see if generalization has occurred.

4. Continue the sequence with *first, second,* and *third* until the student seems secure in the knowledge.
5. Then line up two more objects or persons to teach *fourth* and *fifth*. You can explain the pattern of adding the *th* sound to the regular (cardinal) number to form the "order" number (ordinal). However, the word for the number-five object is not pronounced *five+th* but is changed to *fifth*. Tell the student that our tongues would get twisted up trying to say *fiveth,* so we call it *fifth* because it is easier to say. Use the procedure given in steps one and two to work on *fourth* and *fifth,* alone and then together. When the student is secure, go back and review *first, second,* and *third,* and add *fourth* and *fifth*.
6. Explain the pattern (adding *th*) for making order numbers from sixth through tenth. Write the numbers on paper to show the students. Model saying the ordinal numbers with the student chanting along with you. When he seems secure on the pattern of forming the numbers, have him show you the sixth through tenth objects in the line, as was done in step one. If you are having trouble teaching sixth through tenth at this time, drop it and try again later. Many people only use the first five ordinal numbers with the addition of the word *last.*

Scott, who has Down syndrome, looks at the teacher with bewilderment when she asks him to touch the **first** *object. The teacher gently takes his finger and puts it on the first object (physical prompt). Scott smiles and repeats the touch when asked to touch the second object. However, when the teacher goes to help Kala, who is somewhere along the autism spectrum, she shrinks back and pulls her hand away. The teacher then gets Kala's attention and models touching the object. After a minute of silence, Kala touches the first object herself.*

Although we are told to use a hierarchy of prompts from physical to verbal, it is important to know the individual student's preferences and learning styles when deciding what assistance to give.

Many students already know the meaning of *first*. Teach the meaning of first and *last* as you see opportunities in the student's daily routine. For instance, when the student is looking at a line of students, comment, "Peter is last. He is at the end of the line. He is last." Repeat enough times during the daily routine until the student understands the meaning of *last* in line. Do a similar routine with *first.* Most students want to be first in line or first to get some reward.

CHAPTER 9

Whole Number Addition

Pre- /Post Checklist

Can the student:

1. ____ Combine two groups by counting on from the highest number.
2. ____ Indicate that addition is needed when two or more groups are put together.
3. ____ Demonstrate that the order of two numbers does not matter when adding.
4. ____ Tell total number of items of two groups whose sums are 12 or under.
5. ____ Demonstrate place value of ones and tens with concrete objects when adding two-digit numbers (sums under 50).
6. ____ Demonstrate skip counting (repeated addition) by 5s and 10s with the calculator.

Addition Facts (optional)

7. ____ State some beginning addition facts.
8. ____ Add doubles.
9. ____ State addition facts adding up to ten (complements).
10. ____ Demonstrate patterns in addition facts.
11. ____ State difficult facts that follow no pattern.

If you have done some of the previous activities in this book, you have probably already introduced addition to your student(s). While working with small numbers, we do addition by counting. Working with hands-on materials, it is easy to see that 2 straws added to 3 straws makes a total of 5 straws. However, when working with larger quantities, it becomes tiresome to count all those straws, even if you know how to "count on" from the highest number. Addition is a process that gives us a system for joining two or more quantities in an efficient way.

The focus of this chapter is first to help students understand *when* to use addition. By using hands-on materials and pictures, students should be able to *see* the situation in their minds, realize that items are being put together, and understand that means we need to use addition.

Students need a great deal of experience in combining small groups (sums 10 and under). With experience, many students will just naturally learn the addition facts of groups whose sums are 10 or less. First, they count to find the answer. Later, they may visually remember those small groupings and come up with the answer without counting.

However, the premise of this book is that the student does not have to learn the addition facts if she knows how to set up the problem on the calculator. Some students will always rely on the calculator. Still, if your student(s) can memorize some addition facts, they are very useful for doing mental math or estimating when an exact answer isn't needed. The last half of the chapter therefore covers strategies for learning the addition facts. For most concrete thinkers, especially those with problems with rote memory, this will be an *optional* section.

Of course, memorizing the math facts is not difficult for all students, including some on the autism spectrum. The emphasis for these students will need to be on how to *use* the math facts. Try having your student(s) memorize some of the addition facts, but stop if she is inconsistent in learning or is showing signs of real frustration.

Addition will be addressed in the following ways:

- **Basic addition concepts**
  - counting on
  - using addition to put items together
  - commutative property (order of items not important)
  - addition vocabulary

- **Addition of numbers using manipulatives**
  - with sums of 12 and under
  - place value in addition
  - addition of single-digit numbers

- **Addition facts (optional)**
  - beginning addition facts
  - doubles
  - addition facts adding up to ten
  - patterns in addition facts
  - difficult facts

Should Addition and Subtraction Be Taught Together?

Most schools teach subtraction along with addition in order to emphasize that the addition facts are related to the subtraction facts. That is, they teach "fact families" such as $2 + 3 = 5$, $3 + 2 = 5$, $5 - 2 = 3$, $5 - 3 = 2$. This approach may or may not work for students who are concrete learners. If the student is included in general classes for math and is doing well, it is probably best to go along with the math textbook used in the student's school and have her try to learn the two processes simultaneously. Watch for signs of confusion or frustration, however.

Students who have difficulty learning the two processes together will profit by learning addition thoroughly before starting on subtraction. Worksheets that include both types of problems are difficult for many typical children as well as students who are concrete thinkers, because they mechanically get in one gear such as addition and have difficulty switching.

Hopefully, thoroughly learning to use each process in real and simulated situations, as is done here, will help students think about the process to be used before doing the mechanical operation. Using the calculator forces them to make a decision as to what operation is needed from the very beginning.

Basic Addition Concepts

Before beginning work on these activities, students need to understand the process of counting to find an answer. Learning to add is much more efficient if the students know the process of *counting on*. Another basic understanding is that addition is needed when groups are put *together*. Whether two groups or many groups are being put together, the answer is found by addition. The last basic understanding related to addition is that the numbers that are to be added do not have to be in any special order ($4 + 3 = 7$ and $3 + 4 = 7$). It does not matter which number or group is written first (*commutative property*). The answer is the same.

COUNTING ON

You can teach students to be more efficient in counting two groups of items by using the strategy of *counting on*. Students may *count on* from either number and be mathematically correct. In this chapter, we are going to emphasize *counting on* from the largest number because it is the most efficient way to reach the total.

When the student is combining two groups of items, and knows how many are in each group, she should start with the number of the largest group and *count on*. For example, when we combine 9 and 4, we do not have to count each one of the items starting with 1. We can start with the number of the largest group (9), then count 10, 11, 12, and 13 for the 4 additional items in the second group to get the total. Check to see whether the student counts on from the highest number when she is counting the numbers on two dice or is adding two groups of sticks or straws. If she is unable to *count on* in these situations, review the counting-on section in Chapter 8, More Counting Skills.

OBJECTIVE: The student will combine two groups by saying the largest number and counting on with the second group to get the total (sums under 20).

PROCEDURE:

1. Play the *Top This* game from Chapter 8 to make sure the students understand which number is the largest from a choice of two. Use a number line if the student needs a visual picture of the value of numbers. If she needs practice with spotting the number of dots instead of the numerals, have her pull dominos out of a bag and compare.

GENERALIZATION INTO DAILY ACTIVITIES:

- When the student is counting two groups, emphasize how much faster it is to *count on* instead of starting all over again from the beginning. Games using two dice provide good examples of the efficiency of *counting on.* Have the student look at one die and spot the number of dots there. Then have her *count on* with the dots on the second die. You can always have her check the answer by counting each of the individual dots.
- If the student is already counting dollar bills, you can start her with a five- or ten-dollar bill and count on with any additional dollar bills. She does not have the individual bills to count when she has the five- or ten-dollar bill to begin with. Thus, she must say "10" for the ten-dollar bill and count on each single bill as 11...12...13....

ADDITION NEEDED TO PUT GROUPS TOGETHER

OBJECTIVE: The student will indicate that addition is needed when two or more groups are put together.

MATERIALS:

- An assortment of small, identical items that can be grouped together, such as craft sticks, rocks, straws, counters, buttons, large cereal pieces, raisins, and toys
- Two 4" x 6" index cards, one with a big plus sign and the other with an outline of a stop sign.

SUCCESS STEP: Show the student the plus sign and the stop sign and ask her to show you the stop sign.

PROCEDURE:

1. Show the student the plus sign. Tell her that it is used for addition or putting groups together. You may tell her that it is two little straight lines that cross each other and then make a plus sign with your index fingers. "This is called the plus sign." Ask her, "What is this sign called?" Show her both the stop sign and the plus sign and ask her to point to the stop sign and then the plus sign.

2. Make up some stories that will appeal to the students about two small numbers that must be combined. For example: Two boys cleaned out their jacket pockets. They both found some pennies in their pockets. One boy said, "Let's see if we have enough pennies to buy some gum. Let's see how many pennies we have together."
3. Give the students copies of the plus sign and the stop sign. Have them listen to the stories and indicate which sign is needed. They do not have to add the numbers together at this time. They just need to indicate whether the plus or stop sign is indicated. Of course, the students will choose the plus sign for addition because the stop sign is not relevant to the situation. Jan Semple (1986) has the students chant, "Numbers together are plus, plus, plus." She has the students cross their fingers into a plus sign three times.
4. Repeat the above procedure, making simple stories about some of the following items. If the students do not already know the word "addition," make sure you use it often when discussing these activities. For example, say things like, "Let's do some more addition." or, "Who knows which sign we should use for this addition problem?"
    - two colors of poker chips
    - two students combining pencils
    - two groups of sticks
    - two groups of pennies
    - two groups of spoons
    - two groups of trading cards, beanbag animals, Barbies, or anything else highly motivating to your students
5. To make sure that your student(s) can distinguish the plus sign from the stop sign, slip in a problem that can use the stop sign. Example: when the car is coming up to the dog in the road, what sign do you use?

ADDING REGARDLESS OF THE ORDER OF THE NUMBERS

Students need to recognize that they can *add* numbers in any order. Especially when doing addition with the calculator, the numbers can be entered in any order, as contrasted with subtraction, where the largest number must be entered first.

OBJECTIVE: The student will be able to demonstrate that the order of two numbers does not matter when adding.

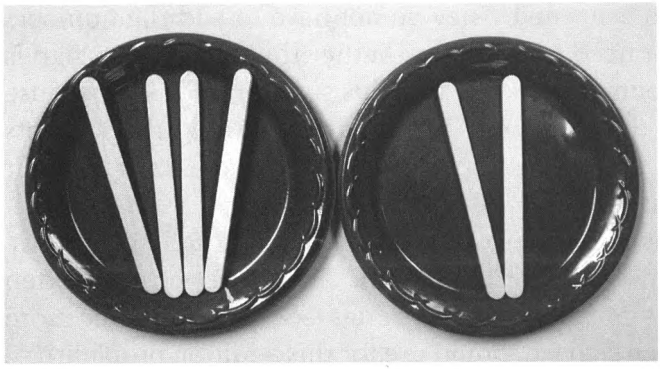
MATERIAL:

- Two paper or plastic dinner plates of differing colors
- Small craft sticks, pretzels, unsharpened pencils, counters, or straws
- Post-It notes and marker
- Paper and pencil
- Teacher's calculator (for a group, it is useful to use an overhead projector and calculator such as Calc-U-Vue, available from Learning Resources)

SUCCESS STEP: Place the two plates about a hand's width apart. Ask the student to put 4 sticks on one plate and 2 sticks on the other plate. Praise the student for correct actions.

PROCEDURE:

- Review the number of items placed on each plate during the success step. Move the plates close together. Count the total number of sticks (6). Then move the plates back to their original positions.



- Pretend that you are a magician and move the plates to opposite their original positions. Ask, "Do we have the same number of sticks?" Move the plates together and count the total. "The total number of sticks is the same, even when I move the groups around." Explain that "total" means how many there are altogether, if necessary
- Write the appropriate numerals (4 and 2) on Post-It notes and fasten them to the appropriate plates. Say, "See, I have written a number on each plate that matches the number of sticks. Now if I move these plates around, are the numbers the same? Is the total the same?" Repeat as necessary.
- Have the student repeat the above with 3 objects on one plate and 5 on the other. If she can write the numbers on the Post-It, have her do it. Otherwise, you write the numbers on the notes and have the student stick them on the plates.
- Have the student act like a magician and move the plates around and call out the total.
- Write the same numbers on a piece of paper as addition problems: 3 + 5 = __, 5 + 3 = ___ . Point out the plus (+) sign that you have previously introduced. Point out the equals sign and explain that it means that the answer number is coming next. The sign is called the *equals* sign. Have the student write in the total for each problem. To reinforce vocabulary, say, "That's right. The ***total*** is 8. After the *equals* sign comes the number that is the *total*."

CALCULATOR ACTIVITIES:

- When you introduce the plus and equals signs, show the students where those signs are on the calculator. Ask them to touch the plus or equals sign when you name it.
- Model doing simple addition problems (numbers under 10) on the calculator. Do some problems that are the reverse of others. Show your student(s) that the answers are the same regardless of the order. Write some of the problems you have modeled on paper and let the students try to solve them with the calculator.

Practice with Addition of Numbers (Sums of 12 and Under)

Concrete learners need to have many experiences with small numbers so they get a visual picture of what addition of these numbers means. It is important to use simple numbers so the student can concentrate on the process of addition. If plenty

of concrete experiences are given, the student may learn the addition facts with sums under ten without being drilled on them.

Families sometimes ask if they should discourage their child from counting on her fingers. My answer is no, at least not initially. Fingers are always with us. Having five fingers on each hand is probably the reason for our base ten system. Almost everyone counts on their fingers at one time or another. However, if the students are older when they are learning to count and add, using fingers to add could be considered "babyish." Instead, you could suggest that they tap on their desks, one finger at a time. If the motion is very slight, others will probably not notice or will think that it is just a mannerism. Or, you could teach them to just move the fingers on the open hand slightly, and it won't be considered inappropriate. Counting on fingers is inconvenient for larger numbers regardless of the system used, so students should be encouraged to use the calculator for large numbers.

PRACTICE, PRACTICE, PRACTICE

OBJECTIVE: The student will say or indicate (point, etc.) the total number of items in two groups.

MATERIALS:

- An assortment of small identical items that can be grouped together, such as sticks, rocks, straws, counters, buttons, large cereal pieces, raisins, and toys
- Calculators for teacher and each student
- Two paper or plastic plates

SUCCESS STEP: Have the student put some of the items on each plate. Praise her.

PROCEDURE:

1. Using some of the small items suggested in the Materials section, put a few on each plate and have the student indicate what process or sign (addition) is needed to find the total. Then find the total number of items by counting on. Repeat until students become confident in visualizing the groups being combined.
2. Reinforce the visual sense by writing the numbers of items on Post-It notes and putting them on the appropriate plates. Then write the addition problem on paper or on a slate so the student sees the concrete items and the numerals. When writing the problems, make a number sentence by writing the numbers horizontally: $4 + 5 = 9$.
3. Give the students as much experience as possible with combining concrete items, labeling the groups with numbers, and finding the total.

4. Next, demonstrate checking each problem by calculator. Tell the students to punch in a number, look at the number in the calculator window to make sure it is correct, and then put in the + sign. Next, punch in the next number, look at the window to check, and end by putting in the = sign. Practice doing problems with sums less than 12 on the calculator. Soon you may see that the student can indicate the correct total by memory from practicing on the calculator.

ADDING WITH A NUMBER LINE

If your student is included in a class that is learning to use a number line for addition, it would be appropriate for her to learn that skill too.

On the number line, the student will touch the largest number that is to be added. She will then count forward on the number line for the amount of the other number to be added. The last number that she touches will be the sum or total of the two numbers. It is important that students understand that they are ***not*** to count the number that they start on. Usually the teacher explains that they are counting the number of hops that are taken, not how many numbers there are.

If a student makes mistakes in counting because she counts the first and last number, give her a green button (or similar marker) and tell her this is her frog. Put the frog on the numeral representing the largest number and ask her to count the jumps the frog makes (which represents the second number to be added).

You can give her additional practice by counting bunny hops or hurdles jumped over by a runner or anything that emphasizes counting hops rather than the numerals on the number line.

Dan never seemed to get the correct numbers when he used his number line. The volunteer who worked with him asked him to show her how he counted, explaining what he did out loud. Dan put his finger on the little slash mark that was labeled 2. He then counted each slash as another number in the counting sequence. Thus, 2 + 3 = 4.

The volunteer said, "You are counting each slash mark. We are really counting jumps or giant steps. Here is a pretend frog. Put him on the number 2. He needs to jump 3 times. Count his jumps instead of the slash marks." Dan did, and came up with the correct answer of 5.

COVER UP GAME

MATERIALS:

- Two dice
- Cover Up game board for each student (Appendix B-13)
- Bingo tokens or pennies (12 per student)

PROCEDURE:

1. Each student should have a game board number line with squares for the numerals 1 to 12. Each player gets a game token or a penny to put on the number-one square.
2. The first player shakes the two dice and calls out the number on each die. She then must add those two numbers to get a total. For example, one die shows 3 dots and the other die has 5 dots. The player says, "This die has 3 and the other die has 5 dots. 3 + 5 = 8." (She may use counting on or a calculator.)
3. The player puts a marker on the square of the game board that matches the total (e.g., on the number 8 of the game board). An adult or a peer who thoroughly

knows her facts should be there to make sure the answer is right. Then the next player takes a turn shaking the dice and covering a square on her own game board.

4. At first, let the students use the calculator if they are not sure of the answer or if they give a wrong answer. If some of the students are not able to learn these simple addition facts, allow them to use their calculators all the time. They must get the correct answer the first time they work the problem on the calculator or they do not get to move their game token.
5. If a player does not say the problem (e.g., "three plus five equals eight"), she loses her turn. Players must say the problem and do the addition even if they can see that the answer is a number they already have covered.
6. If a number has already been covered on a player's game board, the player cannot cover any more numbers, and the next player takes a turn.
7. The game ends when one player has covered all her numbers. If time is limited, the player with the most numbers covered after the time is up is the winner. When the game gets to the point where all but one square is covered, the player will probably be throwing the dice many times until she gets the right combination. At this point, she does not have to say the problem and do the addition each time. She just continues throwing the dice until she gets the right combination of dots.

Place Value in Addition

Since this book advocates teaching students to use a calculator earlier than is customary, it is important to make sure they are not just blindly punching numbers in, without understanding why. For instance, when using calculators, students will find little difference between the addition of one-digit numbers and the addition of two-digit numbers. That is, the process of entering the numerals in the calculator is the same as long as they understand the place values of the numbers. It is important that they do know the place value for each number they enter in the calculator, however. When they are putting in the number 23, the 2 represents 2 *tens* and the 1 represents 1 *unit* (ones).

There are several reasons why understanding the concept of place value is important. First, we are emphasizing the *meaning* behind the use of addition. Mechanically operating the calculator is not useful if the individual does not know how to use the skill in a functional way. Second, understanding place value helps students to have a sense that the answer they get is reasonable, and they may be able to represent a visual picture in their minds of *tens* and *hundreds* (perhaps as bundles of straws or sticks from your hands-on instruction.)

INTRODUCING THE CONCEPT

OBJECTIVE: The student will be able to demonstrate the place value of ones and tens with concrete objects while adding two-digit numbers whose sums are under 50.

MATERIALS:

- Clothespin Box (1 shoebox per student marked **TENS** on the right half of the

long side of the box and **ONES** on the left half of the box.
- 40 or more colored plastic clothespins
- The cardboard holders that the clothespins came on, cut to hold only 9 clothespins. (If no holders are available, cut cardboard strips about one-inch wide and long enough that they will only hold 9 clothespins.)
- rubber bands

SUCCESS STEP: Have the student put 22 clothespins on the teacher's place value box (two groups of 10 pins and 2 single clothespins). Praise her.

PROCEDURE:
1. Ask the student if she can put the group of 22 together with the group of 13. Can she add them? Assist the student to see that now she has 3 groups of 10 clothespins and 5 single clothespins. How many all together? Check the answer with the **calculator.**
2. Repeat with groups of 21 + 24 and other two-digit numbers as needed and check with the **calculator.** (At this stage, be sure not to have the student add two numbers that require regrouping, such as 29 + 13.
3. You may want to have the students do some worksheets with single- and double-digit addition problems. First have the students use the clothespin box to illustrate the addition. Use only numbers under 30. Check each problem with the calculator. When the student does several problems accurately, you can have her use the calculator only.

Skip Counting Review

Now that your students have a beginning understanding of place value, it may be a good time to revisit the skip counting by 5s and 10s introduced in Chapter 8. See if your students are able to remember how to count by 5s and 10s. Then show them that skip counting is actually adding the same number over and over again.

1. Have the students count by 10s on their calculators several times. Start them out putting in the number 10, then press + and put in 10 again and say, "10 more is 20," looking at the answer on the display. Pair up students and have them take turns counting by 10s on the calculator, with partners checking the display for the right answer.
2. Begin the next few sessions by challenging the students to count by 10s with the calculators while you watch them.
3. Next, have the students count by 5s on the calculator several times. Start by putting in the number 5, then press + and put in 5 again and say, "5 more is 10," looking at the answer on the display. Pair up students and have them take turns counting by 5s on the calculator with their partner checking the display for the right answer.
4. Give the students plenty of practice on the 5s. They should be able to see the pattern of the ending digit being 0...5...0...5 from the number lines or the 50 or 100 chart.

Addition Facts (Optional)

With the approach used in this book, the student does not have to learn the addition facts if she knows how to set up the problem on the calculator. Some students will always rely on the calculator. However, the addition facts are very useful for doing mental math or estimating when you don't need an exact answer. Some students will learn some combinations just from the repetition involved in learning about addition. Some strategies for memorizing the facts will also be given below, but learning them is not a necessity for doing addition.

Traditional math textbooks often require students to know the facts before going on to learn uses of addition. In this book, however, students can be learning the addition facts at the same time they are using addition in practical situations.

The optional addition facts will be addressed in the following areas:

- Beginning addition facts
- Addition facts adding up to ten
- Patterns in addition facts
- Doubles
- Difficult facts

BEGINNING ADDITION FACTS

OBJECTIVE: The student will memorize the addition facts with sums up to 5 (later up to 10). (Do only as much of this objective as you see the student can handle. It is a rote memorization task that may be difficult or almost impossible for some learners.)

MATERIALS:

- Addition flashcards or cards you have made with the addition facts on them
- Paper plates
- Craft sticks or straws
- Marker
- Post-It notes

SUCCESS STEP: See if the student can tell you what 1 + 1 is when you illustrate with your fingers.

PROCEDURE:

1. Using flashcards or your written problems, find out what facts with sums up to 5 the student really knows. Students often know simple facts such as 1 + 1 = 2 and 2 + 2 = 4 from their daily experiences. You may already know what facts are known from doing the previous activities.
2. Using the concrete activities with craft sticks and straws that have been used for previous activities on addition, have the student work out the problems using the numbers from 1 to 5 (1 + 1, 1 + 2, 1 + 3, 1 + 4, 2 + 2, 2 + 3). For example, have her put 2 sticks on one plate and 3 on the other, then count or add on to tell you the total.
3. After the student has successfully put the concrete groups together, bring out another plate to serve as the answer plate. The answer plate will not have sticks or

straws on it, but you will have written the answer or sum on it in large numerals. A Post-It note can also be used here.

4. The student will now see two plates with sticks or straws on them showing the numbers being added. She will also see another plate with the number representing the sum written prominently on its surface. You may also make a card with the equals sign (=) so you can show the entire number sentence as the student will see later (3*** + 4 **** = 7).
5. After you have shown the student all the combinations using the numbers 5 and under, you can make 5 answer plates (numbered 1 to 5) for her to choose from, so she can place the correct answer plate down in the number sentences.
6. Transition to writing numerals in the number sentences by gradually replacing the sticks with a Post-It containing the appropriate numeral. For example, the student has one plate with 2 sticks and one plate with 1 stick. She can tell you that that equals 3. You then take away the one stick and put down a Post-It note with a 1 on it instead and ask, "Now how much is it?" See if she can remember that 2 sticks plus the number 1 equals 3. If she can, try putting down a Post-It note for the 2 sticks too and see if she can remember that 2 + 1 equals 3.
7. Using flashcards, see if the student has learned any of the addition facts from the previous concrete activities. Repeat the concrete activities with sticks or straws for those facts that are missed.
8. Once the student can remember addition facts with sums up to 5, work on the facts with sums up to 10 in the same manner.

DOUBLES IN ADDITION

As the name implies, doubles are math facts in which you add two of the same number (e.g., 2 + 2, 8 + 8). Learning the doubles of up to 9 + 9 can be helpful. It seems to be easier for students to learn the sums of two numbers than to learn other fact combinations. Some theorists say that it is because the student doesn't have to remember three different numbers, since two of the numbers are the same.

OBJECTIVE: The student will be able to give the answer to the doubles facts (1–9), or as many as possible.

MATERIALS:

- A number line
- Straws or flashcards
- Dice
- Dominoes

SUCCESS STEP: Have the student throw two dice until she gets doubles. Praise her. Say, "You got doubles! See, you got two of the same number."

PROCEDURE:

1. Continue to have the student throw the dice until she gets doubles. Count up the dots on the dice. Write the numerals on paper or on a chalkboard or whiteboard.

Say, "You threw a 3 and another 3. They are doubles. How many dots in all?" Then write the answer on the board or paper. Continue as long as the student is interested.

2. To make sure that the student understands what doubles are in written numerals, write several number facts, mostly doubles but with a few that are not. Have her cross out the problems that are *not* doubles. (For instance, write 2 + 2..., 3 + 1..., 4 + 4..., 5 + 5..., 3 + 6..., 3 + 3... on the board or paper. See if she can cross out 3 + 1, 3 + 6.)
3. Mnemonics can be used (and created) to help students learn the doubles. For example:
    - A car has 2 wheels on one side and 2 wheels on the other, which makes 4 wheels altogether. Have the student look at a car or picture that shows all 4 wheels of a car so she has a visual picture of that common combination of numbers.
    - Introduce pictures of insects (such as grasshoppers) that have 3 legs on each side for a total of 6, or spiders that have 4 legs on each side for a total of 8.
    - Of course, the student should be able to picture 5 fingers on 2 hands, which equal 10 fingers total.
    - You can use an egg carton to show 6 + 6 = 12.
    - If you use 16-crayon boxes, you can show that there are 2 rows of 8 crayons each, so 8 + 8 = 16. Or you can have the students remember the phrase, "I ate (8) and ate (8) until I was sick-teen (16)."
    - For 9 + 9, you could sketch 9 boys and 9 girls on the "A team" (eighteen).

USING DOUBLES TO FIND ONE MORE, ONE LESS

If a student has learned the doubles facts well, she can sometimes use them as stepping-stones to other facts. She may be able to find *one more, one less* using those doubles. For example, if she knows that 3 + 3 = 6, she may be able to look at the combination 3 + 4 and think, "If 3 + 3 = 6, and this problem has a number that is 1 more than 3, then the answer will be 1 more than 6, which is 7." To do this, the student needs to be able to visualize two equal groups plus one more.

OBJECTIVE: The student will learn to visualize two equal groups plus one more.

MATERIALS:

- Poker chips
- Calculator

PROCEDURE:

1. Give the student an even number of poker chips (4 to 20).
2. Have her make two groups of chips with the same number in each group. Ask her to tell you the total if she knows the doubles fact, or to find the total by counting or by using the **calculator** to add the two groups.
3. Then add one more chip to one of the two groups of chips and ask what the total is now. If the student is correct, do many more concrete illustrations with the chips. If the student does not understand what number is one more than the total she originally counted, go to Chapter 8 and review *one more* using the number line.

This strategy requires more mental math than many concrete thinkers can easily do. However, students who are really secure with doubles facts may be able to use this strategy. You can call the numbers on both sides of a number its *neighbors.* Using the doubles as bases, the student can figure out what the sum would be if the numbers were doubles (e.g., the problem is 5 + 6, so the student figures out that 5 + 5 = 10). The student could then add 1 more to that sum to account for the *neighbor* (6) that is one more than 5. If the student is successful with using the *one more* strategy for doubles, she may be able to use the *one less* strategy (e.g., thinking of 5 + 6 as being one less than 6 + 6). That seems to present a more difficult memory problem for most concrete learners, however.

Addition Facts Adding Up to Ten (Optional)

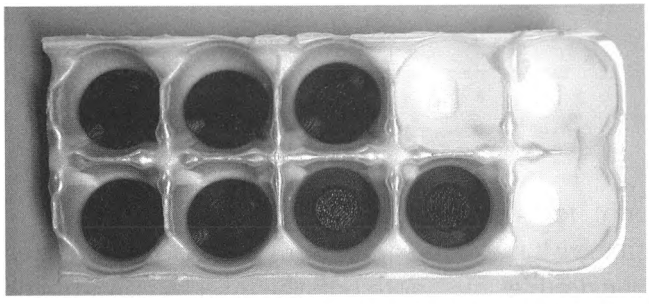
Many adults do not realize that they use the pairs of numbers adding up to ten (complements) when they are doing mental math. Think of the numbers 4 and 6. Perhaps they are easy for you to add mentally because they add up to 10. Some people also use the complements and add one more or one less to mentally add numbers such as 4 + 7. That is, they think, "3 + 7 is 10, so 1 more is 11." This activity is designed to help students learn the pairs of numbers that add up to ten, although they can also play by counting instead of remembering addition facts.

GOTTA BE TEN GAME

OBJECTIVE: The student will be able to tell the pairs of numbers that add up to ten (complements).

MATERIALS:

- An egg carton that has the top two pockets cut off so that there are ten pockets left. (You may want to outline the top of each pocket with a black marker to make the individual pockets stand out.)
- 20 poker chips, 10 each of two different colors
- Dice



PROCEDURE:

1. Set out the egg carton and the poker chips.
2. Have the first player throw the die and call out the number. The teacher should put that many of the same color chips in the pockets of the egg carton. Put the chips in adjacent pockets, not scattered apart, to make a more memorable visual picture and to make counting easier for the players.
3. Say, "How many more chips do you need to make 10?" Have the player count how many empty pockets are left. That will be the number of chips that are needed to make ten. Have her put in the correct number of chips in a different color. For many students, using one color for the first number and another color for the complement will assist the learning.

4. Let the other players, if any, roll the dice and count how many more chips they need to make 10.
5. After the combinations have been counted out several times, some of the students will be able to say, "I got a 2. The other number will be 8," without having to count the empty pockets. They are learning the combinations that add up to ten. If you wish to keep score, award a point each time the student gets the other number of the combination right, no matter how she gets the answer. You might want to eventually give two points if the student gets the number right without counting.
6. Some students will have difficulty remembering these combinations because of their problems with short-term and working memory. Let them feel as if it is perfectly okay to count the empty pockets of the egg carton each time. They will usually try to memorize the combinations because it makes the game go much faster.
7. The combinations will be easier to learn if students play the *Gotta Be Ten* game for a few minutes each day. If they do not make progress over a week or two, it is probably better to go to the calculator and not worry about learning the complements at this time.

Patterns in Addition (Optional)

At the same time you do concrete addition activities with students, you can work on helping them memorize the addition facts that follow a pattern. I call these **Shortcuts to Addition.** When students learn the patterns, they have many facts learned quickly. The patterns used most commonly are:

- *Any number plus 0 equals that number.* (9 + 0 = 9)

  Discuss the principle that if you add nothing to any number, it remains the same number. Do the page on *Shortcuts to Addition Facts* (Appendix B-14) several times to make sure the student has the principle securely.

- *Any number plus 1 is just one number higher on the number line or in the rote counting sequence.* (5 + 1 more = 6, or 1, 2, 3, 4, 5 + 1 more = 6)

  Discuss the principle as an example of *one more,* which was learned earlier. Use the number line and the rote counting sequence to go through adding 1 to numbers from 1 to 12. Do the bottom section of the *Shortcuts to Addition Facts* sheet as many times as needed. The student may write the answers or give the information orally.

- *Any single-digit number plus 10 just needs a 1 (tens) in front of the other number.* (For 6 + 10, just put a 1 in front of the 6.)

- *Any single-digit number plus 9 will result in a sum in which the last number is one less than the original number.* (Example: for 9 + 7, you know the last number will be one less than 7, or 6. The entire answer is 16.)

Using a number line, you might ask the student to put a finger on the number that is not 9 (e.g., for 9 + 5, put your finger on 5). Then ask what is *one less* than that number. The student writes down the number one less in the *ones* place and then writes a one in the tens place to get the answer (14). This is quite a difficult concept so you may have to reteach it after the student has learned other facts. It may not be worth the effort to teach the shortcut to add nines if rote memory is a problem because each answer requires two steps, making the task more difficult.

If the student has learned how to add 10 to any number, she may be able to figure what 1 less will be when adding with 9.

A Word about Worksheets and Written Work

Once students reach this point in their mathematics learning, they will increasingly be called upon to solve written math problems. It is worth putting some thought into how the problems are presented on the page, as well as the amount of writing that is expected of children with Down syndrome, autism, or other disabilities.

Many students with Down syndrome and some students with autism spectrum disorders or other disabilities have low muscle tone. Especially when they are young, their writing hands tire quite quickly. They may also have problems just forming the numbers correctly. If they are asked to copy down problems before solving them, it often takes a lot of time and is physically tiring. When students are learning the process of addition, it is wise to have them concentrate on that learning and not on the copying process. Using preprinted worksheets or having the practice work photocopied onto the students' papers will help them focus on the math skills they need to learn. Consider adding this accommodation to your student's Individualized Education Program (IEP) at school (if she has one) so she can focus on math, rather than fine motor skills, when completing written work.

When students are first learning to solve addition problems written vertically rather than horizontally, I suggest printing the problems on graph paper with large squares, one numeral per square. Tell the students that it is easier to add the *ones* with the *ones* and the *tens* with the *tens* when the numbers are written underneath each other, lining up the similar units.

Adding Larger Numbers with the Calculator

Students need to get a great deal of practice using the calculator for addition. The mechanical process should be almost automatic so the student can concentrate on problem solving. Students will probably need more practice adding with the calculator than can be given here. There are some excellent mass-market workbooks that give practice with addition, and most also provide the answers. You will find these workbooks labeled for kindergarten to grade three. If your student is beyond those grades, you may need to cover up the words that show the younger grade label.

The process of learning to regroup (*carrying* when the sum is over 10) is usually a major step in beginning addition. If your student is progressing satisfactorily in general classroom math, you may want to see if she can understand the concept of

regrouping. Use the materials introduced above to show concretely how to make 10 *ones* become 1 *ten* to be added to the existing *tens*. However, most concrete thinkers will do better using the calculator for adding numbers beyond 9. With the calculator it does not make any difference if the addition involves regrouping *(carrying)* or not. For these students, you should give more attention to helping them learn to enter the numbers accurately and understanding when to use addition.

TWO- AND THREE-DIGIT ADDITION

OBJECTIVE: The student will be able to accurately add two- and three-digit numbers with the calculator.

MATERIALS:

- Copies of math worksheets (Appendix B-15)
- Pencil
- Calculator

SUCCESS STEP: Give the students the first problem on the worksheet and go through the steps with them so that they are successful.

PROCEDURE:

1. Ask the student(s) to try the problems alone. Make sure they are not racing through the problems on the worksheet. Accuracy is much more important than speed!
2. Bear in mind that when adding multiple-digit problems, some students may have to touch each number on the paper as they enter it in the calculator. Other students will need an index card or straight edge to hide the other distracting numbers on the page. You might also need to highlight (or have the student highlight) all of the units (ones) with one color, the *tens* with another color, etc. The highlighting is done before the student starts to enter the problem in the calculator and is faded when the student seems to have internalized the place value concepts. Try some of these adaptations if a student is having difficulty knowing which number to enter next or loses her place in the worksheet frequently.
3. Have students check the accuracy of each answer by redoing the calculation. If possible, ask them to put the numbers in the calculator in the opposite order from their original entry. You will be able to see if they can apply the principle of commutative property (order of items not important in addition).

You can give each worksheet several times. You can also change one number of each pair to make different worksheets with different answers.

MULTIPLE ITEM ADDITION

Adding up several items to get a total is a more difficult task than adding just two numbers—even with a calculator. There are more chances of making an error with multiple numbers, and students often have trouble keeping their place in the list.

OBJECTIVE: The student will be able to accurately add multiple one- and two-digit numbers with the calculator.

MATERIALS:

- Addition of 3 single- and double-digit number worksheets (Appendix B-16)
- Pencil
- Index cards

SUCCESS STEP: Ask the student to show you the addition and equals sign on her calculator. Praise her.

PROCEDURE:

1. Have the students go through the first problem on the worksheet with you. Use an index card to cover up all but the first number to be entered into the calculator.
2. Teach the students to start with the top number. Slide down the index card to uncover one number at a time.
3. Recheck the answer by adding the numbers in reverse order.

Redo the worksheets at a different time. Make other worksheets with three or more numbers to add if more practice is needed.

GENERALIZATION INTO DAILY ACTIVITIES: Adding multiple items is a very useful skill that can be used in many ways in everyday life:

- Shop for several items and have the student add up prices in even dollars (e.g., $5.25 becomes $5).
- Play games such as Scrabble, Hearts, or darts where you get points for every round and have the student add up the scores.
- Miniature or regular golf requires the addition of small numbers for each hole.

LARGER NUMBER ADDITION

When students can accurately add two- and three-digit numbers, they need some practice in adding even larger numbers with the calculator accurately. They should learn to accurately add larger numbers so that they will be able to use addition with dollars and cents when shopping, banking, and budgeting. There are several worksheets with three- and four-digit numbers in Appendix B-17. For additional practice, make your own worksheets (and figure out the answers). You can find workbooks for early math in the coloring book section of most grocery or discount stores.

You may be able to find Internet sites that have sample worksheets or freeware on addition. For example, www.funbrain.com has a variety of games using math. Also, www.kidsdomain.com has download sites for addition and subtraction, as does www.learningplanet.com (e.g., Spacey Math Game).

Addition Word Problems

While students are learning fluency in using the calculator, we want to make sure that they understand when to use the addition process. In this section, we will therefore teach a process in problem solving. We will progress from using concrete objects to using semiconcrete representations to using abstract numbers. Students will learn to draw math problems—first with pictures and later with stamps, tally marks, and numbers.

USING OBJECTS OR ALREADY-MADE PICTURES TO SOLVE ADDITION STORIES

The easiest way to solve a word problem is to use the actual objects that are to be added together. After the student has manipulated the actual objects that are to be added, she may be able to see the parallel in counting pictured objects on a page. The objects pictured should be exactly the same at first. Later, progress to using pictures with slight differences, such as one sheep looking the opposite way from the rest of the group, or people who are wearing different colored hats, etc.

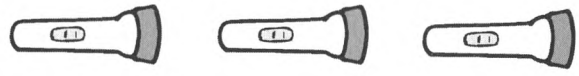
OBJECTIVE: The student will read simple story problems that include pictures and use the pictures to assist in addition.

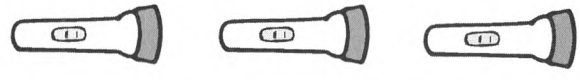
MATERIALS:

- Illustrated addition stories from Appendix B-18
- Mass market workbooks (such as *The Complete Book of Math,* McGraw-Hill Children's Publishing)
- Websites—If you are teaching several students, you may want to use the material on www.mathstories.com. This site has some free math materials but requires payment for the story problems. The advantage is that the site supplies pictures for beginning story problems, uses fairly large print, and explains the problem situations very clearly. Or try the free worksheets with story problems at www.schoolexpress.com/#worksheets
- Pencil

PROCEDURE:

1. Find math word problems that have few words and clear pictures. For example:

Bob has 3 flashlights. 

Ann has 3 flashlights. 

How many flashlights do they have altogether?_____
(or, total number of flashlights: _____)

This type of addition story problem requires that you understand that putting items together requires addition. You will also be using the word *total* for the phrase

"how many do you have altogether?" as an alternative. Use as many of the already illustrated math stories as are needed to make the transition from concrete objects to pictures. Encourage your student(s) to count the pictures if she does not know the appropriate addition fact.

DRAWING PICTURES OF A MATH STORY

You can make a transition from using already illustrated math problems to having the student draw pictures of the items that are to be added. You can have her draw very simple pictures such as balls or cats. However, sometimes the time needed to do the drawing distracts from the math learning. Using a simple stamp or a bingo marker saves time and usually makes the task more fun for the student. For this activity, we will therefore have students make semiconcrete representations (drawing or stamping).

OBJECTIVE: The student will be able to represent (draw or stamp) items in simple addition story problems and solve for the total.

MATERIALS:

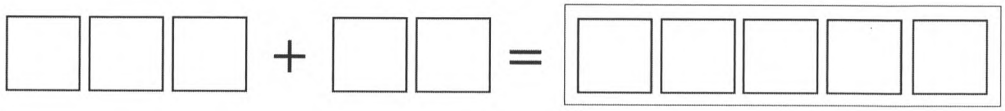
- Story Problems to be Explained from Appendix B-19 and those made up by the teacher or parent
- Washable markers
- Washable ink stamps made for children and washable ink stamp pads)
- or Colored Bingo markers (for older students)
- or Stickers
- Paper

SUCCESS STEP: Have the student show you how she can make a circle or mark with the markers or stamps on paper.

PROCEDURE:

1. Go through the first story problem orally (without pictures) and make sure the students understand what is wanted. Repeat the numerals and write them on the chalkboard or on a tablet of paper that they can see.
2. Have the students illustrate the first story problem by making a drawing or stamp for each one of the items in the story problems. Have them make the marks in a straight line across the paper with a space between the groups of objects to be counted (e.g., make 4 butterfly stamps, space, and then the next 3 butterfly stamps). Make sure that they do not start making other marks or drawing other objects. It may be wise to give them a blank sheet of paper to experiment on with the stamps or markers before you start with the math problem.
3. Ask the students what operation (process) they need to use when they are putting items together or finding a total. Have them put the operation sign (+) between the two groups of stamps on each problem. Then let them count the marks or stamps to find the total. Then they must check the answer with the calculator and write the answer on the paper.

4. Write or type another of the suggested addition story problems on the top of a piece of paper. See if the students can make the stamp marks, count them, and check the answer with the calculator independently. Continue giving them simple addition story problems until you feel that they are able to do the process carefully and well. (They will probably be more motivated to solve your problems if you use their names in some of them.)
5. For students who learn best with visual teaching, you may want to set up a structure for solving an addition story problem. Use a graphic organizer such as connected boxes to show the addition story problem. For example:



If this graphic organizer is already on the paper, the student only has to put a stamp or drawing in each box to solve the problem. The graphic organizer is just a teaching device that can be dropped when the student is more competent in doing story problems.

Wording Your Word Problems

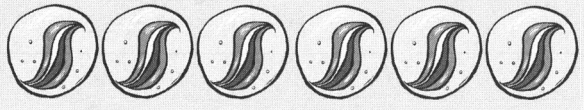
Be careful that you do not make up questions that change the normal word order of the sentence or use terms the students may not know. We are not testing their language comprehension.

At first, use simple declarative sentences that do not change the order of the sentence as "Wh" words do. For example, "He *had* ___ *boxes*" is easier to understand than "How *many boxes did he have?*" Many times students would be able to do the math required in a story problem, but the word order and the language of the problem are the barriers that impede success. The ability to understand the change in word order when making "Wh" questions comes developmentally late in a child's language learning.

Here is an example of a word problem that is written well for a child who has language delays:

Bob and Allison went looking for marbles in their backyard.

Bob found 3 marbles.

Allison found 6 marbles. 

They had 9 marbles altogether.

WRITING SIMPLE NUMBER SENTENCES

OBJECTIVE: The student will write one-digit number sentences for word problems.

MATERIALS:

- Pencil
- Word problems (including some problems previously done in above two activities)

PROCEDURE:

1. You have introduced adding single digits with the problems arranged horizontally in what we call a number sentence. Check to see if the students know the label *number sentence* for problems such as 6 + 9 = 15. Model making *number sentences* for the students using some of the story problems with already made pictures you used previously. Say, "I write the 3 here to stand for Ted's 3 flashlights. I write the 2 here to stand for Anna's 2 flashlights. If we want to know how many flashlights are here altogether, we have to add (as you draw a plus sign). The total (write the equals sign) is 5. That is a number sentence."
2. Have the students make up number sentences for other problems. Begin by using some of the problems they already worked out with pictures in the activities above. That way, they can count the objects to be added and write down the appropriate numbers. You may want to give them a form for a model such as:

   ____ + ____ = ____

   If you are working with several students, have each one make up simple problems to give the others to make as number sentences.
3. Repeat the above process until you are confident the students understand what to do to make a number sentence from the very simple one-step story problems. Later have them make number problems from any real situation or story problem. Making number sentences is not an easy concept. You may not get complete understanding at this time. However, as you teach them about the use of addition for solving problems, keep reminding them, "Let's make a number sentence (equation) out of this before we solve it with the calculator."

GENERALIZATION INTO DAILY ACTIVITIES: You want to make sure that when the students are faced with actual situations requiring addition, they can make visual pictures to illustrate them, set up addition problems, and solve them accurately with the calculator. Parents, you have the best chance to make sure that your child can generalize these skills into daily living tasks. Teachers will have to look at possible ways to use addition during the school day.

- When baking cookies, cupcakes, or muffins, keep track of how many you've made each time you take a pan out of the oven. For example, the first pan had 10 cookies; the second pan had 12 cookies on it. How many total cookies did you bake?
- Play darts, toss a beanbag at a target, or play some other game where you can earn different point scores. Add up your score as you go along.
- At school, if the class is working toward a reward by earning a certain number of points or tokens for accomplishments, you can add the number of points

earned to the previous total to see how many you have now. You can do the same at home if your child has a reward chart or is saving her allowance.

- Count how many items (sports cards, seashells, video games, etc.) you have in a collection and write down the total. Each time you add a few items to the collection, add that number to the total instead of counting each item.

CHAPTER 10

Whole Number Subtraction

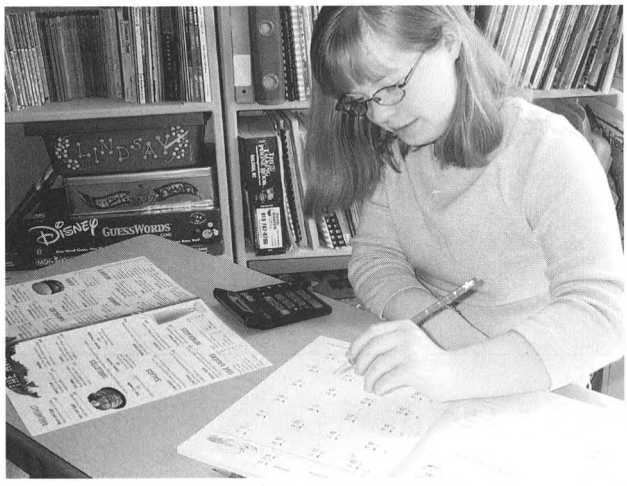
Pre- /Post Checklist

Can the student:

1. ____ Use traditional take-away subtraction.
2. ____ Compare two sets of items and say which set has more items and how many more.
3. ____ Use subtraction to see if he has enough money to buy something, and, if not, how much more is needed.
4. ____ Tell how much change he should get back when he pays more than the cost of the item.
5. ____ Demonstrate with concrete materials how subtraction is the reverse of addition with three fact families.
6. ____ Write or recognize the correct way to enter a subtraction problem when the numbers are given orally (largest number on top or entered first in the calculator).
7. ____ Unbind a tens group of sticks (yielding single-digit sticks) in order to have like units to subtract from like units.
8. ____ Determine which form of subtraction is needed in a given situation.
9. ____ Use subtraction in simple, realistic word problems.

Subtraction is often taught along with addition so that students learn subtraction facts that are in the same fact family as the early addition facts. This book, however, recommends teaching the two operations at different times so that students who are concrete thinkers can thoroughly learn the addition process before moving on to subtraction.

As mentioned earlier, you may want or need to follow your school district's sequence for introducing addition and subtraction if your student is included in the general classroom for math. You might have to do the addition problems with answers under 12 and then skip to the subtraction problems that use those fact families. Use both the addition and the subtraction chapters in a similar way to help the student stay with his class in math. If the student is older or his classmates have already finished addition and subtraction, though, you can work on each process separately.



Take-away subtraction is the most common use for subtraction. Subtraction is also frequently used for comparisons of two groups where nothing is taken away from either group but the difference is calculated. For instance, two uses for subtraction that are often encountered in shopping situations are 1) "How much more is needed?" and 2) "What change should be given back?" Students who are hands-on learners usually have more difficulty with these types of subtraction than they do with take-away subtraction. Word problems in particular can be very difficult for them.

Although take-away and comparison subtraction are often covered at the same time, it is important that students treat them as separate situations. It is also essential that students immediately recognize each of these situations as requiring subtraction.

The following major areas are covered in this chapter:

- **Types of Subtraction**
  - Take-away subtraction
  - Comparison subtraction
  - How-much-more subtraction
  - Figuring change (currency only)
- **Subtraction in (Almost) Real Situations**
  - Determine type of problem
  - Do a simulation of a problem
  - Make a number sentence out of the problem
  - Use subtraction in real situations

Basic Principles of Subtraction

Children are usually aware of the process of subtraction without knowing that it is labeled *subtraction*. For example, they know that if they have four cookies and someone takes away two cookies, they have fewer cookies than they started with.

Subtraction is really the inverse of addition. If you know that $3 + 5 = 8$, you also know that $8 - 3 = 5$. Essentially, addition is bringing two smaller numbers together to form a larger number. Starting with that larger number and subtracting one of the

smaller numbers will yield the other smaller number. The relationships between those three numbers are what math books often call a "fact family."

Although some activities for learning subtraction facts are presented here, remember that students learning survival math skills do not have to rely on their short-term memory to do either addition or subtraction if they are able to use the calculator. Subtraction becomes more difficult when regrouping into different units or borrowing is involved, but the student can jump that hurdle by using the calculator.

WHAT IS SUBTRACTION?

The student has probably already been doing simple subtraction activities in his everyday life such as commenting, "Jason took two of my Legos! Now I don't have enough for a bridge. I had ten Legos and now I only have eight." You just need him to be aware of the process he is already using in small ways and to learn to express the process mathematically.

OBJECTIVE: The student will be able to write a number sentence for simple problems (10 or fewer) which use subtraction.

MATERIALS:

- 10 pennies
- Small slate or whiteboard
- Chalk or whiteboard markers

SUCCESS STEP: Give the student three pennies and ask him how many pennies he has. Praise a correct answer.

PROCEDURE:

1. Take away 1 penny from the 3 you gave the student and ask him how many pennies he has now. He will say 2. Then write on the slate board the number sentence that has been made. Say,

   "You had 3 pennies" (and write the number 3 on the board).
   "What did I do next?" (took away 1 penny).
   "I am writing a minus sign (–) to show that I took it away (or subtracted it)."
   "Then I write the 1 penny after the minus sign to show how much I took away."
   "Then I write an equal sign (=)." (already known).
   " And how many did you have left?" (2)
   "Yes, I write the 2 after the equal sign to show what you have left."

   3 – 1 = 2

2. Tell the student that this is a "take-away" or subtraction number sentence. Read the whole thing out loud for him. "Three minus one equals two."
3. Repeat the demonstration with 5 pennies and take away 1 or 2 and see if the student can write part of the number sentence. Progress to doing simple "take-away" problems with some of the 10 pennies.

4. At a later session, see if the student can make subtraction number sentences for all of the facts using 10 pennies. If he is unable to write the numbers, have him tell you which numbers to write. You can also write the numbers from 1 to 10 on index cards and have him point to them or arrange them into a number sentence.
5. Spend several sessions doing simple subtraction problems with 10 or fewer items and writing their number sentences. You may want to pair two students and have them take turns "taking away" objects from each other. (It can be done in a "funny" way. For example, give one student a number of objects he doesn't especially want, such as worksheets to do, and then have him invite another student to take some away or pretend that the objects are cookies and have one student take some—sneakily.)

Take-Away Subtraction

Unlike addition, subtraction is used for more than one situation. The most common use for subtraction is the type students usually call "take-away." Take-away occurs when some items are taken away from a larger group of items (9 - 5 = 4). Most of the problems that students will encounter in daily living are take-away situations.

SOCK EATER GAME

OBJECTIVE: The student will be able to write a number sentence for a simple take-away subtraction problem.

MATERIALS:

- 9 different-colored socks—none of them pairs
- A small box with one side cut out (can be a large shoebox)
- Index cards with 9 – 1 = 8, 9 – 2 = 7 and all the other 9-based subtraction facts

PROCEDURE:

1. Tell the story of the Sock Eater: Ellen washes some of her clothes in the washer and dries them in the dryer. Somehow, Ellen never seems to get as many socks out of the dryer as she puts in. When she tells her mother, her mother says, "A Sock Eater must be stealing away your socks!" Ellen laughs at the joke with her mother. However, later she thinks, "What if there *is* a Sock Eater in our laundry room?" She counts the socks that she puts in the dryer and she counts the number of socks that come out of the dryer. Here is what happens:
    - The first day she puts 9 socks in the dryer and she only gets 8 socks out of the dryer. The teacher acts out the story with the socks going in the box (dryer). He or she shows the one sock that was stolen, and then the teacher and class count out the number left. (The teacher may pretend to gobble up the one sock or put it behind his back while the students count the remaining socks in the box.) The teacher writes the problem 9 – 1 = 8 on a piece of paper or chalkboard so the students can see what the number sentence looks like.

- The teacher then takes all the socks out, shakes them, and repeats the story for the second day as he puts them back in the box. The second day the teacher (Sock Eater) takes another sock out, so now two socks are missing. He writes the number sentence $9 - 2 = 7$ and shows the students.
- The teacher continues eating socks until there are no socks left in the dryer or until the students get bored—whichever comes first. You can "ham" up the story to grab the students' attention and even wear a mask to better play the part of the Sock Eater.

LARGEST NUMBER FIRST

In addition, it does not matter which number comes first, but in subtraction the largest number needs to be given first. Whether you are writing the problem horizontally or in columns vertically, the largest number must come first or at the top of the problem.

OBJECTIVE: The student will be able to write or recognize the correct way to write a subtraction problem when the numbers are given orally.

MATERIALS:

- Paper and pencil or small chalkboard and chalk for teacher and student
- Paper plate
- Craft sticks or pencils

SUCCESS STEP: Show the student a simple take-away subtraction problem using sticks and paper plates. Put 5 sticks on a paper plate and then take some away. Ask him which is the largest number—the number you started with (5) or the number that is left after you have taken some sticks away. Praise the correct answer. Repeat if the first answer is not correct.

PROCEDURE:

1. Call out some simple subtraction problems, and ask the student(s) to write them or choose the correct equation from a set of index cards with the problems prewritten (horizontally) on them.
2. After practice with simple problems, show the student how the problems can be written vertically. Give him practice with solving problems written vertically. Remember to give the student graph paper if he has difficulty lining numbers up.
3. If a student has difficulty writing a problem, ask him which is the largest number of the numbers you called out. When he correctly identifies the largest number, tell him that is the number he needs to start with. If he has trouble understanding this, demonstrate with objects why you can't take away more than you have.

TAKE AWAY GAME

OBJECTIVE: The student will be able to demonstrate at least 5 one- and two-digit take-away subtraction problems (without regrouping) with concrete materials.

MATERIALS:

- 19 craft sticks or straws for each student
- Large paper plates
- A slate, whiteboard, or piece of paper for the teacher

SUCCESS STEP: Ask the student to put 11 sticks on his plate. Praise him for doing it correctly.

PROCEDURE:

1. Divide the group into pairs—either pairs of students or the student and the teacher as a pair. Have one student hold the large plate with 11 sticks. Have the other student (or teacher) be the grabber. The grabber takes away the number of sticks that the teacher calls out. For example, the teacher says, "Take away 1." The grabber takes away the craft stick, saying, "I am taking away 1." The teacher asks the other student how many sticks are left. The student says, "I have 10 left."

2. The teacher then writes the number sentence 11 – 1 = 10 on paper or a slate so the students can see the symbols that represent that situation.
3. Then the teacher says, "There are 10 sticks left. Take away 4 sticks." The grabber takes 4 sticks, leaving 6 sticks on the plate.
4. The teacher asks the students what she or he should write for the number sentence on the paper. Help them if they are not sure how to write it.
5. Reverse the pair of students and have the other person be the grabber. Make it a little like a game of take-away. Make sure the students do the dialogue as given in step 1. It will help them to learn the steps in subtraction with this creative practice.
6. Make up one- and two-digit subtraction take-away problems with and without regrouping. Give the students as much practice as necessary to visualize the take-away problems.
7. At this point, students should be ready to learn to subtract on the calculator. See "Subtraction with the Calculator" below. Have the student check the take-away subtraction problems with the calculator.

SUBTRACT ONLY LIKE UNITS (REGROUPING)

The important concept for the student to understand is that *ones* can only be subtracted from *ones, tens* from *tens, hundreds* from *hundreds,* etc.

OBJECTIVE: The student will unbind a group of tens in order to have like units (ones) to subtract from like units.

MATERIALS:

- The shoebox labeled ONES and TENS from Chapter 9.

- 9 individual clothespins and 20 additional clothespins grouped in *tens* on cardboard strips,

OR

- 9 individual craft sticks and 20 additional sticks bound together as two groups of tens

SUCCESS STEP: Have the student point to the groups of tens and to several single clothespins or sticks. Praise correct identifications.

PROCEDURE:

1. Put the number 14 on the Clothespin Box using one strip of tens and four single ones (or lay out one group of 10 craft sticks and 4 single sticks). Tell the student that you need to take away 6 single clothespins right now. Say, "There are only 4 single clothespins on the box. What should I do?"
2. Encourage the student to figure out on his own that he needs to take apart the group of 10. You may want to tell him that breaking down one ten into ten ones is called *regrouping,* if he will need to know that word in the math curriculum in the general classroom.
3. Take away the 6 single clothespins (or sticks) and thank him for solving your problem. Ask him how many single clothespins or sticks are left on the table (8). Then write the problem in numbers so he can see what it looks like.
4. Emphasize that you can only subtract *ones* from *ones.* If the student has more than 9 clothespins for the *ones* column (on the box), put the extra ones on the table in front of the *ones* space. Ask the student what should happen next. Encourage him to bundle 10 together and pin the loose pins (ones) to the box.
5. Repeat this sequence with a few different numbers until the student feels secure.

Subtraction with the Calculator

Your student(s) needs plenty of practice in doing the mechanical process of subtraction with the calculator. This process should be practiced until it is almost automatic. If the student feels secure in subtracting on the calculator, he will be able to give his attention to the more difficult task of deciding when the operation of subtraction is needed.

Regrouping (borrowing) in subtraction has always been difficult for some typical students in general classrooms. However, when students use the calculator, the regrouping is done for them. It is still important that they understand the concepts in regrouping and place value so they can understand real-life subtraction problems. Once the concept of regrouping has been introduced, I suggest that teachers and parents review simple take-away problems involving regrouping with the Clothespin Box for about five minutes before each math session in this chapter.

SUBTRACTION WITH THE CALCULATOR

OBJECTIVE: The student will be able to accurately subtract one- and two-digit numbers with the calculator. Later, he will solve three- and four-digit problems with the calculator.

MATERIALS:

- Subtraction practice sheet from Appendix B-20
- Pencils
- Calculator

SUCCESS STEP: Work out the first problem on the worksheet for the student, talking as you solve it. Ask the student to do the same problem. Praise his success.

PROCEDURE:

1. By now, the students should be very familiar with writing subtraction problems horizontally. Point out the way the problems are written vertically on the worksheet. Explain that just as when the numbers are written horizontally, you enter the largest number first. Explain that the top number is entered into the calculator first, then the minus (–) sign , followed by the second number.
2. Have them practice putting some of the problems in the worksheet into the calculator and check the calculator windows. Get in a rhythm when using the calculator. You can even chant the sequence:
    - **Five** (or whatever the number is) as you push the number 5.
    - **Look** (check the number in the window)
    - **Minus** (or whatever the operation is)

    - **Three** (or whatever the number is)
    - **Look** (check the number in the window)
    - **Equals** (as you press the = sign)

3. Then have the students do the problems on the subtraction worksheet from Appendix B-20.
4. You will probably have to give them many more problems to solve on the calculator before they can use the calculator smoothly. This book cannot give you enough practice materials to teach the skills that take years to learn in school, but once you have introduced the concepts, it will be easy to make up or buy materials to practice subtraction. This would probably be a good time to use some of the mass market workbooks for primary level math.
5. Gradually increase the size of the numbers that are used. Using the calculator, the student should be able to work with numbers in the hundreds or more.

Comparison Subtraction

Aside from take-away subtraction, the most frequently used type of subtraction is comparison subtraction. Comparison occurs when two groups of items are brought together and matched one-on-one to see which group is the largest (or smallest) and by how much. For example, if Peter has 8 baseball trading cards and Nelson has 6 baseball cards, who has the most baseball trading cards and by how many more? The matchup shows that Peter has 2 more cards. Students usually need practice before they can understand that comparison situations require subtraction.

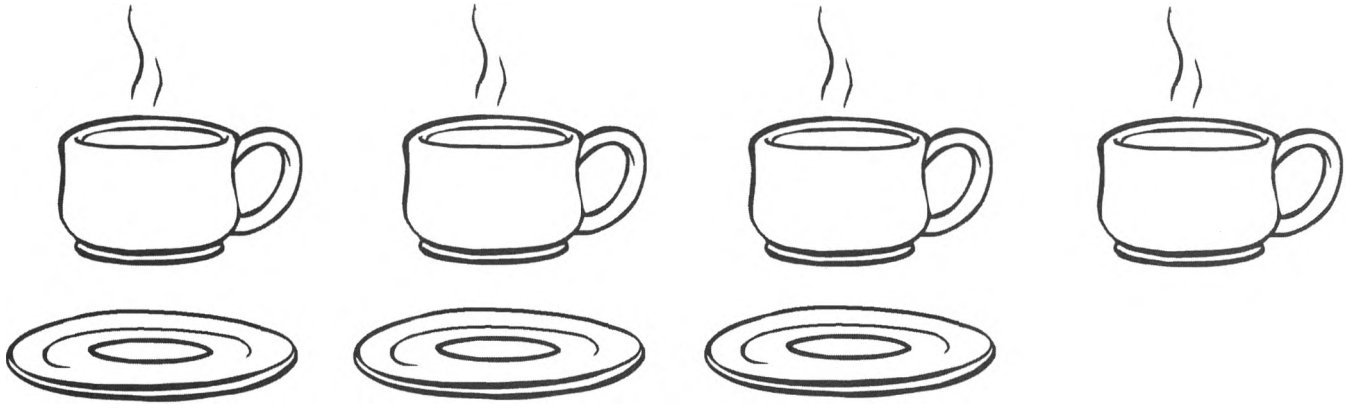
WHO'S GOT MORE?

OBJECTIVE: The student will compare two sets of items and be able to say which set has more items and how many more.

MATERIALS:

- 5 cups and saucers
- 5 empty jars or bottles with lids
- Poker chips or checkers (6 of one color and 9 of another)

SUCCESS STEP: Ask the student to count the number of items in the two sets, jars and lids. Praise him for counting correctly.



PROCEDURE:

1. Give the student four jars, and show him four lids. Ask him if every jar has a lid. Then put your three lids on the matching jars. He will probably say, "One does not have a lid" or something similar. Ask him, "Are there more lids or jars? How many lids do I need?" Remove the lids from the jars.
2. Have the student line up the jars on the table. You put one lid next to each jar. Ask, "Which poor jar does not have a lid?" The student should point to that jar. Bring out the hidden lid and say, "I need one more lid," and put that lid by the empty jar.
3. Repeat the procedure with the cups and saucers, varying the number of items, ending with six saucers and four cups.

4. Then, have your student get out the calculator. Tell him to enter the number of saucers (6). Have him push the operation (subtraction), then put in the number of cups (4), and then push equals (=). The answer should be the same as when he lined up the cups and saucers to compare. Tell him that he can use subtraction when he **compares** things. The answer will be the difference between the two groups of items or how much more one group has than the other does.
5. Now give the student 6 poker chips or checkers that are the same color. You line up 9 chips or checkers in a contrasting color on the table. Ask him who has the most chips. The student should lay his chips by each of your chips in one-to-one corre-

spondence. (If he doesn't, line up a few to show him how.) Then ask, "How many more chips do I have compared to your chips?" (3). Have I taken away any chips from you?" (no). Tell him again that even though we are not taking away any chips, we use subtraction to get the answer when we **compare** amounts.

6. If the student is not motivated to figure out the difference between two groups of checkers or poker chips, try giving him two or three fewer pretzels (or stickers, if you don't want to give food) than you give yourself or a classmate. Ask him if both people got the same amount of pretzels (or stickers). When he says no, compare the two groups.

7. Redo the previous problems with the calculator. Remind the students about using the minus sign and putting the larger number first. Using the poker chips (or more desirable items), make up a few simple comparison situations and have the students compare with each other or with the instructor. Then redo the problems with the calculator.

The major idea here is to have the students recognize that both take-away and comparison situations require subtraction. Discuss the vocabulary words used, such as *compare, minus, subtraction,* and *difference.*

Note: *Use sets that fit together, such as the jars and lids or cups and saucers when first teaching comparison subtraction. Other examples include: pencils and eraser caps, CDs and clear plastic cases, envelopes and mailing labels, drinking glasses and straws, shoes and laces, dolls and hats. Matching those pairs up is usually easier than just comparing like items.*

Moira was able to do take-away subtraction problems accurately, including checking with the calculator. However, when she was asked how many more tokens she had compared to her partner, she didn't know what operation to use. When her teacher suggested that she use subtraction, she indignantly said, "No, no! I can't take away her tokens." To Moira, subtraction could only be used if someone was "taking away" something from someone else. She needed to experience other uses of subtraction.

How Much More Is Needed?

Some authorities include the how-much-more category with the comparison type of subtraction. Strictly speaking, however, there are differences. In comparison subtraction, we can line up two sets to see which is larger and by how much. In the category of how-much-more, we know the final number and one of its parts, but have to imagine the other part. The most frequent occasion for using the how-much-more type of subtraction comes when we need to know if we have enough money to buy something, and, if not, how much more money we need. For example, "I have saved $4, but the action figure or Barbie costs $10. Do I have enough money? If not, how much more money do I have to save before I have enough money to buy it?"

WHAT CAN I BUY?

OBJECTIVE: The student will be able to use subtraction to determine if he has enough money to buy an article and, if not, how much more money is needed.

MATERIALS:

- Dollar bills (simulated or real) for student and teacher
- Pictures of desired items from newspaper ads or catalogs (glued to index cards, if that will make them easier to handle)
- Post-It notes with prices in whole dollars (currency) only

SUCCESS STEP: Have the student look at ten or more pictures of desirable items and decide on his three favorite items. Praise his selection.

PROCEDURE:

1. Give the student twelve single dollars. Put a Post-It note with a currency price that is higher than $12 on the pictures of the three most desired items. Start with prices that are just a little too high, such as $13, $14, and $15.
2. Ask the student if he has enough money to buy any of these three items. Discuss how much money is still needed to buy the item. Have the student hold out the $12 and put it next to the picture and price of the desired item. Ask the student to guess how much more money is needed. Then put down one dollar (from your money) at a time, asking if there is enough money now. The student should count every time.
3. When the student does have enough money to buy the item, ask him how many dollars you had to give him. This answer is in the how-many-more category of subtraction. Point out that if you take the price and subtract 12 from it, you will get the amount of money you need in order to have enough to buy it. Show the same problem on the calculator.

4. Repeat the procedure with different amounts of dollar bills and different desired items. Make the practice fun by adding some silly items and some ridiculous prices. Use the calculator to check each answer. Make sure that the student is entering the largest number first and pressing the minus sign to subtract. Repeat as needed.
5. For motivation, it is important to have pictures or ads of things that the student would really like to have. Individualize the items according to the student's age and likes. The student may want to count on or use a number line to find out how much more is needed. That is fine. There isn't only one way to solve these types of problems, but using subtraction is the most efficient.

Figuring Change (Currency Only)

The other vital life situation that requires subtraction occurs when we pay for an item with more money (larger bill) than needed and require change back. Many individuals give the next highest dollar and receive change back. I seldom count the change I receive back. For the many small purchases I make, I trust the cashier to give me the right change. For larger purchases, I do check the change, often with a calcula-

tor. Students need to know how to figure the amount of change they should get back, even if they do not check their change every time.

EARN AND PAY GAME

OBJECTIVE: The student will be able to accurately tell how much change he should get back when he pays an amount over the price of the item.

MATERIALS:

- *Earn and Pay Game* from Appendix A-1
- *Earn and Pay* game cards from Appendix A-2
- pennies, small toys, or other objects to use as game tokens
- paper and pencils
- realistic-looking currency

SUCCESS STEP: Give each student 3 ten-dollar bills and 4 twenty-dollar bills. Ask the student how much money he has in ten-dollar bills. Praise success.

PROCEDURE:

1. Prepare the Earn and Pay game as described in Chapter 4. Place a stack of *pay* cards on the pay spot and a stack of *earn* cards on the earn spot.
2. The first person rolls the die and moves his or her marker that number of spaces. The space landed on will be either a pay space or an earn space. The player picks up the top card from either the pay pile or the earn pile. Be sure the player reads the description of what he must pay or earn and have him say how much money he must pay out or earn. The player receives or pays out that amount and receives the pay or earn card.
3. The purpose of playing the game at this time is to have the students use big bills to pay for items, and then figure out the amount of change that they need to get back. As the banker, you can give out single and five-dollar bills in change, but the students have only bigger bills (at first) that require change when making a purchase.
4. As the teacher/banker, you ask, "How much money do you owe me?" The student says the amount of money written on the pay card. The teacher asks, "How much money did you give me? Wow, that's a lot! Do I have to give you money back?"
5. Help the student to enter into the calculator the largest number (the amount he gave you) first. Then have him subtract the amount that he really owes the bank. The difference will be how much change he needs to get back.
6. The first player to cross the finish line is Winner I and the player with the most money left is Winner II.

Subtraction in (Almost) Real Situations

This book places more emphasis on students understanding when to use subtraction than on the mechanics of subtracting or on learning the subtraction facts. Subtraction facts are useless if you don't know when to use them in a real-life situa-

tion—which I have found is often the case, even for young adults. Students need to learn to recognize that subtraction is needed when we compare quantities of things, when things are taken away, when we are making change, or when we are determining whether we have enough money to buy something. They should be able to decide what operation to use in a real situation or to listen to a description of a situation, determine what operation is needed, and then do the calculation on the calculator.

DETERMINING THE TYPE OF PROBLEM

OBJECTIVE: The student will be able to determine which operation (+ or –) is needed for a problem situation, including those that require take-away, comparison, how-much-more, and figuring change (currency only).

MATERIALS:

- Situations in which various types of subtraction are needed
- Craft sticks or pencils
- Paper plate

SUCCESS STEP: Put 6 sticks on the plate and ask the student to subtract 2.

PROCEDURE:

1. Do a few concrete subtraction problems with sticks or pencils to see if the student remembers **take-away** subtraction. For example, put 8 sticks on a plate, and have him grab 3 sticks away to illustrate 8 – 3. Then ask him how many are left.
2. If necessary, review the other types of subtraction with the student using concrete materials.

   Comparison: Two groups of items compared one-to-one.
   How-much-more: Total compared with smaller amount of money (student's) to find how much more money is needed.
   Figuring change: Total amount given is larger than amount needed (price) so amount of change can be found by subtraction.

3. Discuss the following math problems orally and illustrate with concrete items or drawings. Ask what kind of subtraction is needed.

   6 cars in the parking lot
   2 cars leave
   How many left? (Take-away)

   Alice had 8 library books.
   Kyle had 6 library books.
   Who has more books and how many more? (Comparison)

   Sam wants a game that costs $17.
   Sam has $8.
   Does he have enough money to buy it?
   If not, how much more money is needed? (How-much-more)

Todd wants to buy a $13 baseball cap.
He gives the clerk $15.
How much change should he get back? (Figuring change)

Tina buys a cake for $8.
She gives the clerk a ten-dollar bill.
How much change should she get back? (Figuring change)

Alex has 6 trees in his backyard
His father cuts down 2 of them.
How many trees are left? (Take-away)

Mark has 18 model cars in his bedroom.
Juan has 7 cars.
Which boy has the most cars and how many more? (Comparison)

Emily has 13 CDs.
Sophie has 8 CDs.
Which girl has more CDs and how many more? (Comparison)

Addie wants shoes that cost $22.
She has saved $17.
Has she saved enough?
If not, how much more money should she save? (How-much-more)

At first, do not have the student solve the above problems. Ask him what type of subtraction is needed. Or have index cards available for each type of subtraction so he can point to the appropriate card. After many examples, the "light bulb" may turn on, and he will get the pattern.

4. If the students need more of a challenge in naming the operation needed, mix in some addition examples. Julie has $3. Katie has $5. Maria has $4. How much money do all the girls have together?

NUMBER SENTENCES OF PROBLEMS

MATERIALS:

- Pencil, chalk, dry erase markers
- Paper, chalkboard, or whiteboard

PROCEDURE:

1. Work out the above problems on the chalkboard or paper with the students participating in the discussion. You model writing a number sentence for each problem.

Cake problem: $10 – $8 = $2 change needed

2. As you jointly work out the number sentences with the students, have them find the answers with their calculators.

Word Problems

Word problems using subtraction are more difficult than word problems using addition. In addition, the student just has to add all relevant numbers and get a total. In subtraction, the student has to decide whether subtraction is the appropriate operation for "take away" or for comparison, how-much-more, or change situations. Then he needs to determine which number is the largest and subtract the smaller number from it. Students need much more experience in the various types of subtraction before it becomes useful for them.

Chapter 9 suggests some strategies in addition word problems that are also useful for subtraction word problems. In particular, drawings of the objects mentioned in a problem can be very useful for concrete learners, especially with take-away and comparison subtraction. Students can also use rubber stamps or tally marks to represent items in the word problems.

USING SUBTRACTION IN LIFELIKE SITUATIONS

OBJECTIVE: The student will be able to listen to a real or simulated situation being described, determine what operation needs to be done, and then do the problem correctly on the calculator.

MATERIALS:

- Paper and pencil
- $50 in ten-dollar bills (realistic play money) for each student
- Different-colored pieces of construction paper for each student
- Props suggested by scenario (optional)

SUCCESS STEP: Ask the student(s) if anyone has ever given them a lot of money. Ask them if they have ever bought a big present for themselves. Praise the students' answers.

PROCEDURE:

1. Read this story to your student(s):

   Ann's grandma had been sick for a few weeks. She was not able to come to Ann's birthday party. But Grandma did send a birthday card with $50 for Ann.

   She wrote, "Dear Ann, You can use this money to buy yourself a big birthday present or several smaller presents. Let me know what you buy. Love, Grandma."

   Ann was very excited to have the money to spend. Her mother sat down with her to discuss the ways that she could spend the money.

2. Now tell your student(s) that they are going to pretend to be Ann and figure out how she can spend her $50. Have them put the 5 ten-dollar bills on a piece of construction paper so that they can see each bill. For each problem, have them start with the total amount and then "take away" the amounts suggested. By looking at the paper, they can tell how much money is left. Then have each student do the problem on the calculator to check. They learn the routine of putting in the largest number first, then "taking away" (subtracting) the smaller number and pushing the equals (=) sign.
3. Continue with the story: At first Ann wanted to buy a smartphone. The one she liked costs $99. Does she have enough of Grandma's gift money to buy the phone? When the student says that there is not enough money, say, "No, that will not work!" Take back the money and say, "We will have to start all over again." Give him the money as if you are just giving it to him for the first time and say, "You need to pick a gift that costs less money."
4. Then Ann thought about getting a small present. She could buy a CD that she really liked for $10. Does she have enough money to buy the CD?
5. If Ann buys the CD, how much money will she have left from Grandma's gift? Do the problem with the currency and determine how much will be left. Check the problem with the calculator.
6. Ann bought the CD, but then her brother Nick stuck his head in the door and said, "Great, now that you have some money, you can pay back the $10 you borrowed from me last week. Come on, pay up!" Ann reluctantly gave her brother $10. Now how much money does she have left from her grandmother's gift?

GENERALIZATION INTO DAILY ACTIVITIES: It is crucial to learning for you to find daily situations involving activities that are motivating for your student and that involve subtraction. Identify the different types of subtraction as you encounter them, but do not require the student to know those names. If your child/student hasn't learned subtraction facts, be prepared to whip out a calculator so he can get the practice and satisfaction of figuring out real-life situations with a calculator.

- Playing simple bowling games with plastic pins or lightweight cans can be a subtraction exercise. You can set up ten cans and have the student roll a ball at them. You say, "You had ten pins up, and you knocked down five. How many are left standing? Let's count to see if you are correct." You can make the task more or less difficult by varying the number of cans you set up. You may tell the student how many pins he knocked down because it is the subtraction operation that is important, not counting the fallen pins.
- As the student saves money for some desired item, you can have him count his money frequently and subtract to get the answer to how-much-more he needs.
- If you use an illustrated chart to record daily activities toward a goal, ask him how many more stars or stickers are needed to reach his goal.
- If the student enjoys watching sports, discuss final scores and ask him how many points his favorite team won or lost by.
- Deliberately give him less food or play materials than a peer or sibling. When he asks for more, have him tell you how many more you need to give him to make the amounts equal.

- Have him figure out how many pages he needs to read to finish the chapter or book (for instance, if he is on page 48 and there are 64 pages).
- If he is selling items for a school, scouting, religious, or other fundraiser and will get a prize if he reaches a certain level, help him subtract to find out how many more items he needs to sell (for example, if he has sold 18 candy bars and will get a prize if he sells 20).

CHAPTER 11

Introducing Multiplication

Pre- /Post Checklist

Can the student:

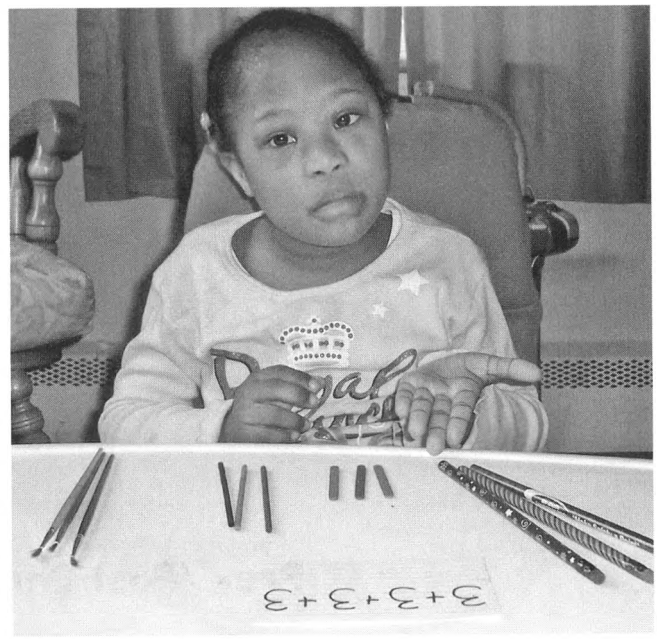
1. ____ Show, using concrete materials, that multiplication is repeated addition of like amounts.
2. ____ Use the calculator (or mental math) to multiply single-digit numbers.
3. ____ Show with manipulatives and/or the calculator that numbers can be multiplied in any order (commutative principle).
4. ____ Show with manipulatives that 0 times any number equals 0.
5. ____ Multiply single-digit numbers by 10 without using the calculator.
6. ____ Multiply single digits times multiple digits using the calculator.
7. ____ Decide whether an answer from the calculator is reasonable.

If a student understands the concepts of addition, multiplication can be introduced as a faster way to add numbers that are the same. Using the same colored plates and craft sticks that were used to teach addition, the student can experience how much more efficient it is to multiply numbers that are the same—even when using the calculator.

The student needs to have a clear, vivid picture in her mind of what it looks like when you multiply numbers. She should see groups that have the same number of items, and she should understand why we count the number of groups and the number of items and then multiply. Activities in which the students actually manipulate objects to find totals are the best way to anchor those pictures in their minds.

As with addition and subtraction, the question of memorizing the multiplication facts is a big factor in learning. Most people use a calculator for multiple-digit

multiplication. Some hands-on learners may be able memorize the single-digit facts with the help of rhymes, mnemonics, flashcards, and just plain hard work. However, if you have persevered, but the facts don't seem to stay with a particular student from day to day, teach her to conceptualize multiplication as described in this book and then allow her to use a calculator. Also, don't think that it must be an all-or-nothing situation with the facts. For instance, it can be very useful for a student to know the 2s, the 5s, and the 10s without having to resort to the calculator if she can't memorize the other facts.



Multiplication as Repeated Addition

INTRODUCING THE CONCEPT

OBJECTIVE: The student will be able to show, using concrete materials, that multiplication is repeated addition of like amounts.

MATERIALS:

- 5 or 6 brightly colored plastic dessert plates (same color)
- 40+ small craft sticks
- A calculator for each student
- Index cards and marker

SUCCESS STEP: Put out 2 plates and put 3 sticks on each. Ask the student to count the number of craft sticks and tell you the total.

PROCEDURE:

1. Using the same plates used in the success step, put out 4 plates with 3 sticks on each. Ask for the total number of craft sticks. Let the students count each stick on each plate and get the total. They may have to touch each stick to do the counting.
2. Put an index card to the right of each plate. Say, "How many sticks on this plate?"(3) Write the numeral 3 on the left edge of the index card. For each of the 4 plates, write 3 on the left edge of the index card and place it next to the plate. Looking at the line of 3s, say, "When we need to put numbers together, we use what operation?" (addition)

3. Say, "3 + 3 + 3 + 3 is our number sentence." Have the students punch in 3 + 3 + 3 + 3 on their calculators and then push the equals sign for the answer. The answer should be 12. Tell them that they can find the answer without counting by adding the numbers together. Point out that each of the numbers to be added is the same.
4. Then tell them that you know a shortcut way to get the total—a way that is faster than counting or adding—and it is called **multiplication**.
5. Say, "I have 4 plates here. What number of sticks do I have on each plate? (3) For my shortcut way, I put the number of plates (4) in the calculator, then put in the operation sign for multiplication (×) and then the number of sticks on each plate" (3). Put up a chalkboard or whiteboard where the students can see as you write "4 plates" and "3 sticks on each plate." Write the number sentence 4 × 3 = 12.
6. Demonstrate how to multiply 4 × 3 on the calculator and compare the answer to the answer you got by counting or by repeated addition.
7. Repeat the above step using 3 plates and 2 sticks on each plate.
8. Emphasize that multiplying is faster than repeated addition and counting.
9. Then say, "Let's see what happens when the numbers get larger." Have the students repeat the process using 7 sticks per plate and 5 plates. Repeat with other larger numbers. The students should be able to see that larger numbers take more time when counting or doing repeated addition, but not when multiplying.
10. Each time, have the student tell you how many plates are in front of her, what operation to use (multiplication), and how many sticks are on each plate. Then put the numbers in the calculator and solve the problem. Sometimes count the total number of sticks before using the calculator; sometimes do it after you have the calculated answer.
11. When the students are quite proficient at multiplying the number of sticks using the calculator, ask one of them to set up a multiplication problem for you or for the other students using plates and sticks. This is one way to check whether they understand that the number of sticks on the plates has to be the *same* in order for them to multiply. If they do it wrong, say, "OK, remember that to *multiply*, you have to have *groups* of the same number. How many sticks do we have on this plate? And this one? Is that the same number?" Practice for 10–15 minutes each day until you feel that the students have the concept.

Season was quite proficient at addition. When she was asked to set up a multiplication problem with plates and sticks, she quickly put down the plates and sticks. However, when the other students tried to solve her problem, they were unable to multiply because she had a different number of sticks on each plate. Her instructor asked her how many sticks she had set up. She quickly counted them correctly. The instructor told her that she did the addition problem correctly, but she had not done multiplication. The instructor then showed her what the multiplication problem would be by having her put the same number of sticks on each plate.

Multiplication of Single-Digit Numbers

INTRODUCING THE CONCEPT

OBJECTIVE: The student will be able to use the calculator (or mental math) to multiply single-digit numbers.

MATERIALS:

- Poker chips
- Colored plates
- Construction paper of 3 different colors
- White paper
- Calculators

SUCCESS STEP: Repeat a problem done successfully with the craft sticks and plates and have the student figure out the answer. Praise her or show her how to correct her error and give her an easier problem.

PROCEDURE:

1. Using the teaching method described above, put poker chips instead of sticks on the plates to see whether your student(s) can generalize the concept using different materials.
2. Have the students make problems out of the poker chips for you or the other students.
3. Tell the students that the plates are used to make it easier to see the *groups* that are on the table. The number of plates tells us the number of groups that we have. Ask the students what would happen if we put out pieces of construction paper instead of the plates. Would the answer of the number of poker chips be the same?
4. Put out 3 different colored pieces of construction paper (representing groups) to put the sticks or chips on. Use the word *groups* instead of plates while making multiplication problems using numbers 5 and under. Repeat with only white pieces of paper.
5. Fade out the pieces of paper and just put distinct groups of items in front of the student to introduce the use of the phrases *number of groups* and numbers in each group for multiplication.
6. With a variety of other kinds of plates and papers, have the students set up multiplication problems for each other. Then use straws or pencils or even chocolate chips so they can generalize multiplication to different objects. Use only numbers below the number 9. For example:
    - 2 teams with 5 balls per team = how many balls in all?
    - 4 groups of 3 different kinds of candy = how many pieces of candy?
    - 4 plastic bags with 6 cookies in each bag = how many cookies in all?
    - 2 rows of eggs having 6 eggs in each row = how many eggs in all?

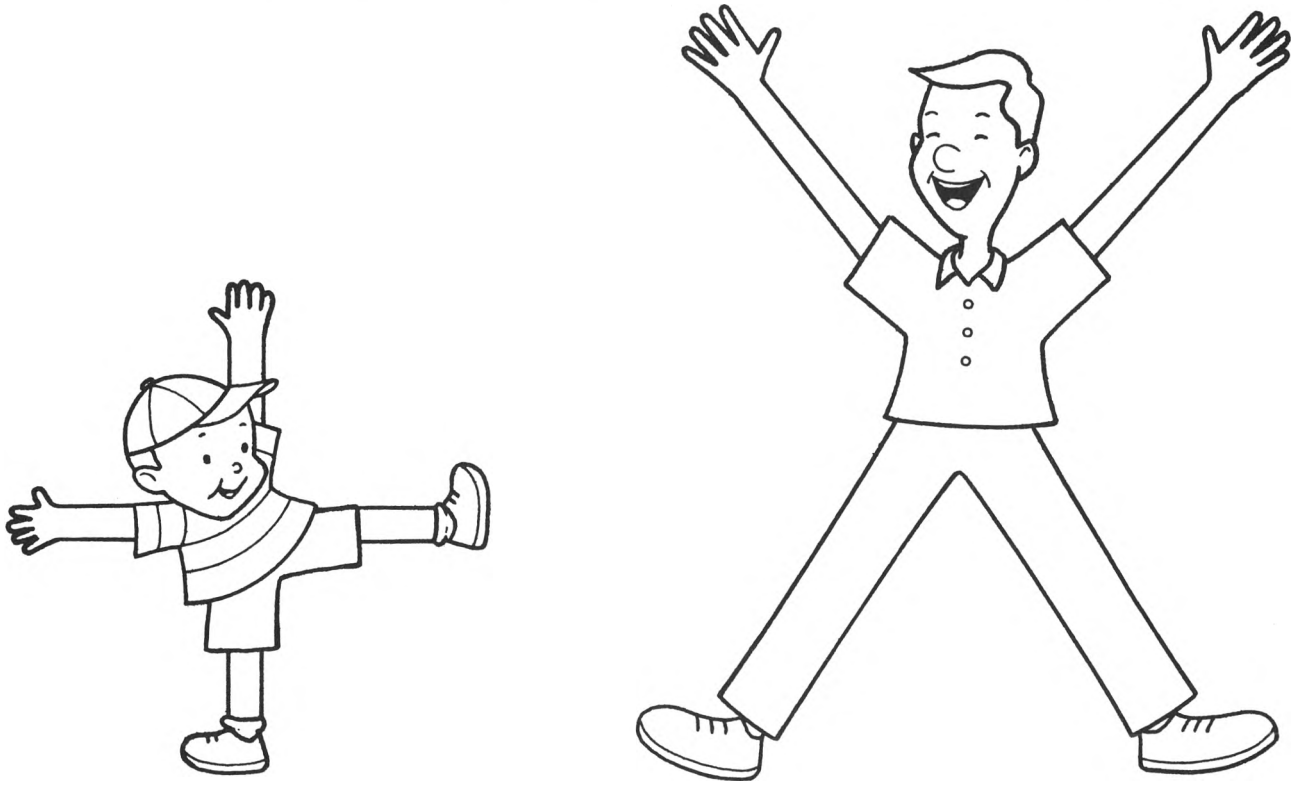
Math Vocabulary

*Heather understood the term **multiplication**, but she got mixed up on the word times as used in connection with multiplication. "I thought times was about the clock," she said. She needed to learn the different meanings of the word **time**.*

Directly teach the vocabulary of multiplication, if needed.

- The student should know that the word *multiplication* means a quick way of doing addition with equal groups.
- She should understand that the symbol × signals multiplication and is read as *times*.
- If the student attends a general education class where learning correct terminology is important, teach the mathematical terms *factor* (one of the numbers being multiplied) and *product* (the answer in multiplication).
- Distinguish the symbol + for addition as contrasted with the × for multiplication.

In Chapter 9, Jan Semple's chant was used to teach the addition sign: saying, "Numbers together go plus, plus, plus," while striking the two index fingers together to form a plus symbol. Likewise, you can teach the symbol for multiplication by crossing your arms to show a slanting **x**. The manipulation of fingers versus arms may help the students tell the difference between the addition and multiplication signs.



Students must be able to discriminate between the addition and multiplication symbols on the calculator. First try writing the two symbols on index cards with a marker. As you hold the symbols up, have the students call out either "plus" or "times." Then reverse the activity and give the students the index cards. Now when you call out either *plus* or *times,* the students should hold up the correct index card. You can also play a second version of the game by using the words *multiply or add.* If you want to challenge some of the students, you can use both sets of words.

If the students are still having difficulty choosing the correct symbol, tell them that the addition sign is a child who stands with one leg on the ground (+). The multiplication sign is a grown-up who has two legs firmly on the ground (x). They learned about addition first (when a child) and multiplication later (when they were more grown-up).

DETERMINING WHETHER TO USE ADDITION OR MULTIPLICATION

OBJECTIVE: The student will be able to tell when to use multiplication instead of addition given simple, real-life situations.

MATERIALS:

- Slate or whiteboard and chalk or marker
- 9 pencils or craft sticks

SUCCESS STEP: Arrange the 9 pencils in front of the student in groups of 3, 4, and 2 pencils. Ask if we can multiply to find out how many pencils there are altogether. If the student correctly answers no, praise her. If not, emphasize that you can only multiply if the groups are equal.

PROCEDURE:

1. Rearrange the pencils into 3 groups of 3 pencils. Ask the student if she can use multiplication now to figure out how many pencils there are in all.
2. Have her use the calculator to determine how many pencils there are using multiplication. Then have her use addition to solve the same problem.
3. Emphasize that you can only use multiplication when there are groups of items that are the same or equal.
4. Talk over the following problems with the student. Ask her to decide whether or not multiplication could be used (don't actually solve the problems for now). Draw pictures to help her understand the problems, if that is helpful. When multiplication is an option, point out that all the numbers could also be added to get the right answer, but it would be much more work.
    - Pablo promised to bring fruit to the picnic. He brought 3 oranges, 7 apples, 9 pears, and 5 peaches. How many pieces of fruit did he bring?
    - Abby made 4 potatoes for Ty, 4 pancakes for Sam, 4 pancakes for Liz, 4 pancakes for Josh, and 4 pancakes for Ashley. How many pancakes did she make?
    - Kyle had 2 gerbils. Those gerbils had 2 baby gerbils in July, 3 baby gerbils in October, and 5 more baby gerbils in December. How many baby gerbils does Kyle have now?

- Sarah gave 2 pencils to each of her 7 friends. How many pencils did she give away altogether?
- LaShawn collects toy pigs. She put 5 pigs in each of the 3 windows in her bedroom. How many pigs did she put in the windows?

5. Go over the problems with the student again, and this time have her solve them with the calculator. If you want to emphasize that multiplying is quicker than adding, have her do the problems using both operations.

GENERALIZATION ACTIVITIES: It is important for students to see the value of being able to multiply in real life.

- Practice figuring out how many items total there are with things that come in groups. First count them and then multiply to get the answer. Suggestions of items to multiply include the following:
  - Several bundles of juice boxes
  - Several packages of pencils
  - Packages of peanut butter or cheese crackers
  - Any other common items that are packaged with more than one item and can be viewed without opening the package (2 or more packages)

  Have the students count the items in each of the packages and record and label the final number on paper (e.g., 6 socks). Then ask them if they would like to try a shorter way to find the total. Write the number of items in a package times the number of packages; for example:

  3 (packages) × 2 (number of items in each package) = 6

- Figure out some situations in your students' everyday lives where they *need* to use small figure multiplication. For example, when it is appropriate, give each person two or three pieces of a particular food for a treat. Ask one student to get the treats and distribute them. First, however, the student needs to figure out how many treats to get out. She needs to multiply the number of people times the number of treat items that each person is to have.

  5 (people in family) × (times) 2 (the number of gummy bears that each person can have) = 10 (total gummy bears).

  This activity can be repeated over and over when your student is responsible for handing out silverware or foods at home. In school, the student could be responsible for handing out construction paper or other materials when each person needs more than one item.

MULTIPLYING WITH FOOD

OBJECTIVE: The student will be able to transition from doing multiplication with concrete objects to doing semiconcrete or representational multiplication.

MATERIALS:

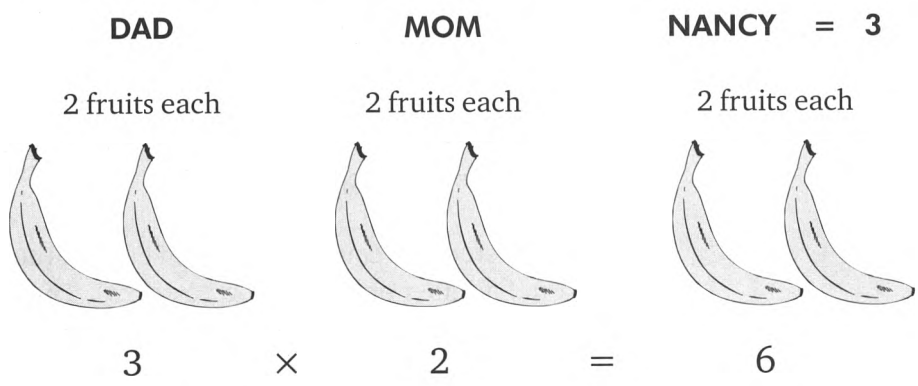
- Colored construction paper
- Marker
- Scissors
- Fruits such as grapes, strawberries, or bananas

SUCCESS STEP: Ask the student to identify the fruit. Praise success or help her be successful.

PROCEDURE:

1. Have the students do a simple problem with the actual fruits:

   Mom wanted to give each member of the family 2 pieces of fruit. There are 3 people in the family. How many pieces of fruit should Mom get out of the refrigerator?



   This problem should be easy to solve. The students can count the bananas.
2. Next, give each student 8 long strips of yellow paper (or blue or purple paper, if you are using grapes.)
3. Then tell the student that she should pretend that each strip is a banana. Write the word *banana* on each strip, if necessary. Hand out a white piece of paper with the words Dad, Mom, and the student's name on the top.

4. Ask the students how many bananas Dad was supposed to get (2). Repeat, asking about Mom and the student. Then have the students put 2 yellow pieces under the heading of Dad, 2 yellow pieces under the heading of Mom, and 2 yellow pieces under the student's name. Then write the equation on the white piece of paper:

$$3 \times 2 = 6$$

   3 (number of groups) x 2 (fruits per person) = 6 fruits

5. Have the students notice that the equations and the answer for the construction paper bananas are the same as for the real (or pictured) bananas.
6. Spend some time doing real (or pictured) small number problems and matching them up with construction paper strips.

You can see that drawing out multiplication problems quickly gets tedious as the numbers get larger. Since my son has trouble with fine motor skills and drawing, I had him practice using straws (singles and bundles of ten). The numbers need to be small when working on multiplication with manipulatives.

CALCULATOR PRACTICE

1. Write the following problems as large as the student needs on unlined paper. Leave plenty of room between problems.

$2 \times 3 =$ ____	$1 \times 2 =$ ____
$3 \times 3 =$ ____	$2 \times 2 =$ ____
$2 \times 4 =$ ____	$4 \times 3 =$ ____
$4 \times 1 =$ ____	$4 \times 4 =$ ____
$5 \times 5 =$ ____	$1 \times 5 =$ ____
$5 \times 4 =$ ____	$5 \times 3 =$ ____

2. Have the student solve the problems with a calculator. Check the answers by using the calculator and calling out the problem and the answer. Have the student check her own paper. Repeat as many times as necessary, mixing up the order of the problems.

Rowan was having trouble understanding the concept of multiplication. She was a strong tactile learner but a poor visual learner, so her teacher figured out a way to show her tactilely. She had Rowan close her eyes. She took her finger and said, "Let's tap out groups of four with your finger." The teacher then tapped Rowan's finger on the table four times. She said, "That's one group of four." She then tapped out another group of four taps and said, "That is two groups of four." She repeated the motions with a third group of fours. Then she said, "Now we have tapped three groups of four. And we tapped twelve times."

Later the teacher had Rowan count on her fingers for each group that was tapped. She followed up with having Rowan combine three groups of four straws and then count them. After several tactile sessions, Rowan understood what multiplication was compared to addition.

MULTIPLY THIS GAME

OBJECTIVE: The student will be able to use the calculator (or mental math) to multiply single-digit numbers.

MATERIALS:

- Large dice (or blank dice) and a marker
  - (Blank dice may be found at teacher's stores or on the Internet. You can also make dice using the number cube pattern in Appendix B-21.)
- Index cards
- Opaque tape such as masking tape
- Calculators

PROCEDURE:

1. Cover the dots on the dice with tape. With marker, number the sides of each die from 0–5. You want the student to get a visual picture of the numerals. You may also make cardstock dice from the model in the appendix.
2. Use any number of students. If the student is alone, the teacher will have to be her partner.
3. The first student throws both dice. The teacher then writes out a multiplication number sentence on an index card using the two numbers the student has thrown as the student reads the numbers on the dice. All the students then do the problem on their calculators, but only the first student is allowed to give the answer. If she is correct, the teacher writes the answer on the index card and gives it to the student. If the student gets the answer wrong, she does not get the card.
4. The next player throws the dice and play continues. If a student throws the side that says 0, she loses her turn. (This will introduce the fact that any number multiplied by 0 equals 0.)
5. After about 15 minutes, call time. The winner is the person with the largest total when she adds up the totals on her index cards. For example, if Karla had the index cards $2 \times 3 = \mathbf{6}$, $\mathbf{5 \times 2 = 10}$, and $\mathbf{3 \times 3 = 9}$, for a total of **25**, she would win over a student whose total was only **20.**

   Note: *Notice that the students must* ***add*** *up the numbers from each index card to get their own total. You can then see if they still understand multi-item addition.*

More Digits:

6. Next, introduce multiplying the numbers 6 through 9. For this version of the game, you will have to write the numbers 6–9 on the dice, as well as 0 and 5.
7. Encourage the students to memorize some of the facts as they are doing them on the calculator. They will find that it is faster if they memorize some of the easier facts rather than putting them in the calculator every time.

ORDER OF MULTIPLICATION

OBJECTIVE: The student will be able to show with manipulatives and/or the calculator that numbers can be multiplied in any order (commutative principle).

MATERIALS:

- Plates and sticks as used before
- Turnarounds worksheet (Appendix B-22)
- Calculators

SUCCESS STEP: Using a problem already mastered above (numbers 5 and under), have the student find the answer on her calculator. Praise her correct answer or give her an easier problem.

PROCEDURE:

1. Have the student set up a multiplication problem for you to solve on the calculator.
2. Ask her to do the following problems (which you have written on paper with plenty of space between them). Do the problems *(turnarounds) 2 at a time.*

$2 \times 3 = ____$	$1 \times 2 = ____$
$3 \times 3 = ____$	$2 \times 2 = ____$
$2 \times 4 = ____$	$4 \times 3 = ____$

and others, if needed.

3. Ask the student if she sees a pattern with these pairs. Lead her to notice that if you reverse the order of the two numbers, the answer is the same. Call them *turnarounds*.
4. Do a chant by crossing your arms above the wrist to make an X and saying: "Multiplied numbers can turn around." Repeat three times.
5. Demonstrate with plates and sticks that you can switch the plates and sticks around and come up with the same answer. For instance, if you have 4 plates with 3 sticks on each one, you have the same number of sticks as you do with 3 plates with 4 sticks on each one.
6. Have the student do the Turnarounds worksheet from the appendix.

Multiplication by Zero

Katri consistently had trouble multiplying with more than one digit. She had learned her multiplication facts easily and did not need to use the calculator. After looking at quite a few examples of Katri's work, however, her teacher found that she did not understand that 0 times any number is 0. Whenever she encountered a 0 in a multiplication problem, she would add it rather than multiply by it. So she multiplied 4 times 50 and got 204.

$$\begin{array}{r} 50 \\ \times\ 4 \\ \hline 204 \end{array}$$

Katri's teacher spent some time reviewing the addition facts with 0 as contrasted with multiplication facts, and Katri didn't make that pattern of mistakes again.

Multiplying by 0 can often confuse students. They may confuse the problem with addition, where adding 0 to any number leaves the number the same. Acting out the situation may help them visualize the principle.

INTRODUCING THE CONCEPT

OBJECTIVE: The student can demonstrate with manipulatives that 0 times any number equals 0.

MATERIALS:

- 3 paper bags
- Small pieces of candy
- Small chalk- or whiteboard or pad of paper with appropriate marker
- *Multiply This game*
- Large dice (or blank dice) and a marker
  - (Blank dice may be found at teachers' stores or on the Internet under *blank dice. You can also make dice from the model in Appendix B-21)*
- Index cards
- Opaque tape such as masking tape
- Calculators

SUCCESS STEP: Put some candy inside one of the bags. Ask the student to look in the bag and tell you what she sees.

PROCEDURE:

1. Put 2 pieces of candy in each of the 3 paper bags. Let the students look in the bags and figure out that there are 3 groups of 2 each, or 6 pieces.
2. Next, take the candy out of the bags. Let the students look in the bags. Ask the students how much candy is in the bags? (0) Show them on the board that $3 \times 0 = 0$.
3. Put your arms above your head with your arms rounded to make a zero. Chant the rule, "Zero times anything is zero." Repeat the zero rule as much as is necessary to have them learn it.
4. When all have had a chance to tell you what 3×0 equals, give them some of the candy as a reward.

Multiple-Digit Multiplication

I introduce multiple-digit multiplication by teaching students to multiply by 10. Multiplying by 10 by just adding a 0 to the end of the other number can be a useful skill and can help students realize what happens when multiplying by a two-digit multiplier. If possible, have the student discover the pattern herself.

MULTIPLYING BY TEN

OBJECTIVE: The student will be able to multiply single-digit numbers by 10 without using the calculator.

MATERIALS:

- Multiplying by Ten worksheet (Appendix B-23)
- Straws or craft sticks banded together as tens (50)
- Small chalk- or whiteboard or pad of paper with appropriate marker
- Calculators for each student
- Cards with the numerals from 1–9 written one per card

SUCCESS STEP: Have the student put the problem 4×10 in the calculator and find the answer. Praise the correct answer.

PROCEDURE:

1. Show the problem $4 \times 10 =$ _____ on the board.
2. Say, "We are going to need groups of tens. How many groups of tens do we need?" Point to the number 4 on the board. "That's right—we need four groups of tens." Have a student bring you four groups of ten from the pile of 50 straws or sticks sitting on the table.
3. Ask the students if they can figure out how many straws or sticks are here all together. Some may think of skip counting by tens to forty. Others may want to take off the rubber bands and count each group one by one. If no one figures out a way, you can demonstrate skip counting by tens or counting each straw.
4. Write the answer (40) on the board in the appropriate place.
5. Write the problem $2 \times 10 =$ _____ on the board.
6. See if someone can figure out how to do the problem without much coaching from you.
7. Repeat with $3 \times 10 =$ ____, having the students do as much of the hands-on work as possible.
8. See if someone can figure out the pattern after the demonstrations. Do not tell the students what the pattern is until they have completed the Multiplying by Ten worksheet and the following discussion.
9. Fold the worksheet so that only one problem at a time can be seen. Have the student do each problem one at a time with the calculator, unfolding the sheet as she goes. Check each problem to see if it is correct.
10. When the worksheet is finished, have the student look at each problem with its answer. Circle the important parts with highlighter, if necessary. Point out that the

other number starts out in the ones or units place, but multiplying it by 10 moves that digit into the tens place.

$$5 \text{ (in ones place)} \times 10 = 50 \text{ (5 in the tens place).}$$

11. See if the student can see that when you multiply a number by 10, you just add a 0 to the right side of that number.
12. One by one, hold up the cards with the numerals 1 through 9. Have the students multiply each numeral by 10. Have them use their calculators to find the answers and show you the calculator answer window or write the answers using mental math on the back of their worksheets.

Kyle got the correct answer for the problem 3 × 56 = 168. However, he got the same answer for the problem 30 × 56. He had not learned the pattern for multiplying by higher multiples of 10. His mother had him use the calculator to do pairs of problems like 5 × 77 and 50 × 77 until he could demonstrate the pattern.

Single Digits Times Multiple Digits

The next step is to teach your student(s) to multiply the numbers 1–9 by two-digit numbers. Remind the students that they already know how to multiply one two-digit number—the number 10. Then use manipulatives to give them a tactile and visual picture of how multiplying with two-digit numbers is done.

INTRODUCING THE CONCEPT

OBJECTIVE: The student will be able to multiply single digits by multiple digits using the calculator.

MATERIALS:

- Colored plates and sticks (50 sticks and 4 plates for each student)
- Calculators for each student
- A slate and chalk or whiteboard and markers
- Calculator Multiplication worksheets from Appendix B-24

SUCCESS STEP: Hold up 3 bundles of 10 sticks and ask one student how many sticks you have in your hand. If she is not correct, show her physically that you have 3 groups of 10 sticks each. Ask her how to write the same numbers on your board. You then write 3 × 10 = 30. Praise her for her help

PROCEDURE:

1. Ask the students if anyone remembers the shortcut way to multiply 10 times a number. Hopefully, they can tell you that you can multiply by 10 by adding a zero

to the original number. Do several examples. Tell the students that they already know how to multiply by two digits because the number 10 has two digits.

2. Using the same example from the success step (3 × 10), ask the students what would happen if each of the 3 groups had 10 sticks and 1 stick more. Ask a student to add 1 stick to each of the 3 bundles of sticks. Have the students count the total. Encourage them to find a shortcut by saying they already know that 3 bundles of 10 equal 30 sticks and that they could just add the 3 extra sticks to make a total of 33.
3. Have another student make 3 bundles of 10 sticks and then add 2 more sticks to each group, making groups of 12. Write the numerals on the board: 3 × 12 = ____. Count the manipulatives and fill in the answer (36).
4. Repeat step 4, having each student do the problems of 3 × 13, 14, and 15 using manipulatives. Have the students work in pairs if you are working with a small group.
5. As needed, make up new problems and write them on the board, using numbers where the product does not exceed 50 (the number of sticks each student has). Use the manipulatives until you are sure the student has a visual picture of multiplying multiple digits.
6. Explain to the students that the calculator just multiplies the number of groups times the number of ones and then multiplies the number of groups times the number of tens and adds them both together to get the answer.

   Show the students how this is done on the board.

$$3 \times 14 = 3 \times 4 \text{ and } 3 \times 10.$$

$$3 \times 4 = \underline{12 \text{ and } 3 \times 10 = 30.}$$

7. You then add 12 and 30 together to get the answer of 42. Have the students repeat your demonstration.
8. When the students have a clear picture of what happens in multiplication, tell them that their calculator does the two multiplications and the addition for them and gives them the final answer. Have them do some of the preceding problems with their calculator and see the shortcut.
9. Have the students do the Calculator Multiplication worksheets, using their calculators.
10. At this point, you may want the students to do some more problems from the mass market math workbooks that are sold in grocery and discount stores. The students should be able to use their calculators and write the answers in the workbook.

Using the Calculator Accurately

We need to be sure that students are entering problems into the calculator correctly, without undue speed, and checking for accuracy frequently. A printing calculator may be useful, so the student can look back at the paper tape and see if she entered the numbers correctly. Or you may want to invest in a calculator such as the See 'N' Solve Visual Calculator or the TI-15 calculator (Texas Instruments), which display the entire problem, including the answer, on the screen.

In addition, we need students to look at the answers they get on the calculator and determine if they are reasonable. General mathematics textbooks often ask stu-

dents to estimate the answer to see if it is reasonable. In my opinion, estimating the answer can be more difficult for hands-on learners than actually doing the problem. I have the most luck in getting students to look at the reasonableness of the answers when they have a real-life situation or a well-understood story problem. For example:

"Jake, do you really think you are going to need $219 for bus fare? That's an awful lot of money. I could probably fly to Chicago for that."

"Do you think that I could bring in only two marbles and yet have enough marbles to give one to everyone in the class?"

Encourage students to ask themselves whether the answer seems reasonable, given a hands-on or real-life situation. See if the students can apply the principles learned in single-digit multiplication to multiple-digit multiplication. Have them explain in words, if possible, how they did the problems with their calculators. If their understanding is secure, it is not necessary to have them do many multiple-digit worksheets just for practice, but a few problems introduced at the beginning of each lesson should help them maintain that skill.

MULTIPLICATION BASEBALL GAME

OBJECTIVE: The student will be able to multiply using the calculator in a game situation.

MATERIALS:

- Batter Up game board (Appendix B-25)
- Batter Up game cards (Appendix B-26)
- Game markers/tokens
- Answer sheet (Appendix B-27)

PROCEDURE:

1. Put players' markers on home plate. Shuffle game cards and place in a pile.
2. First player picks a card from the pile.
3. If it is a game call card (one with instructions), she follows the directions, and the play goes to the next person.
4. If she chooses a challenge card (one with word problems), she can choose either of the two problems.
5. She solves the problem, with or without using the calculator.
6. She or the teacher finds the answer on the answer key.
7. If the player is correct, she may advance one base. If the player is not correct, she strikes out and her turn is over. Then the play goes to the next player.
8. The object is to keep answering the cards correctly so you advance around the bases and make a run. The winner is determined by chance (which game call cards are drawn), as well as accuracy in solving the problems. The person with the most runs at the end of play wins the game.
9. The game can also be played with teams, alternating the turns between teams.

GENERALIZATION ACTIVITIES:

- Get your student to help you figure out what you need to buy for school lunches for a week. If you pack 3 juice boxes a day, 5 days a week, how many do you need for the week? If you use 4 slices of bread a day, 5 days a week, how many slices do you need?
- Make s'mores. Everyone needs 2 graham crackers, 3 marshmallows, 4 squares of chocolate. How many of each do you need if there are 2 or 3 or 4 people?
- When shopping, point out products that come in multiples—for example, Pop Tarts come in 3 packs of 2 ($3 \times 2 = 6$); a box of packaged peanut butter crackers might contain 6 or 8 packs of 6 crackers (6×6 or 6×8).
- If your student is learning about calories per serving, look at packages and determine things such as: if one cookie has 90 calories, how many calories will you get if you eat 2 or 3 cookies? If one serving of juice has 110 calories, how many calories will you consume if you drink 2 or 3 servings in a day?
- When you are watching football on TV or at a game, talk about how many touchdowns (plus field goals) it takes to make 14, 21, 28, 35, etc. points.
- At school, figure out how many craft or school supplies the class needs. For example, if everyone needs 4 pieces of construction paper and there are 20 students in the class.
- Use commercial workbooks for multiplication practice as needed.

CHAPTER 12

Measurement and Time

Pre- /Post Checklist

Can the student:

Length:

1. ____ Order objects by length and use comparison terms such as big, bigger, and biggest.
2. ____ Measure objects with a ruler marked in inches or centimeters (whole inches or centimeters only).

Capacity:

3. _____Compare relationships among units of capacity such as cups, pints, and quarts.

Weight:

4. ____ Estimate which of two objects is heavier and check with a balance or scale.
5. ____ Read the temperature on a thermometer.

Time:

6. ____ Use class and individual schedules.
7. ____ Tell time with clocks to the hour
8. ____ Tell time to the half hour and quarter hour

If we consider the nonacademic uses of mathematics, it becomes clear that concepts involving money are the most useful in our daily lives. However, the concepts of measurement come in a close second.

Jean Piaget's (Copeland, 1984) concept of *conservation* is as important in learning about measurements as it was in the concept of early numbers. The student needs to

understand that however we cut up, arrange, or move an item, its total quantity is the same as before (if nothing is added or removed). If we divide a broom into four parts, the sum of the four lengths is the same as the total length of the undivided broom. If we measure the four sides of a rectangle individually, the total length of the sides should be the same as when we bend a measuring tape around to measure all four sides at once. Students do not have to be able to describe conservation as a formal rule, but it should be a basic assumption behind their measuring.

Vocabulary is an important part of measurement. We use different terms for similar concepts, such as *bigger* and *larger* or *small* and *little.* Students have to be able to *recognize* these terms even if they don't use them in their own speech.

You should probably not try to teach everything in this chapter at one time. There are too many concepts to grasp and remember in a short period. In addition, you need to take the time to teach measurement with actual hands-on experiences to show its practicality. Many of the concepts can be taught with science lessons at school or at home when cooking, when shopping for food, or when dealing with weather or even with a fever.

This chapter will cover the following categories:

- **Length**
- **Capacity**
- **Weight**
- **Temperature** (simple concepts)
- **Time**

Length

In this section of the book, we are exploring ways to give students an understanding of linear measurement with commonly used units of linear measurement (inches, feet, centimeters). Note that I am not using the word *length* in its narrowest sense—that is, to refer just to the greatest of the dimensions of an object (in contrast to its width or height). Instead, I use *length* to mean the distance that anything extends in any direction.

Length is one of the most noticeable attributes of an object or item. One issue that often confuses all students, but especially those with language delays, is the vocabulary used in talking about length. There are many different words that convey length, such as *height, width,* and *depth,* and *big, tall, long,* and *wide,* as well as their opposites *short, narrow,* etc. All of these words mean something slightly different, depending on the context in which they are used. Therefore, teachers must give hands-on experiences with measuring length and labeling with the word that is most often used in that setting. The comparison forms of the words such as *tall, taller, tallest* must also be learned.

USING COMPARISON WORDS

OBJECTIVE: The student will be able to order groups of three different persons or objects by size and use comparison words such as big, bigger, and biggest. Sample words could include tall, long, wide, deep, high, low, short, small, little, etc.

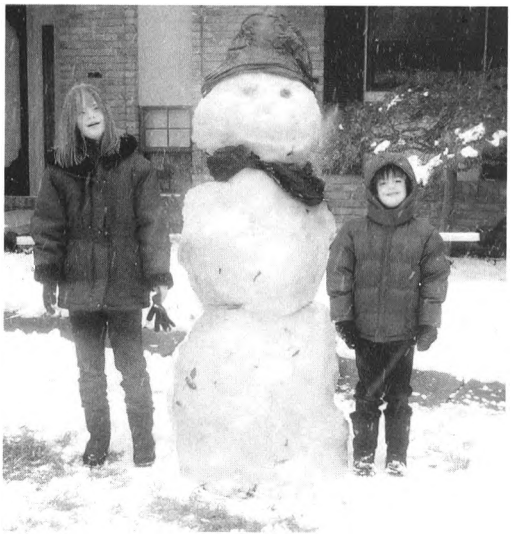
MATERIALS:

- Class or family members who are three different heights
- Shoes of three different lengths
- Wash cloth, hand towel, and bath towel that are similar, three size of balls, three different lengths of chalk, or a set of three other everyday items that are three different sizes

SUCCESS STEP: Ask the student which one of you has the bigger hand. Compare hands. Praise correct answer.

PROCEDURE:

1. Have the class or family members stand up and compare their heights. Ask the student to put them in order from small to large. You label them with comparison terms such as *big, bigger,* and *biggest.* Then mix them up and have the student put them back in order and point out the big, bigger, and biggest. You can also try other terms such as *tall, taller, tallest.*
2. Find many things in the environment whose size can be described with three comparison words. Get as much variety as possible. Model the variety of comparison words that can be used so the student can recognize what the words mean.
3. After you've labeled a few sets of items for the student, show him a novel set of items and see if he can generate the comparison words himself. For example, give him a soup spoon, a tablespoon, and a ladle, and say, "This one (soup spoon) is *large.* How about these two?"



GENERALIZATION INTO DAILY ACTIVITIES:

- Play a game like Jenga and talk about building your stack of blocks *taller.* What is your family record for tallest stack?
- If it snows in your part of the country, make a big snowball, and then a bigger one. Should you put the biggest snowball at the top or the bottom of your snowman? Where is the snow deep, deeper, and deepest?
- Make long, longer, and longest snakes with clay or play dough.
- If your student is going to get a haircut, talk about how short he wants his hair—shorter than Dad's? The shortest ever? Longer than his friend's?
- Draw with chalk or crayons and talk about making thick vs. thin, wide vs. narrow lines.

Using Standard Units of Length

This chapter will focus on inches, feet, and yards as units of measurements. Activities using these units can be adapted to the metric system, including centimeters and meters.

Note: *Students can get confused by the very small markings or slash marks on a regular ruler. Use a simple ruler with only the whole inches or centimeters showing at first. After practice with the simple ruler, you may be able to introduce a regular ruler without confusing your students. You may want to draw a colored line with a marker on the whole inches or centimeters so students can easily pick them out.*

MEASURING LENGTH

OBJECTIVE: The student will be able to measure 5 common objects less than a foot (30 centimeters) long using a ruler marked at one-inch (or one-centimeter) intervals.

MATERIALS:

- Cardboard or poster board rulers marked by hatch marks from 0 to 12 inches or 0 to 30 centimeters. (The students will write in the numbers on the rulers.)
- Various items in the environment under 12 inches (30 centimeters) in length
- A standard ruler for each student

SUCCESS STEP: Have the students write the numbers from 1–12 on the marked cardboard rulers. Students who have difficulty writing can point to the already written numbers as they are named. Praise correct responses.

PROCEDURE:

1. Have the students take off one shoe. Have them compare each other's shoe length. Ask the students if their feet are the same size. Whose shoe is longer? Shorter? Tell that in the old days, people used their feet to find out how long something was. In the US, we still use "the foot" to measure things, but we use rulers, which are always the same size, instead of feet, which are all different sizes, to measure a "foot."
2. Have each student compare the length of his foot with the cardboard ruler and say whether his foot is longer or shorter than the foot on a ruler.
3. Point out that some things we want to measure are smaller than a foot. So, we need to measure them with a smaller unit called inches (or centimeters). Point out the inch/centimeter markers on the cardboard ruler. Demonstrate how to measure a small item with the ruler. Point out that you line up the left edge of the ruler with one edge of the item you're measuring.
4. Have the student(s) measure a pencil, a paperclip, the width of a sheet of paper, a book, and other objects *under 12 inches.* (This way, they do not have to mentally add several measurements to get the length of something that is longer than a foot.) If possible, have the students pair up to check each other with their measurements.
5. Introduce a simple regular ruler. You may want to draw colored lines marking the whole inches on the ruler itself so the student can transition from the cardboard ruler more easily. Then use the regular ruler to measure some of the same things you measured with the cardboard ruler. If the measurement falls between the inch marks, have the student go to the next higher number

ROAD RACE GAME

MATERIALS:

- Road Race game board (Appendix B-28) mounted on a file folder
- Road Race game cards (B-29)
- Game tokens (Matchbox cars or paper cars made with pattern from B-30)
- Simple poster board rulers, 5 inches (or 12 centimeters) long, marked off by inches (or centimeters)

PROCEDURE:

1. Photocopy the Road Race game board and game cards in Appendix B-28 and B-29. (Gluing the game to the inside of a file folder makes it more durable.) Decide what kind of cars you want the players to use to drive past the buildings of the town on the game board. You may make the cars from the pattern and instructions given in Appendix B-30. You may also use Matchbox-type cars that really roll around the raceway but can also be used for play that has nothing to do with the math game). Decide which are appropriate for the students you are teaching.
2. Cut out the small game cards, turn them face down, and mix them up on the board. Give each player a poster board ruler that is 5 inches long, marked off in inches.
3. Players follow the instructions given on the game card, such as "Go three inches forward," or "Go one inch backwards." Players must use the ruler to measure three inches and place the front of the car at that exact point. Players can challenge each other's measurements or where a player places his token by measuring themselves.
4. If players have trouble measuring around the turns of the track, start the race using just one side of the track. Put a paper "finish" sign at the end. Later, progress to using the whole race track as students get more proficient at measuring.
5. The first car to cross the finish line is the winner. Be sure to supervise the first few road race games to make sure that all the students follow the rules and measure correctly.
6. If more than two students want to play, several students can work as a team for each car.

MORE MEASURING

OBJECTIVE: The student will learn to measure common objects that are over one foot long (American units only).

MATERIALS:

- A standard ruler and a piece of chalk for each student
- Objects approximately 13 to 19 inches long (e.g., long file folders, manila envelopes, shoe boxes)
- A yardstick (for American units only)
- A small tape measure (optional)

PROCEDURE:

1. Have the students take off their shoes and measure their feet using the regular 12-inch rulers, as before. Review that even if their feet are smaller than 12 inches, the standard measurement we call "one foot" is always 12 inches.
2. Then have the students measure the length of an object that is larger than 12 inches (between 13 and 17 inches would be the simplest). Explain that you lightly mark the object with chalk at the far edge of the ruler (by the 12). Leaving your finger at the chalk mark, move the near edge of the ruler to that mark and count the number of additional inches. Show the students how to add the 12 inches to the second number of inches by counting on to get the total number of inches. Show them how you can count on by saying 12 and then counting up. (Have them round any fractions of an inch up to the next highest inch.)
3. Give the student some objects that are between 13 and 19 inches long to measure in the above manner. If he has a lot of difficulty using a small ruler and turning it over, use a yardstick or tape several poster board rulers together to measure longer objects.
4. After the students are secure in measuring feet with rulers, tell them that they are going to learn to measure more accurately. Explain how we read to the largest number of feet and then count by inches. For example, the chair is over 2 feet tall but not 3 feet tall. We say that it is 2 feet and (counting the inches) 6 inches tall. Have the students redo the measurements for the things in their environment, being more accurate in feet and inches. Practice as needed.
5. Some students may easily make the transition to using a tape measure. A small tape measure that has a push button to rewind the tape may have great appeal for some students. Using this interest, the student can be taught to measure both short and long lengths.

GENERALIZATION INTO DAILY ACTIVITIES:

Measuring length can easily be included in home and school activities.

- Both at home and at school, heights can be measured on a height chart or on an interior door. Families should be sure to date the height marks of each child every time they are measured. At school, heights can be made into a bar graph or other visual record.
- Sometimes students get the idea that length is only a tall, thin attribute. You can use a string to measure your child's waist and then measure the string, giving an expanded picture of what length is. You can also use a fabric tape measure in the same way.
- Measuring length can be integrated into shopping excursions. Fabric can be measured for home furnishing and clothes. Or you can measure a photo or picture at home that needs framing and then shop for a frame that will fit.
- When shopping for new jeans, measure the inseam on an old pair that fits well and show your student how to look for jeans with the right inseam.

Capacity

The term *capacity* is used here for liquid volume such as water and sand. Students often have use for the concepts of cups, pints, and quarts (or liters) in daily life before they understand what they are.

Children are often fooled by appearances when it comes to capacity, according to Jean Piaget. They often think that a tall, thin quart bottle holds more than a short, squat quart bottle, for instance. Typically developing children acquire conservation of capacity/volume in stages from ages seven through eleven years of age. That is, they understand that if you pour the contents of one container into another container, you still have the same amount, regardless of the shapes or sizes of the containers. Students who have not reached that developmental level will have difficulty understanding this concept. They still can learn to use liquid volume in many practical ways throughout the day, however.

Again, we are going to work with the units of measurement used in the United States for these activities. If your area uses the metric system, you should use those units instead. Then you can explain the beauty of conversion of the metric measures based on 10, while Americans spend much more time memorizing the relationships of our units to each other.

INTRODUCING CONCEPTS OF CAPACITY

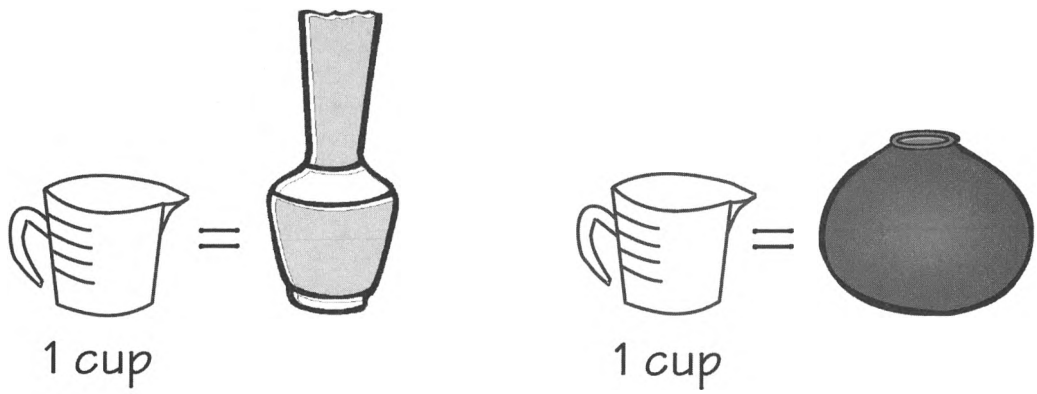
OBJECTIVE: The student will demonstrate understanding that the liquid volume of water stays the same despite being put in different-shaped containers (conservation of capacity).

(This objective may not be developmentally appropriate for young students or those who are not at the developmental level of a seven-year-old.)

MATERIALS:

- Glass or clear plastic measuring cup
- Clear bottles of different shapes
- Food coloring
- Water
- Funnel (optional)

SUCCESS STEP: Ask the student to point to the cup. Praise correct identification.



PROCEDURE:

1. Put food coloring in a cup of water. Choose two clear bottles that are different in shape. If possible, have the student pour the cup of colored water into the tallest container. Use a funnel if that will help. Then have him pour the same water into the short bottle. Point out that the same amount of water looks different in the different bottles.
2. Put a different food color in another cup of water. Pour that cup of water into the tall bottle. Compare the two bottles. Emphasize that the amount of water in both bottles is the same even if they look different. Pour the water back into measuring cups to show that the amount of water is the same. Try again with different-sized bottles.

Jake enjoyed working with Play Doh. He usually rolled it into balls and rolled them around on the table. One day when he had rolled two balls that were approximately the same size, his friend taught him how to roll a ball into a snake. "Oh, look!" he said. "I made a real big ball." When the teacher talked to Jake, she realized that he thought he had made one ball bigger by stretching it out into a long snake. The teacher asked him to make the snake back into a ball. Jake did, but he still thought the snake had more Play Doh than the ball. Jake was not developmentally ready to understand that the amount does not change even if the shape does—the concept of conservation.

STANDARD UNITS OF CAPACITY

The student will now learn how to measure with cups, pints, and quarts. Even if he does not completely understand the concept of conservation of capacity, he can learn to use cups, pints, and quarts in play or in meal preparation.

OBJECTIVE: The student will be able to compare the relationships among units of capacity measurement such as cups, pints, and quarts.

MATERIALS:

- Standard measuring cups
- Pint and quart canning jars or clear plastic pint and quart storage bottles
- A quart jar or more of uncooked rice
- A sheet cake pan (rectangular) for each student
- Colored water
- Clear jars or glasses

SUCCESS STEP: Model the names of the standard measures as you hold them in front of the student, saying cup, pint, and quart. You may want to have the names labeled clearly on the measures. Then ask the student to point to the cup and then the pint. Praise correct recognition or put the student's hand on the correct item and praise him.

PROCEDURE:

1. Cover the floor under the table or desks with newspapers or a drop cloth. Give each student a rectangular cake pan or cut-down cardboard box and measuring utensils. Explain that the rice must stay in the cake pan or box at all times. Good luck! Give each student a container of rice. Let the students feel the rice and move it around.
2. Model filling and dumping with the measuring utensils. Ask them how many cups of rice will fit in the pint jar? Then have them try to put that many cups in. Let them spill the rice into the cake pan as they pour and tell them that the measurements will not be perfect. Dump the rice out of the pint jar and have them see how many cups will fit in the quart jar. Let them practice making the comparisons as long as they are filling the cup up completely each time and counting the cups that are needed.
3. See if the students can tell you how many cups make a pint and how many make a quart. Write the numbers on a chalkboard or paper so the students can see them visually: 2 cups = 1 pint, 4 cups = 1 quart. If the student is not verbal, have him point to the correct number out of 3 or 4 numbers written before him.
4. Do not introduce the comparison of pints to quarts at the early sessions. When the student is secure in the number of cups in a pint and quart, you can see if he has figured out by himself how many pints are in a quart. If not, you can teach that concept in a separate session where you practice with pints and quarts, pouring water or rice back and forth.
5. You want your student(s) to generalize these units to anything that flows, so also do these activities with sand or very small pieces of macaroni, if possible.

Give the students many opportunities to measure capacity in everyday activities. They will usually enjoy the hands-on activities, and they will need many repetitions in different settings to maintain the skills.

GENERALIZATION INTO DAILY ACTIVITIES:

- Of course, the best generalization activity would be in cooking. However, most recipes use fractions that have not been introduced yet. Making Kool-Aid or Jell-O or a simple recipe such as for trail mix using only whole cups would be most appropriate.
- When shopping, ask your student to get a quart of milk or a pint of whipping cream or half-and-half and point out the different-sized containers for dairy products.
- When eating foods that can be easily measured in cups such as popcorn or breakfast cereal, measure out a cup to show how much it is. This would be especially useful for older students who are learning about serving sizes as part of eating a healthy diet.

Weight

When children are first learning about weight, they need to be able to lift two objects to compare their weight. When the objects are very different, the student will be able to feel that one is heavier than the other is. When the objects are similar in weight, you can help him understand that some standard type of measurement is

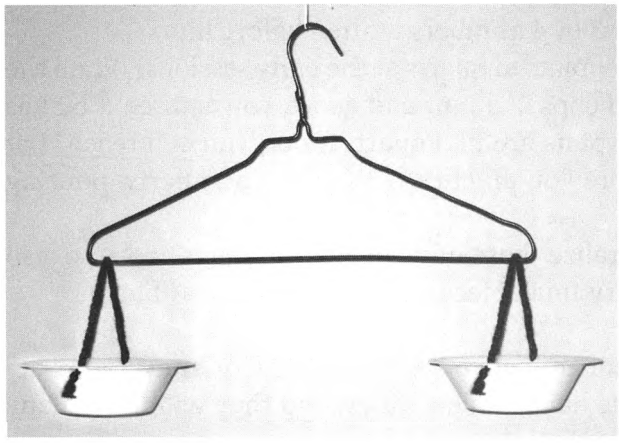
needed to show differences. Children also need to realize that appearances can be deceiving. Some big objects may weigh very little. Only the unit pounds (kilograms) will be discussed in this book.

THE BALANCE GAME

OBJECTIVE: The student will be able to estimate which of two objects is heavier and test his predictions with a pan balance.

MATERIALS:

- A commercially built pan balance or a pan balance made with a coat hanger, wire or string, and two paper or Styrofoam soup bowls
- Small objects of different weights that will fit in the soup bowls
- Unopened packages of food that weigh 1 to 5 pounds (or kilograms), for example a package of butter or bags of rice, sugar, or flour
- Bathroom scale



SUCCESS STEP: Hold up two items, one of which is definitely heavier than the other. Ask the student which one is heavier. Let him hold the objects and judge for himself. If he is correct, praise him. If not, model the correct answer and have him try again. Tell him that you are going to find a way to measure which object is heavier.

PROCEDURE:

1. Demonstrate how the pan balance works. Balance one small object in one bowl on the left side of the balance and the other in the right bowl. The heavier object will pull that side of the balance down. Therefore, that object is heavier. Let the student put one object in each bowl and see which is heavier.
2. Have the students form two teams or pit the teacher against one student. You hold up two objects, and one student or team must guess which object is the heaviest without touching.
3. After guessing, the student puts the items on the balance and sees if he is correct. He gets one point if he is correct, and the other team or the teacher gets one point if he is incorrect. The game is more fun if some of the objects do not look as light or heavy as they are. Possible "foolers" would be heavy brass or pewter objects, a small box full of light packing materials, a box of Splenda (a sugar substitute), glass vs. plastic drinking "glasses" or utensils, or a baseball vs. a tennis ball.
4. Reinforce weight-related vocabulary, if necessary, while playing the game. For example, say, "Yes, that one is lighter than this one. You can tell it weighs less because the bowl didn't go down as far."
5. Tell the students that weights are measured in pounds (kilograms). Show them some food products that have their weights written in pounds (kilograms). By looking at the weight in pounds, we can tell how heavy something is and know if

it is heavier or lighter than something else. Have them place the food items on the bathroom scale and check that the weight given on the food package matches the weight given on the product.

6. Explain that we also gauge our weight in pounds. Let the students weigh themselves if it can be done without embarrassment.
7. If the students are older, you may want to show them how the word *pound* is written, and the crazy abbreviation for pound, which is *lb*. Point out that the abbreviation is commonly used in store displays and ads. A chant could go:

   A pound's a pound,
   The world around.
   I tell you no fib,
   Why write it as lb.?

GENERALIZATION INTO DAILY ACTIVITIES:

- When grocery shopping, let your student weigh the produce: tell him you want two pounds (kilos) of bananas or three pounds of tomatoes, and let him add or subtract items from the scale to approximate that weight.
- Ask your student to get you a five-pound bag of potatoes or a two-pound bag of sugar when shopping. Show her where to read the weight on the package.

Temperature

Most children come to school with some understanding of temperature. They probably know the meanings of *hot, cold,* and *warm* in general terms. Students living in zones where the temperature changes substantially with the seasons are constantly bombarded with talk about temperatures in conversation or when watching the weather news on TV.

In the United States, students mostly hear about Fahrenheit units of temperature. Since Fahrenheit has 32 degrees as the freezing temperature of water and 212 degrees as the boiling point, students need some kind of benchmarks to determine what type of clothing to put on and what activities are appropriate, given the temperature. Guidance may also be necessary for students using the Celsius scale. Even though the freezing and boiling temperatures are easier to remember, the degrees are almost twice as large as Fahrenheit degrees. There is less numerical difference between temperatures that are cool and cold and warm and hot. Families and teachers may want to talk about temperatures when discussing the daily weather and indicate appropriate temperature benchmarks.

USING A THERMOMETER

OBJECTIVE: The student will demonstrate understanding of the measurement of temperature by correctly moving the temperature band up and down on a simulated thermometer for various temperatures given by the teacher. The student will also be able to read the temperature on a dial or red alcohol thermometer.

MATERIALS:

- A thermometer with a moveable red band, made commercially or by the teacher. (Draw a thermometer on poster board, with degrees numbered from one to one hundred or just spanning the temperature range common in your area. Cut an opening at the top and the bottom of the thermometer and thread a long paper strip partly colored with red in those openings that can be slid up and down.)
- Sources of temperature news, such as weather websites, newspapers, or radio and TV broadcasts
- Real thermometers that are large enough to read clearly (dial, red alcohol, etc.)

SUCCESS STEP: Show the student some ice cubes and a bowl of very warm water. Ask the student to point to the water that is cold. If the student does not recognize ice as very cold water, explain it to him. Show him how water melts off the ice.

PROCEDURE:

1. Show the student the large, moveable-band thermometer. Tell him that it is a *thermometer* and that it tells you how cold or hot the air is. Also tell him that the red band goes higher when it's hot and lower when it's cold.
2. Using the large, moveable-band thermometer, ask where he thinks the red band will be if it is that cold out (pointing to the ice cubes). If he points anywhere below the middle of the scale, praise him and move the band around to show winter temperatures. If not, say that the upper part of the red band shows when it is warm out like the water in the bowl. Move the red band around to show what it would be if it were hot or just warm outside.
3. If you are a teacher working on temperature at school, put a big paper thermometer on the bulletin board so you and the students can demonstrate what the temperature is each day. Some TV weather stations have temperature gauges in schools and report their temperatures as part of the weather news. If your school does not have such a gauge, consider hanging a thermometer outside a large window (or check the current temperature on the Internet if you have a computer in the classroom). Every day, ask one or more students to mark the temperature on the paper thermometer and tell you whether that temperature feels *cold, cool, hot,* or *warm* to them.
4. If you are a parent working on temperature at home, hang a large dial thermometer outside a window. Ask your child to read the temperature on a daily basis. Talk about how warm or cool that is and what kind of clothing or outerwear would be best to wear.

GENERALIZATION INTO DAILY ACTIVITIES:

- Make a game of seeing if your child can guess the temperature when you walk from your house to the car. The adults might guess first and let the children guess some numbers above or below their estimates. Then when you turn the car on, see what the car thermometer says.
- Show your older child how you can use your home thermostat to check the temperature inside the house.
- For students who are allowed to use the Internet, show them one or two

websites where they can find out the local weather. If you are planning an outing or event where the weather matters, sometimes ask your child to go to a weather site and tell you what the temperature is going to be on a certain day.

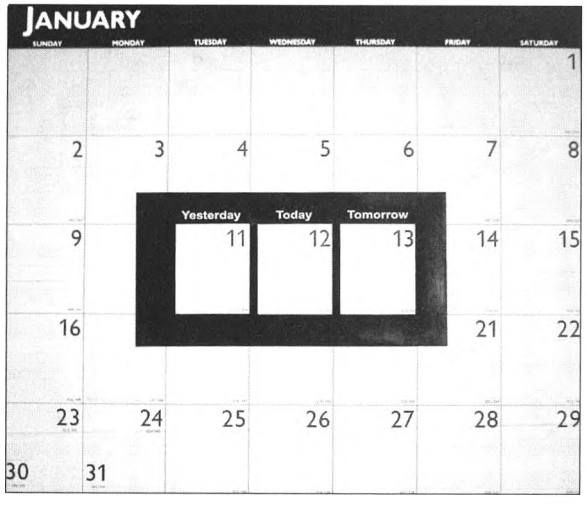
Time

Time does not just involve being able to read the hours and minutes on a clock. Time as a larger concept includes an inner awareness that time is passing and a rough concept of the duration that certain activities take.

Understanding time concepts is often difficult for concrete thinkers. Time is abstract; you can't really see what you are measuring. On top of that, the measurement of time does not proceed in an orderly fashion by tens or twenties. With 60 minutes in an hour, 24 hours in a day, 7 days in a week, and 4 or more weeks in a month, is it any wonder that these and other measures may seem to have no numerical relationship to each other in the minds of concrete thinkers? An individual not only needs to understand the names of these time periods, but also needs to acquire an intuitive feeling for their size or duration.

*When young, Allison often confused the words **yesterday** and **tomorrow** when speaking. She would say things like, "Yesterday we are going to Grandma's house" or "Tomorrow I went to the pool." She clearly understood that **yesterday** and **tomorrow** were time words, but didn't know how to use them correctly.*

Think of the time words *yesterday, today,* and *tomorrow.* A day passes, and what was tomorrow becomes today, what was today becomes yesterday, and you have an entirely new tomorrow. How do you explain those terms to someone who does not understand? Most children with Down syndrome and some other concrete thinkers also have delays in general language learning. It helps if abstract language terms for time concepts are introduced and repeated long before children are taught to tell time.



Some teachers use their morning calendar routine to teach the concepts of *yesterday, today, and tomorrow.* They use a little frame made of poster board with three windows labeled *yesterday, today,* and *tomorrow.* The frame is put on the days of the week on the calendar, with the dates showing. "This is *today*—Tuesday, October 22. Let's see, what was *yesterday?* Monday, October 21. And what is *tomorrow?* Wednesday, October 22." The next day the teacher moves the frame to show the new *yesterday, today,* and *tomorrow.* Just showing the student the time words will not teach him the real meaning behind them, however. You need to relate the words to the student's life.

Parents can weave those time words into their student's world. Make a point of telling your child about events ahead

of time. For example, tell him that *tomorrow* he is going to the zoo. When he wakes up on the day of the event, remind him that he is going to the zoo *today.* On the day after, have a conversation about what you did at the zoo *yesterday.* If he wants to do the same activity, day after day, comment that "We read this book *yesterday.* Do you want to read it *today* too?" Teachers can also use time words in talking about special activities such as field trips or assemblies that will happen tomorrow or today or that happened yesterday.

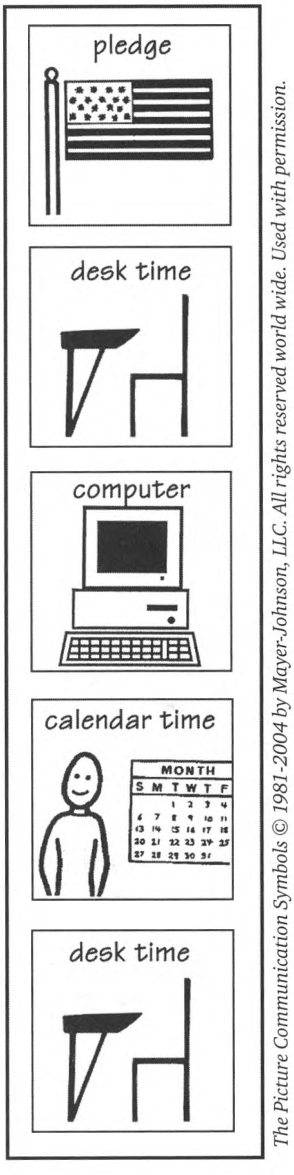
Some people with Down syndrome have problems using tenses correctly (for instance, always speaking in the present tense or leaving off some verb endings such as "-ed."). For instance, Amanda has generally good speech and language skills, but seldom uses anything other than present tense. When she is recounting what she did on the weekend, for example, she says, "Well, it's my mom's birthday, so I eat dinner at a restaurant with her." For students who have difficulty expressing tenses, it is especially import to learn to use the words *yesterday, today, and tomorrow* correctly. That way listeners can tell when something happened, regardless of verb endings.

Schedules

A classroom teacher told me that she has the class schedule on a pocket chart so the cards with activities can be moved around. Since changes may have to be made in the daily schedule, she has a laminated card with a symbol and the words ***"Something Different"*** *written on it. She makes sure that the* ***Something Different*** *is sometimes a treat, free time, or a favorite activity so that the card is not always the signal for an unwelcome change.*

A picture chart of the daily schedule should be present in all elementary school classrooms. Because some activities such as P.E. happen only once or twice a week on specific days, the teacher also needs to have a weekly chart or a monthly calendar where the weekly events are highlighted. Having a visual schedule is very important to some children on the autism spectrum or others who need the concrete evidence of the structure in the classroom. These students often need an individual schedule personalized for them.

For young students, most teachers use a simple picture or an icon (small line drawing) from a software program such as Boardmaker™ (Mayer-Johnson) to represent each event or subject area. The time may also be written on the pictures. If the picture schedule is laminated, the student can check off each time on the schedule with washable marker as he finishes the activity. Pictures can be arranged from left to right to indicate



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the order of activities, or from top to bottom (especially if the child has trouble distinguishing left from right). Some teachers use a pocket chart where the student moves a card from one pocket to another when he moves to another subject or area. Some middle and high schools require all students to have an individual planner, which can be adapted with pictures to help concrete thinkers.

Developing a Sense of Time

Long before children are expected to learn to tell time, they are exposed to the units we use to measure time. In addition to the larger units discussed above (days, months, years), they constantly hear about seconds, minutes, hours. If they ask their parents to do something for them, they may hear, "Wait just a second" or, "In a minute." On a car trip, they may hear that it will take two more hours to get to Grandma's house.

For children who can't tell time, these words are usually not that informative. If words such as *minute* or *hour* are still basically meaningless to a child, you may want to describe time in other terms that make sense to him. This can reduce frustration or anxiety if he really wants to know how long it will be before something will happen or how long something will go on. I have been successful with using the TV schedule as a reference for when time has passed and for how long activities take to accomplish. For instance, you can say, "We will eat dinner as soon as *Nickelodeon* is finished," or some other TV program that comes on near dinnertime. You can also use other frames of reference, such as "right after Dad gets home" or "after we take the dog for a walk." Eventually, of course, children need to develop a feel for the length of a second, minute, and hour.

To further help students understand what fifteen minutes (or some other length of time) means, you can use a kitchen timer that has a dial that keeps turning until the bell rings. My favorite timer is called the Time Timer (see Resources section), which has a red ring around the dial that shows visually how much time is left (the less red that is visible, the less time is left). The student can easily see the passage of time and get an idea of when "time will be up." This Time Timer does not have a bell signaling that the time is up. Some students dislike buzzer sounds, so this timer would be appropriate for them.

Here are some ways to use a timer during daily activities:

- Use the timer to signal the student when his time on the computer, phone, TV, etc. is up. Say, "I'm setting the timer for twenty minutes" as you are setting it, and then inform him, "Your twenty minutes is up" when the timer goes off.
- Ask the student to set the timer for a certain number of minutes when you begin cooking something or when you begin an activity that you want to stop after a set period of time.
- Challenge your child or student to "beat the timer" when he is doing routine activities such as getting ready for bed, writing his spelling words, or sorting silverware into the kitchen drawer. Make it easy for him to beat the timer and say, "Wow! You did that in less than ten minutes!"

Time Telling with Clocks

When digital watches and clocks became widely available, I thought that the problem of telling time was solved for students who are concrete learners. I was wrong.

Although students can be taught to read digital clocks by the hour and minute fairly easily, this is not a substitute for telling time with a round-faced analog clock. A major reason is that students do not get a concept of the movement of time by reciting a digital number. Seeing the minute and hour hand move on an analog clock apparently gives a better visual picture of the movement of time. It is much harder to figure out how much time until an event occurs if the student only sees a digital number such as 11:50. With an analog clock, the student can see that the hour and minute hands have only a little distance to go before it becomes 12:00. In addition, we do not have digital clocks everywhere, and students have to be able to read the time in many different places.

One thing I learned from working with students on digital time telling is that it is efficient to have a consistent way to describe the minutes. Naming the hour and then giving the minutes after the hour is the most understandable way to express time for young children and concrete thinkers. Expressions such as "quarter to 9" or "half past 8" or even "10 to 9" are difficult for students to remember or for teachers to teach at the beginning. For students who are just beginning to tell time, do not use these more colloquial phrases. When students have thoroughly learned time telling, you can acquaint them with these phrases.

At this level, we teach the hour and fifteen-minute increments after the hour. The student should say the hour and then 15, 30, or 45 minutes after.

TELLING TIME BY THE HOUR

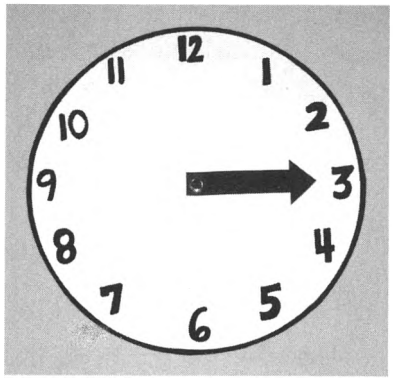
Clocks are made with many different shapes of hands, placements of the numbers (or sometimes no numbers), colors, and markings for minutes. It helps to use only one clock when beginning to teach about time and to use a simplified clock at that. At first, use a clock or a model of a clock that has all the numbers in the standard positions, but only has the hour hand.

OBJECTIVE: The student will be able to tell what the hours are on a clock with only an hour hand.

MATERIALS:

- A large model of a clock for the teacher with only the hour hand (can be made of cardboard or a plain paper plate with the hour hand fastened on with a paper fastener)
- Student clocks with the numbers and hour hand (a commercial teaching clock with hour hand only, or a clock made from a paper plate with a cardboard hour hand-colored black and fastened in the center with a paper fastener)

SUCCESS STEP: Ask the student(s) to point to a clock or clocks in the room. Praise correct answers.



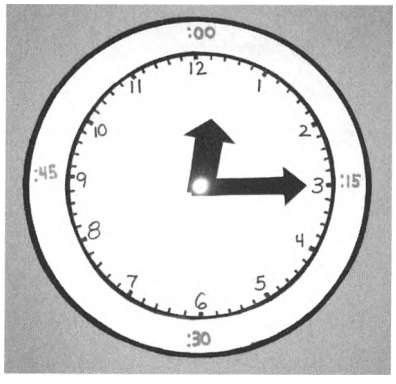
PROCEDURE:

1. Have the students move the hands on their own clocks to each hour. Note, if a paper fastener attaches the hour hand, it may need to be adjusted so that the hour hand moves freely. At first tell them to move the hand to the number, "Put the hour hand on the 5." Later you may change your wording and say, "Make the hour hand show 5 o'clock."
2. Relate the different hours to your daily schedule. "When the hour hand is on the number 12, we go to lunch."
3. At random, give the hours orally (6 o'clock, 3 o'clock) to the students and have them move the hour hand to that hour.

READING THE MINUTE HAND BY QUARTER HOURS

Learning to read the minute hand can be tricky for any child, but can be especially difficult for concrete learners. When the hour hand is on the 3, you say "three," which is relatively easy to understand. But when the *minute* hand is on the 3, you say "fifteen." Even if a student is able to understand *why* you say fifteen (for instance, you count off the minutes on the clock leading up to the 3 and show him there are fifteen), it can still be very difficult to remember how we count off these fifteen-minute increments.

OBJECTIVE: The student will be able to tell the correct hour and the minutes by fifteen-minute intervals on a teaching clock.



MATERIALS:

- A large cardboard teaching clock with both the hour and minute hand. (Add a black minute hand to the teaching clock used in previous activities. Make sure the minute hand is considerably longer than the hour hand. Also add an outer ring to the clock indicating the minutes by fifteen-minute intervals. The outer circle should have :00 above the 12, :15 near the 3, :30 below the 6, and :45 near the 9. Write the minutes in a different color than the hours. The colons should be shown so that the student will be familiar with how the minutes are written.)
- A paper plate copy of the teaching clock for each student

SUCCESS STEP: Ask the student to read the numbers (15, 30, and 45) on the outer ring of the teaching clock. Praise him or model the correct numbers if he is incorrect.

PROCEDURE:

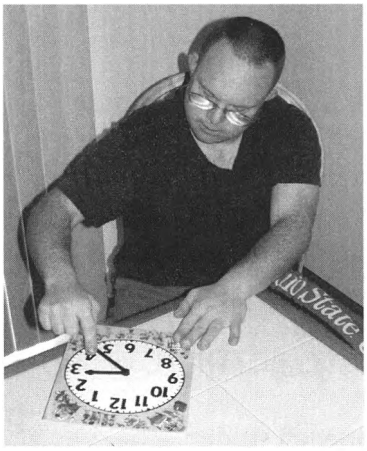
1. Tell the students that we sometimes want to know more than just the hour. Point out how slowly the hour hand moves toward the next hour. We can tell if it is a little after the hour or almost to the next hour by looking at the hour hand. We can tell even better with the long hand on the clock that we call the minute hand.

2. Let the students move the minute hand around the clock for a little while to get them used to how it moves.
3. Explain why the numbers are written on the outer ring of the clock. Show them the number 3 on the regular clock face. Then explain that the number 15 on the outer ring means that it is 15 minutes after the hour if the minute hand is on the 3.
4. Put dots to represent minutes between the numbers on the clock. Tell the student that the dots stand for minutes. Have the student count the dots between the 12 and the 3. (Make sure you have placed the right number of dots!) It should be 15, just like the number on the outer ring.
5. Repeat the procedures with the 6 (30 minutes) and the 9 (45 minutes). Next, explain that you read the number :00 above the 12 as *o'clock.*
6. Practice showing times that are hours and 15-minute intervals on your teaching clock. See if the student can correctly read the times that you show.

If your student(s) can learn to identify the quarter hours quickly and easily, he will probably be able to function quite well in his environment. For now, if the student recognizes that it is "almost" 15, 30, or 45 when the minute hand is not exactly on the quarter hour, that will be precise enough.

PRACTICE WITH CLOCKS

OBJECTIVE: The student will be able to read the hours and the minutes by the quarter hour on a teaching clock (without the outer ring) and on a working clock (all numbers visible).



MATERIALS:

- Student teaching clocks with hour and minute hands (homemade as above, or plastic teaching clocks available from Learning Resources or teacher supply catalogs)
- Drawings of clock faces without hands from Appendix B-31

SUCCESS STEP: Ask the student to point to several hours on the teaching clock.

PROCEDURE:

1. Write some times (hours and fifteen-minute intervals) on index cards or write them one by one on your slate with chalk. Have the students make the times on their individual clocks.
2. Students may have problems when the minute and the hour hand both point to the same number (at 3:15, 6:30, 9:45, and 12:00). It can be difficult to tell that both hands are at the same spot. Explicitly show your student(s) these times and tell them that the hands are trying to play a joke on them. They are both hiding in the same spot. Show students now how those times look on the teaching clock, where the hands are more clearly different, and, later, show them how those times look on real clocks.
3. Make copies of the page with clock faces (without hands). Ask the students to draw the hour and minute hands on the clock to show the numerical times you give them.

(Make enough copies of the page that students can practice for several sessions.) Give a review of clock faces at the beginning of each session until the students are secure in the learning.

TIME BINGO GAME

OBJECTIVE: The student will be able to read the hours and the minutes by the quarter hour.

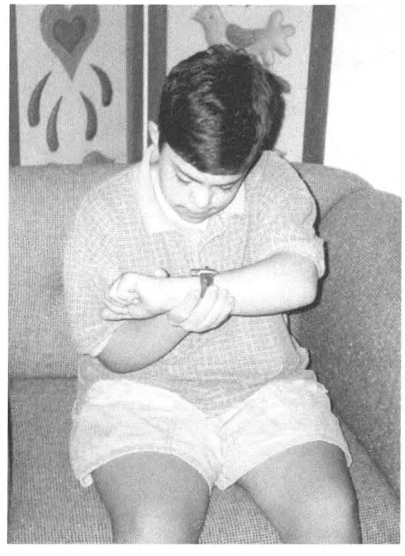
MATERIALS:

- Time Bingo cards from Appendix B-32
- Index cards: 12 showing the time on a clock and 12 showing the same time written with numerals (Cut out clock faces and times from Appendix B-33 and glue to the index cards)
- Magnetic wand and chips (or pennies to use as bingo markers)

PROCEDURE:

1. Give each player a bingo card from Appendix B-32, a magnetic bingo wand, and about 12 chips.
2. One by one, show the clock face cards that match possible numbers on the bingo sheets, saying the letter above the column, e.g., M. Players look at the M column on their bingo cards to see if they have a time that matches the clock face being shown. (Call the letter column first so they do not have to search the whole card for the possible answer.)
3. Players put a bingo chip on the time that matches the clock face. The winner is the first one to fill one row or one column. The winner also gets to pick up his bingo chips with the magnetic wand first.
4. For an easier variation, use the cards with the times written in numerals. Call out the times without saying the letters above the columns.

GENERALIZATION INTO DAILY LIVING: At this point, students need to get plenty of practice reading the time on different clocks and watches. Do not use clocks or watches that do not have all the numbers on them for this practice. See if students notice time on digital clocks and relate it to time on analog clocks. The digital clock time is read with the hours first and then the minutes.



Parents should give the student many occasions to need to tell time at home:

- Remind him what time his favorite TV program comes on and have him look at a clock or watch so he knows when to turn it on.
- Ask him to remind you when something that is cooking or baking should be ready. ("Come and get me at 4:00.")

- Tell him he can leave for the park, his friend's house, the movies, the library, etc. at a specific time.
- Instead of telling him you will do something he asks for "in a minute" or "soon," give him a specific time that you will do it and ask him to remind you then.
- Praise him when he comes home at the correct time or is dressed and ready at the right time for school or other occasions. Sometimes students are only taught about time when they are late or in some other situation when the teacher or parents are angry. The phrase "Catch them doing something good" certainly applies to time learning.

CHAPTER 13

Division

Pre- /Post Checklist

Can the student:

1. ____ Divide a total number of items (under 25) into separate, equal-sized groups.
2. ____ Demonstrate division with concrete and representational materials
3. ____ Correctly read a division problem and solve it on the calculator.
4. ____ Tell when a remainder is needed in a division problem (rather than show a long string of decimal fractions).
5. ____ Divide with the calculator to two decimal places.
6. ____ Accurately do simple division problems with or without the calculator and check them with multiplication.
7. ____ Recognize the three ways of writing division problems.
8. ____ Solve simple division story problems.

In multiplication, equal-sized groups are combined. Since division is the inverse of multiplication, division is separating groups into equal-sized parts. Children often experience division in situations where they have to distribute food, toys, etc., equally among friends or family. For example, a student has a package of five cupcakes to share with two friends. She gives herself and her two friends one cupcake each and finds she has two cupcakes left over. She has divided 3 into 5 and found that the answer is 1 with a remainder of 2.

There are two types of division—sharing division and grouping division. In sharing division, we know the total number of items to be shared and the number of groups and we want to find out how many items to put into the group. For example:

10 apples (total number of items)
5 friends (number of groups)
How many apples per person? (2)

In grouping division, we know the total number of items and the size of the groups. We want to know the number of groups that can be served. For example:

12 candy bars available
Each child gets 2 candy bars
How many children can receive candy bars? (6)

The sharing type of division is used most frequently in daily life. Students need to have many concrete experiences with sharing division so they know when it needs to be used. One way to remember sharing division is to imagine dealing out cards for a game. You know the number of groups (people playing) and the number of cards in the deck. You deal out one card to each player until you have dealt out all the cards so that everyone has the same number of cards. The number of cards left over is the remainder.

MAT DIVISION

OBJECTIVE: The student will be able to demonstrate division with concrete and representational materials.

MATERIALS:

- Various small items that can be divided into up to five groups, such as pennies, buttons, M&Ms, straws, pretzels, pencils, etc.
- Five pieces of construction paper to be used as mats (half sheets or full size, depending on the size of the materials)
- Twelve brown paper circles to represent cookies, if needed
- Pencil or marker

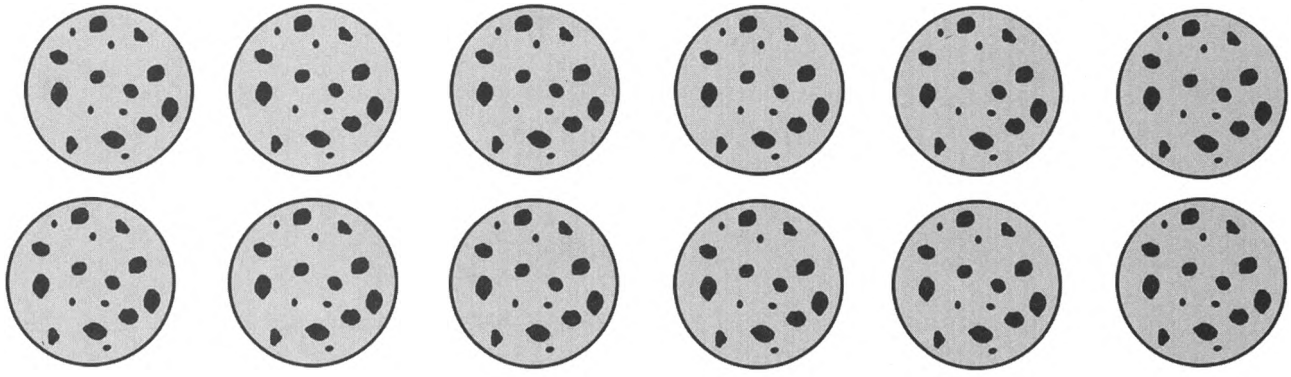
SUCCESS STEP: Ask the student to put each construction paper mat on the table in front of her working area, counting them as she does so. Have her number the mats from 1 to 5 with a pencil or marker.

PROCEDURE:

1. Hold up a handful of pennies (or other small objects). Say, "I want to share these pennies between my two friends (not counting me). I am going to put down two paper mats to represent my two friends. How can I make sure each one gets the same number?" The students should be able to say that you can give first one friend a penny and then another friend a penny until the pennies are all on the mats.
2. Give all the students more pennies. Tell them that this time they are going to share the pennies between three friends so they should put three mats in front of them. To make sure they can divide the pennies evenly, make sure you give them multiples of three.
3. Monitor whether they can divide the pennies independently.
4. Repeat the sharing division with various numbers of friends and other materials. Tell them that another way of saying "sharing equally" is "dividing equally."
5. Show the following pictures and demonstrate how the sharing can be done on paper.

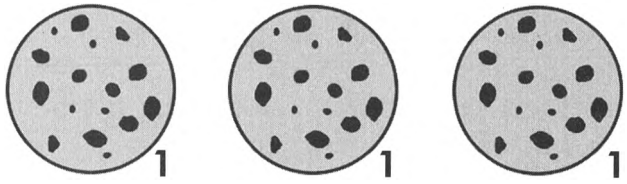
MAKE IT FAIR

PROBLEM A: You have 12 cookies to share with 4 friends. How many cookies does each person get (if each gets an equal amount)?



Each friend gets _____ cookies.

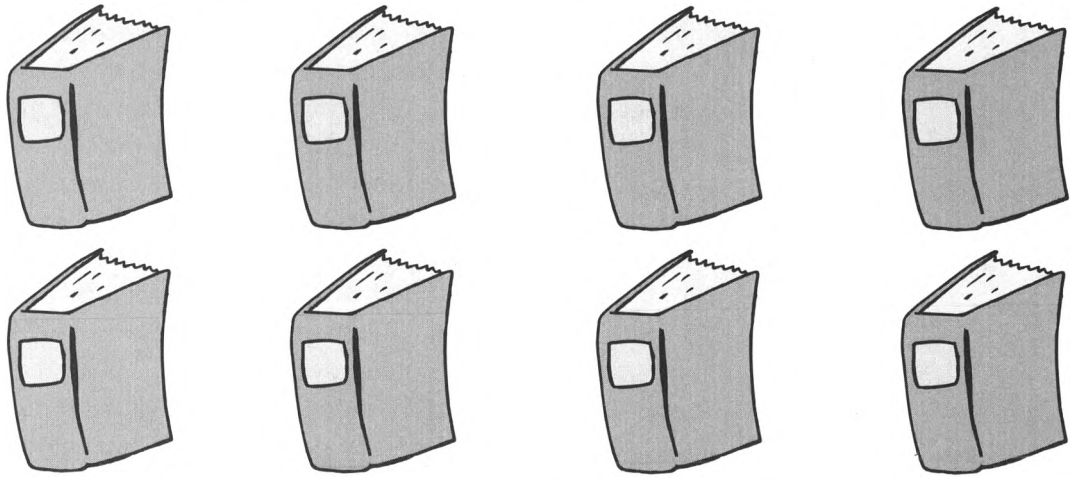
Each friend got _____ cookies. Let's see how many cookies friend number 1 got.



3 cookies each

6. If the students have trouble using the picture representation, use brown paper circles for cookies that can be manipulated, or perhaps use real cookies.
7. Do the next problem, having the students explain each step as was done in the above problem.

PROBLEM B: There are 4 people in the Jackson family and 8 books to be carried into the library. How many books should each person carry if they share/divide them equally?



Each family member will carry _____ books.

INTRODUCING THE DIVISION SIGN (CALCULATOR ACTIVITY)

OBJECTIVE: The student will be able to divide the total number of items (under 25) into separate, equal-sized groups using the calculator.

MATERIALS:

- 25 or more counting chips (poker chips, all of one color)
- Deck of cards
- Calculators
- Whiteboard or slate and markers or chalk
- Wrapped candies

SUCCESS STEP: Ask the student to bring you 10 or more (an amount that can be shared equally with the number of students in the group) counting chips. Praise her for the correct number or assist her to bring the correct number.

PROCEDURE

1. Using the counting chips that the student has given you, tell the students that you want to divide the chips equally among the people who are there. Only use amounts of chips that can be divided evenly. Deal the chips out as if you were dealing cards.
2. Ask if everyone has the same amount. Tell the students that division is about sharing items equally.
3. Give one student some more chips and ask her to share the chips evenly with the number of people there. Make sure that the number can be divided evenly. Then ask how many chips everyone has received. Say, "Right. We *divided* 8 chips into 4 groups and each person got 2....8 chips *divided* by 4 equals 2 chips each."
4. Teach the students what the division sign looks like. Write the division sign on the slate or whiteboard. Point out the dash with the dot above and below it. Show them the division sign on the calculator and contrast it with the subtraction sign. Also point out the position of the + and the × signs. If your student has trouble finding the division sign, put a small dot of tape on it on the calculator.
5. Repeat the chip problems you presented earlier using the calculator. Say, "I have a total number of ___ chips. I always enter the total number in the calculator first. Then I divide (push in the division sign on the calculator) that number by the number of people here. Then I push the equals sign. Is my answer the same as what we got when we distributed the chips to everyone?" For example, 10 (total number of chips) divided by 5 (number of people) = 2 chips each (modify the numbers for the number of chips and people in the room).
6. Pair off the students, if possible. Give one of the partners some more chips.
7. Have that partner divide the chips between herself and her partner. The first person should check the division by putting the problem in the calculator and solving it. At this time put any leftover chips (remainders) down on the floor.
8. Teach the students the chant for entering division problems in the calculator and have them practice entering problems in the calculator as they chant.

   The biggest number goes in first. *(Spread your arms out wide as if holding something big.)*

Then the division sign.
Put the number sharing.
And press the equals sign.
We have our answer met.
How many do **I** get? *(point to self several times)*

You can sing the first four lines to Row, Row Your Boat, then just say the last two lines with emphasis. You will need to repeat the chant (song) at every session until the words become automatic.

9. Now, give each pair of students an even number of wrapped candies. Have one of the pair divide the candies and the other person check the problem on the calculator with both students doing the chant. If you do not want to use candy, use individual stickers, small erasers, marbles, or sticks of gum as an alternative to candy.
10. Adjust the number of wrapped candies that the pair holds and have the partners reverse jobs.
11. Repeat as often as is necessary.
12. At another session, see if the students can come up with experiences that require sharing division such as sharing a batch of cookies or M & M's.

INTRODUCING THE DIVISION HOUSE (CALCULATOR ACTIVITY)

The language describing division can be difficult to understand for some hands-on learners, including students with Down syndrome, autism, or other language delays:

$6 \div 3$ is read **6 divided BY 3.**

$3\overline{)6}$ is often read **3 into 6** or **Divide** 6 **BY** 3

OBJECTIVE: The student will be able to correctly read a division problem written two ways and solve it on her calculator.

MATERIALS:

- A calculator for each student
- Simple Calculator Division Problems from Appendix B-34
- A slate or whiteboard and chalk or marker
- Sheets of paper and a pencil for each student and teacher
- Poker chips

SUCCESS STEP: Write $8 \div 4 = 2$ on the slate or chalkboard. Ask the student to read it aloud. Repeat her words if she is right. "Yes, this says 8 divided by 4 equals 2." If not, have her model your words.

PROCEDURE

1. Write the following problems on the white board:

$6 \div 3$ $\quad$ $3\overline{)6}$

2. Tell the student that these are two different ways of writing the same problem. In the United States the $\overline{)\quad}$ is often used to show the process of division. We will call this a "division house." The number to be divided (dividend) is placed inside the division house. The number to be divided by (divisor) is placed outside the house at the door. The answer (quotient) is placed on the roof of the house. Draw the division house on the slate or whiteboard and explain the parts.
3. Write a large, blank division equation on a piece of paper using a division house.

$$_\overline{)\quad}$$

4. Put 12 poker chips on the table. Ask the students how you can divide these chips among three people.
5. Now model how you can do the same problem with dots on the paper instead of poker chips. You can model putting 12 dots on the paper and then drawing the "division house" over the group of 12 dots. Put the number you are dividing by (3) outside the "division house." See how many groups of 3 each you can make out of the 12 dots and circle each group. Count the number of groups that you have made and put that number on top of the "division house."
6. Now do the same problem on the calculator. Remind students that they have to put the largest number in the calculator first. Say the chant:

   The big number goes in first
   Then the division sign
   Put the number sharing
   And press the equals sign.
   We have our answer met.
   How many do I get?

7. Have the students do the page of Simple Calculator Division Problems from the appendix. All of the problems will come out even. Remind the student that if the problem is written in the house form, the big number will be inside the house.

Remainders

For division, as for the other math operations, this book advocates teaching students to rely primarily on their calculators, rather than on mental math. Using a calculator for division, however, requires a judgment call that is not needed when doing the other operations. That is, when the answer does not come out even, you have to decide whether to express the answer as a decimal or a remainder. We have not introduced decimals yet, so decimals will be meaningless to the students at this point.

For now, when you solve a division problem that results in a remainder, explain to the students that the calculator doesn't know what to do when a number can't be divided evenly. It just goes on and on with numbers that are smaller than one whole number. For example:

Mrs. Meyer bought a bag of balloons for her 4 children to share. There are 10 balloons. How many can each of her children have? (2 balloons)

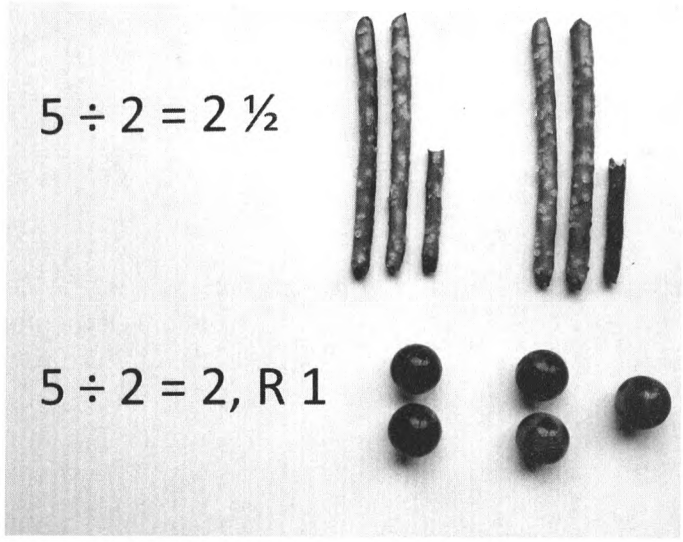
Can you divide the leftover two balloons equally between 4 children? (Not if you want to put air in them.) However, if you do the problem on the calculator, the answer comes up 2.5. For now, we will drop the .5 and say the answer is 2 with a remainder.

SHOW YOUR REMAINDERS

OBJECTIVE: The student will be able to tell when she needs to show a remainder.

MATERIALS:

- 10+ pencils
- Small candies or other small items that cannot easily be divided such as M & M's, Reese's Pieces, marbles, or erasers (enough to give 3 to every person, plus one extra)
- Items that can be divided such as licorice sticks or straight pretzels



SUCCESS STEP: Ask one student to divide an amount of pencils evenly among the other students or the rest of the family. Say, "Divide up the pencils evenly. Give me back any pencils that are left over." Praise her for doing it correctly. If she is incorrect, model the actions for her, collect the pencils, and have her try again.

PROCEDURE:

1. Show the students wrapped candies (one more than three candies for each person). Tell the student(s) that you want to be fair and give everyone the same number of candies.
2. Deal out three candies per person with one left over. See if they can tell you what the remainder is going to be before you deal out the last set of candies.
3. Ask the students what to do with the one left over. Can you cut it easily into pieces? Hopefully, they will answer no.
4. Explain to them that you just have one candy left over. It is a remainder.
5. Let them eat their candies or give them the marbles and tell them that you are saving the remainder for an absent classmate, sibling, or parent.
6. Repeat the procedure with items such as pretzels that can be divided. Distribute the pretzels so each person has two but there are enough left over to give each person half of a pretzel. See if you can get the students to come up with the idea that the pretzels could be broken in half and everyone else could have an equal amount. Later, distribute a different (uneven) number of pretzels to see what the students will do with the one that is left over.
7. Make the point that sometimes you just need to leave a remainder and sometimes you can further divide the items so each gets an equal part.

Time for a New Calculator?

As your students begin working with division and getting into answers that involve decimals or fractions, they may benefit from using a calculator with more features. Some useful models are the Math Explorer (model TI-12, Texas Instruments), the Fraction (Casio), and the Explorer Calculator (model TI-15, Texas Instruments).

Long Division

As early as 1981, a textbook on teaching mathematics suggested that solving long division problems by hand might not be the best use of students' time:

> *Increased availability and lower cost of hand calculators, together with the infrequent need to use long division in day-to-day situations, has caused mathematics educators to question whether we should continue to teach the long-division algorithm. (Bley & Thornton, 1981)*

Several years later, the National Council of Teachers of Mathematics (as quoted in Reys, Suydam & Lindquist, 1984) noted:

> *For most students, much of a full year of instruction in mathematics is spent on the division of whole numbers—a massive investment with increasingly limited productive return.... For most complex problems, using the calculator for rapid and accurate computation makes a far greater contribution to functional competence in daily life.*

To do long division, we have to use several different processes and concepts, for example:

$$56 \div 4$$

1. **Estimate** how many times the divisor (4) can be divided into the dividend (56): 5 (tens) divided by 4 = 1 (tens)
2. **Multiply the** 1 times the 4
3. **Compare** remainder
4. **Subtract** the 4 from 5 to get 1 (ten)
5. **Bring down** the 6 (ones)
6. **Divide** the 4 into 16 = 4
7. Then **multiply** $4 \times 4 = 16$
8. **Compare**
9. **Subtract**

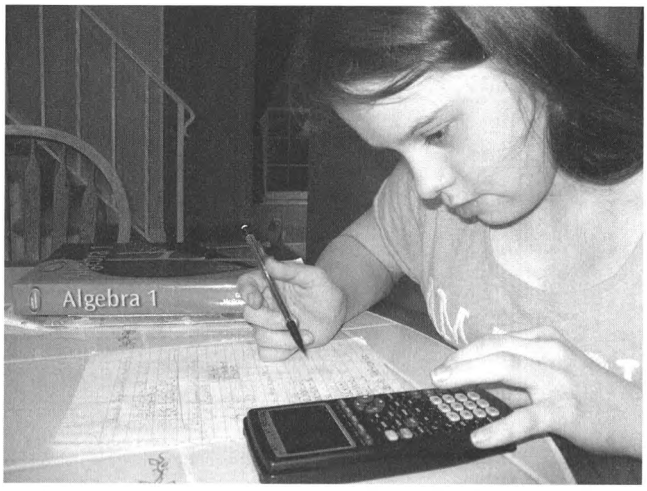
Most students who are hands-on learners have great difficulty estimating, remembering so many steps, lining up the numbers, etc. Therefore, in this book we will only discuss using a calculator for long division. Showing remainders as decimal fractions will be addressed, however, because of their use with money

USING A CALCULATOR FOR LONG (MULTI-DIGIT DIVISORS) DIVISION

OBJECTIVE: The student will be able do multi-digit division with the calculator.

MATERIALS:

- Calculators
- Calculator Division Problems (Appendix B-35)



SUCCESS STEP: Ask a student to do a simple one-digit division problem, using the calculator. Praise her successful answer or help her to correct the problem.

PROCEDURE:

1. Go through the first problem on the worksheet with the students.
2. Have them do the second problem and check it before assigning the sheet to be done independently.
3. Have the students complete all the Calculator Division Problems.

What about Division Facts?

If the student has learned most of the multiplication facts, she can probably learn the associated division facts. For example, if the student knows the 4s multiplication facts, she can probably learn to reverse them to come up with the division facts. Think of how you use the basic math facts. Many adults recall the multiplication fact when doing division. For example, in the problem 42 ÷ 6, most adults, think "what number times 6 equals 42?" Then they recall the multiplication fact of 7 x 6 = 42 rather than recalling a division fact.

REVERSING MULTIPLICATION PROBLEMS

OBJECTIVE: The student will be able to demonstrate with objects that division is the reverse of multiplication.

MATERIALS:

- Bingo chips, poker chips, or pennies (20 per student)

SUCCESS STEP: Ask the students to answer a simple multiplication problem (like 3 × 4 = 12). Keep giving examples until you have an easily recalled fact (answer under 20).

PROCEDURE:

1. Start with the multiplication fact that the students recalled in the Success Step.

2. Have them show you how to represent that fact with bingo or poker chips (making 3 groups of 4).
3. Ask them to tell you how many total chips there are (12).
4. Then say, "What if we *divided* these same 12 chips into 3 groups? How many chips would we have in each group?" See if they realize that the answer is already right in front of them.
5. If not, talk them through it... "How many chips total do we have here? (12). How many groups of chips are there? (3). So, if we divide 12 chips into 3 groups, how many chips are in each group?"
6. Show them on a whiteboard how multiplication is the reverse of division:

$$3 \times 4 = 12 \qquad 12 \div 3 = 4$$

7. Then do more examples of multiplication sums that they know very well, modeling with chips. See if they can understand that you should know the answer to the division problem if you can do the multiplication problem.

DIVISION WORKSHEETS

OBJECTIVE: The student will accurately do simple division problems with or without the calculator and check them with multiplication.

MATERIALS:

- Division Problems Checked by Multiplication worksheet (Appendix B-36)
- Pencil
- Calculator

SUCCESS STEP: Ask the student what kind of problems are shown on the worksheet. Praise the answer.

PROCEDURE:

1. Have the students do the first problem on the worksheet. Then compare answers.
2. Show the students how to check division problems. That is, after the students have solved the problem, have them multiply the two smaller numbers (the divisor and the quotient) to find the big number (dividend). For example, $16 \div 4 = 4$; $4 \times 4 = 16$.
3. Have the students do the entire worksheet and check each problem with multiplication.
4. Give the students more practice in using multiplication to check division by making up your own problems or using simple division problems from mass-market workbooks.

Division with Story Problems

Students need to have real-life and simulated experiences with sharing and grouping division. Some experiences will be simulated now and repeated when all types of story problems are discussed.

SAMPLE PROBLEMS A, B, AND C

OBJECTIVE: The student will be able to solve simple division story problems.

MATERIALS:

- A bag with 20 marshmallows in it
- Word Problem form (4 forms for each student) from Appendix B-37
- Post-It notes, if needed
- 35 straws or craft sticks per student
- Calculators

SUCCESS STEP: Have the student count the marshmallows that are in the bag out loud. Praise correct counting.

PROCEDURE:

1. Read Sample Problem A with the students:

SAMPLE PROBLEM A:

Emily's family wanted to roast marshmallows. There were 20 marshmallows left in the bag. There were 4 people in Emily's family. How many marshmallows could each person get (if they were divided evenly)?

2. Have the students work out the first problem, step-by-step, with you.
   - **a. Read and understand the problem.**
     - Write the numbers of the problem:
       4 in family, 20 marshmallows
     - Write the answer sentence (subject, verb, object) with a blank for the answer.
       Each family member could have ____ marshmallows.
   - **b. Draw (or act out) the problem (See step e, below.)**
   - **c. Find the operation:**
     - Will the answer be **larger than the largest** of the numbers in the problem?
       No, the largest number is 20, and no one in Emily's family could have more than the total number of marshmallows left in the bag. Therefore, the operation will be either subtraction or division.
     - Are the numbers for each person in the problem the **same or equal**? That is, do we need to make equal groups?
       Yes, each person will get the same amount. Therefore, the operation is division.
   - **d. Solve the problem.**
     - Using the calculator, enter the 20 marshmallows divided by 4 (number in family) = 5 marshmallows each. Remember to put the largest number in first.
   - **e. Check the above problem** by choosing 4 people and handing out real marshmallows or construction paper circles to represent marshmallows. If you do not

have 4 people, use Post-It notes to represent each member of the family. Deal out the marshmallows to each family member and show that each member gets 5 marshmallows.

The word problem form above is being introduced with division. However, this same word problem form can also be used with addition, subtraction, and multiplication problems. It is best to teach your student(s) how to use it once they are able to use all the operations and need to choose among the four of them. When students confidently use the form for division, you can have them use it with addition, subtraction, and multiplication word problems.

3. Use blank copies of the following word problem form (Appendix B- 37) for sample problem B (and for all word problems until the student does the steps automatically). If you are working with several students, you may want to print the form on colored paper so you can easily see if they are using the form.

SAMPLE PROBLEM B:
LaNell bought 18 American history stamps on her trip. If she gives the *same* number of stamps to each of her 6 friends, how many stamps will each friend get?

1) READ AND UNDERSTAND THE PROBLEM *(Write labeled numbers; e.g., 4 desks)* 18 stamps 6 friends **Write the answer sentence with a blank.** Each friend gets ____ stamps.	**2) DRAW (OR ACT OUT) THE PROBLEM** 1. ☐ ☐ ☐ 2. ☐ ☐ ☐ 3. ☐ ☐ ☐ 4. ☐ ☐ ☐ 5. ☐ ☐ ☐ 6. ☐ ☐ ☐
3) FIND OPERATION: Will the answer be larger than the biggest number in the problem? Are the numbers in the problem the same or equal? (Are their equal groups of items?)	**4) CIRCLE OPERATION** equal groups ⬇ Larger ➔ Addition Multiplication Smaller ➔ Subtraction Division
5) WRITE THE NUMBER SENTENCE AND SOLVE THE PROBLEM (e.g., 18 ÷ 6 = 3)	**6) WRITE ANSWER IN THE BLANK OF THE SENTENCE IN SQUARE 1.**

4. Try sample word problem C using the above form.

SAMPLE PROBLEM C:
Jake took 33 photos on his trip. If he divides them equally between 3 friends, how many photos does each friend get?

5. Have the students use manipulatives such as straws to check sample problems B and C.
6. Photocopy the following problems from A through F. Have the students do the problems independently using the structured word problem form.
    a. Caroline has baked 28 chocolate chip cookies. She is going to give them to 7 Cub Scouts to eat. How many can each Scout get (if she gives them each the same amount)?
    b. Heidi bought 24 potted flowers. She and her two children planted the flowers in the front yard. If the 3 of them each planted the same number of flowers, how many flowers did each person plant?
    c. Declan ordered a pizza that was cut into 12 pieces. Counting Declan, there are 4 guys who want some pizza. How many pieces of pizza should each guy get—assuming they each get the same amount?
    d. Katri bought some large beads to make a bracelet. The package says that there are 40 beads in it. She thinks that she may have enough beads to make 2 bracelets. If she separates the beads into 2 equal groups, how many beads will she have for each bracelet?
    e. Jeff has 15 tools. He wants to put an equal number of tools in each of his 3 toolboxes. How many tools should he put in each toolbox?
    f. Paul read a 60-page book on his vacation. He read 10 pages each day. How many days did it take him to read the whole book?

THE JOURNEY GAME

OBJECTIVE: The student will be able to do simple addition, subtraction, multiplication, and division number and word problems in a game context.

MATERIALS:
- The Journey game board from Appendix B-38
- Die
- Journey game cards for all operations from Appendix B-39
- Game markers

PROCEDURE:
1. Place the game markers on Start.
2. One student throws the die. She moves her marker the number of dots she has thrown with the die.

3. The teacher hands the appropriate card (addition, subtraction, multiplication, or division number and word problems) to the student. If the student can solve the number problem, with or without the calculator, she moves her marker one place ahead. If the problem is a word problem, the student moves ahead two spaces for each correct answer.
4. The players must follow the instructions given on the spaces where they land. If a player lands on an instruction space a second time, she does not have to obey the instructions.
5. The winner must land on the finish by exactly the right number. She may also win by doing a problem correctly and moving ahead one or two places.
6. You can control which problems the student gets, or you can put the appropriate cards face down on the game board and let the student choose.

GENERALIZATION INTO DAILY ACTIVITIES:

- One of the most common uses for division in everyday life is distributing a group of items equally (sharing). Teachers and families can label these situations as making it fair by division. Examples include the following:
  - handing out food such as cookies or candy,
  - dealing out cards in various games,
  - passing out arts and crafts or school supplies,
  - dividing up tickets for carnival rides or arcades.
- When your student asks you how many slices of pizza, pieces of bread, muffins, ears of corn, etc. she can have at a meal, have her do the division: "There are 8 slices of pizza and 4 of us.... If we share the pizza fairly, how many should we each get?"
- At school or home, divide tasks by number of days. For example: "We need to learn 15 new words this week. How many do we have to learn a day?" Or, "You need to read 20 pages in 4 days. How many pages should you read a day?"
- At school, when dividing students into groups, figure out how many people there should be per group: "We need to make 3 groups. There are 21 students. How many per group?"

COMMERCIAL GAMES TO BUY OR TRY

- ***Division War:*** Barnes and Nobles bookstores sells decks of cards with division facts for playing the popular game War. All the cards are dealt out to players. Then players flip over the top card in their stack and announce the answer of their division problem. The player with the highest answer takes the other players' cards. (You can also make your own division war deck with index cards.)
- ***Multiplication & Division Quizmo*** (World Class Learning Materials): A bingo-like game where one player reads out division (or multiplication) problems and the other players check their boards to see if they have the answer. If they do, they cover it with a plastic chip. The first player to fill in a line horizontally, vertically, or diagonally is the winner.

CHAPTER 14

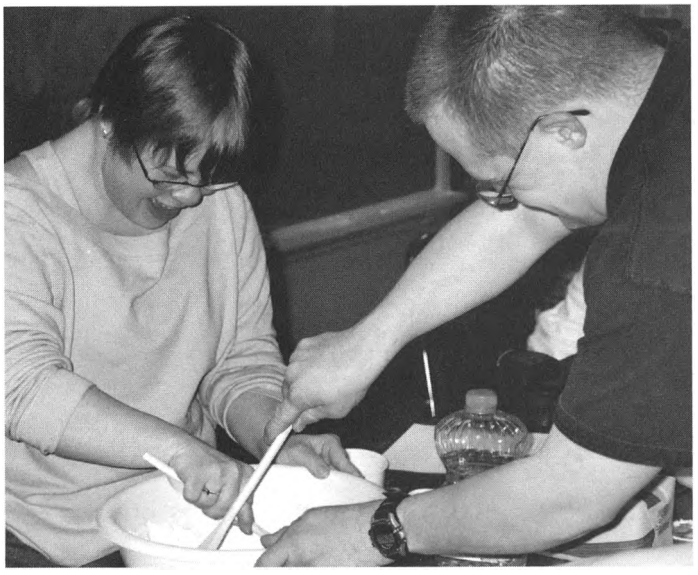
Simple Fractions

Pre- /Post Checklist

Can the student:

1. ____ Recognize one-half of an object and read the fraction as one-half (1/2) when written.
2. ____ Read the fraction 1/4 and demonstrate its meaning.
3. ____ Use halves and fourths in baking.
4. ____ Read the fractions 1/3 and 2/3 and demonstrate their meanings.
5. ____ Use thirds in baking.
6. ____ Read the common fractions (1/2, 1/3, 1/4, 1/5, 1/6, 2/3, 3/4, 2/5, 3/5, 4/5, and 5/6) correctly.
7. ____ Show that 1/2, 2/4 (and 4/8) are the same amount.
8. ____ Demonstrate that when a fraction has the same number on the top and the bottom, it means that the fraction is really the whole number 1.
9. ____ Read mixed numbers (1- to 2-digit whole numbers and simple fractions) as well as demonstrating understanding with real objects.
10. ____ Add fourths together and later add thirds.
11. ____ Use fraction strips to demonstrate and compare common fractions.
12. ____ Bake—using whole number fractions and mixed numbers.

Children often come to school knowing generally what half of an object is. They usually realize that both halves need to be the same size—or at least they will protest if their half of something desirable is smaller than the part given to another child. We can work from that simple concept to broaden a student's understanding of fractions that are necessary in daily life.



First we will work with common or simple fractions expressed as one number on top of another number such as $\frac{1}{2}$ (or, as typed, ½ or 1/2). Initially, the students need to be able to read common fractions accurately. Then they need to be able to read mixed numbers that contain a whole number and a fraction, such as 2 ½.

In everyday life, fractions are most often used in measurement. Cooking, using fractions of teaspoons, tablespoons, and cups, is probably the most common practical application of simple fractions. Measuring length or weight may also require the accurate use of fractions. Even if your student is included in a regular math class and will be expected to learn more complex math involving fractions, I recommend that you introduce him to fractions through the hands-on activities in this chapter. If a student has a concrete, visual understanding of what a fraction actually means, he is more likely to understand why the denominator stays the same when you add ¼ plus ¼, or why it makes no sense to add ½ to ¼ and get ⅙.

Different Styles of Writing Fractions

Handwritten fractions often look different from those in books or on a computer screen. We usually handwrite a fraction by putting one number on top of a line and the other number below the line; e.g., $\frac{1}{2}$. Handwritten fractions usually have smaller-sized numerals than whole numbers do because the two numerals in the fraction are written in the same space as one regular numeral is written. The same is sometimes also true of printed fractions. This difference in size can help students understand that fractions represent smaller quantities than regular whole numbers do.

In some computer and printing fonts, the first number (numerator) is printed just like a whole number, followed by a slash and then the denominator: e.g., 1/2. Some word processing programs automatically change the 1/2 to ½. Therefore, fractions can be written different ways in the same text. Your students need to learn that the different ways of printing do not change the value of the fractions. Be sure to show them visually how fractions can be written differently, but named and used the same—depending on how they are written in the student's text or tests.

Introducing Halves and Fourths

HALVES

OBJECTIVE: The student will be able to recognize one-half of an object and read the fraction as one-half (½) when written with numerals.

MATERIALS:

- Apples, pears, or some other fruit that can be divided into halves
- Other rewarding whole objects such as candy or granola bars
- Sharp knife
- Construction paper
- Slate, whiteboard, or tablet of paper
- Marker or chalk

SUCCESS STEP: Cut the fruit in half and put both halves in front of one student. Ask the student to give you one-half. Praise him for correct action or help him give you one-half of the fruit.

PROCEDURE:

1. Show your student(s) the half fruit used in the Success Step and ask him how much of the fruit this is (one-half). Then take another fruit and cut it so one of the pieces is much smaller than the other. Hold up the smaller piece and ask the student if this is one-half of the fruit. He should reply that the smaller piece is not one-half because the pieces are not the same. If he does not reply correctly, compare the smaller piece with half of the original fruit that you cut previously.
2. Write the fraction $\frac{1}{2}$ (with a straight line rather than a slash) on a slate or a whiteboard. Point out that the fraction is written with a 1 on the top and the 2 on the bottom but we call it *one-half.* Have students repeat the words *one-half* several times while looking at the written fraction. Show them on the board that sometimes one-half is written 1/2 because it is hard to type ½. They should practice reading $\frac{1}{2}$, ½, and 1/2 as one-half.
3. Tell the students that you are going to give one-half of a fruit (or candy bar or any whole object) to each person and they are to watch you so you give them each one-half. Cut fruit, granola bars, or candy bars, etc. in half and give them the halves. Make a "mistake" and cut one of the objects very unevenly. The students should protest that cut if they understand what one-half is. Cut another object correctly to give them equal halves. If you have only one student, you will have to have several different objects to cut in half.
4. Review that ½ is read as one-half.
5. Give each student a piece of construction paper and scissors and ask him to give you one half of his paper. If necessary, cut a piece of construction paper in half to provide a model.
6. Use other things in the environment that can be divided in half (pieces of chalk, pretzel rods, sandwiches, licorice, fruit leather, lengths of ribbon or string) and give the students chances to make two halves and give you one-half. Remind the students that breaking an object in half is the same as breaking it in the exact middle.

When Kayli was reading a list of common fractions, she correctly read 1/6 as one-sixth and 1/7 as one-seventh, but she read 1/2 as "one twoth" and 1/3 as "one threeth." She had correctly figured out the pattern for naming fractions from 1/4 to 1/9 and applied the pattern to 1/2 and 1/3. She needed to be taught how to read 1/2 and 1/3 as separate fractions that do not fit the pattern.

FOURTHS

OBJECTIVE: The student will be able to read the fraction 1/4 and demonstrate its meaning.

MATERIALS:

- Three pieces of construction paper for each student
- A marker or crayon for each student and the teacher
- A slate or a tablet of paper for the teacher
- Approximately 1 cup of uncooked rice for each student placed in a large bowl on the table
- An empty bowl for each pair of students
- A one-cup measuring cup
- A one-fourth-cup measuring cup

SUCCESS STEP: Put the construction paper on the table in front of the student and ask him to choose *one* piece of paper. Praise the student for only taking *one* piece of paper.

PROCEDURE:

1. Explain that each student has *one* big sheet of paper. Say, "I am going to make my one piece into smaller pieces."
2. Fold and tear the paper into fourths. Say, "I have four pieces of paper now. Is each piece of small paper as big as your one big paper?" (no)
3. Put the four pieces down on the table and fit them together into ONE BIG piece. "I have four parts of this one big piece of paper—1, 2, 3, 4." Pick up one of the smaller pieces and say, "Now I am holding one piece of the four pieces of paper. This piece is called one-fourth."
4. Write the fraction 1/4 on the slate and point to the 1, telling them that it stands for 1 piece out of the 4 pieces that make up the BIG piece of paper. Point out that the numeral 4 stands for the 4 pieces that make up the parts of the big piece of paper.
5. Ask the students to fold and tear their paper into fourths as you did. (Note: Students can also use scissors or someone else can tear the creased lines if a student is unable to tear it evenly.)
6. Ask students, in turn, to give you one-fourth of their piece of paper. Say, "Thank you for the one-fourth piece of paper."
7. Then give each piece back to the student, repeating "I give you back your one-fourth piece of paper." Use enough repetition so the students can repeat the term one-fourth easily. Write the fraction on the slate several times.

8. Review the fraction ½ by tearing a piece of paper in half and contrasting it with the ¼ pieces of paper that they have made. Tell them that the bottom number of a fraction tells you how many pieces or parts there are in all.
9. Show the students a one-cup measure and a ¼-cup measure. Using the rice, show them that four ¼ cups of rice fit in the one-cup measure. Point out that the ¼ cup only holds one-fourth as much as the whole cup.
10. Have the students put four ¼ cups of rice into the one-cup measure.

MORE FOURTHS

OBJECTIVE: The student will be able to read the fractions ²⁄₄ and ³⁄₄ and demonstrate their meanings.

MATERIALS:

- A piece of construction paper for each student and the teacher
- A slate or a tablet of paper for the teacher and chalk or marker
- 1 teaspoon and ¼ teaspoon for each pair of students
- 2 or 3 cups of uncooked rice in a bowl to be used for the group of students
- An empty bowl for each pair
- Three different colored full sheets of paper (for each student) labeled ¼, ²⁄₄, and ³⁄₄ on top of each piece
- A one-cup measure and a ¼-cup measuring cup for each pair
- Three clear glasses or bottles for each pair

SUCCESS STEP: Ask the student to fold and tear a piece of construction paper into fourths as he has done previously. Praise his success or assist him, if necessary.

PROCEDURE:

1. Have all students fold and tear the construction paper in fourths. Ask them to give you *two-fourths* of their colored sheet of paper. Then repeat with them giving you *three-fourths* of the paper.

   Note: *Do not attempt to teach them that ²⁄₄ equals ½ at this time. If a student who already knows about ½ figures it out, praise him, but do not explain at this time.*

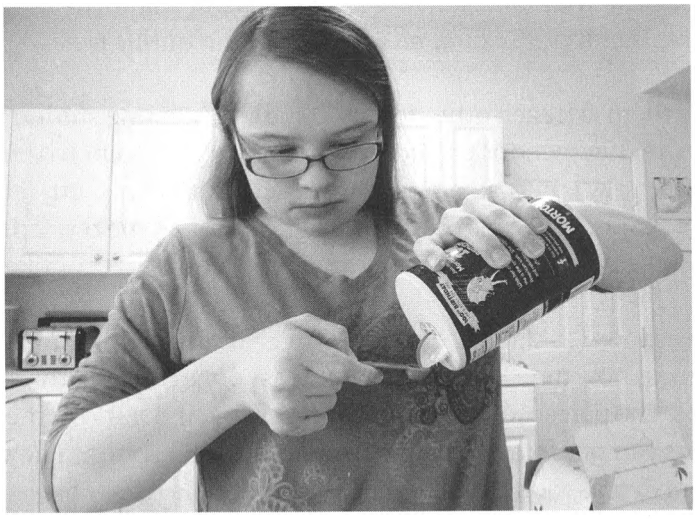
2. Pair the students up and have them ask each other for ¼, ²⁄₄, and ³⁄₄. If one student is not able to speak clearly, write the fractions on the slate and have him point to the fraction that he wants the other student to give him. After each pair is finished, let each one gently let the pieces of paper fall on his partner's head, as a joke-type reward.
3. Give each pair of students one teaspoon. Remark that this teaspoon is sort of like the spoon you use for eating cereal.
4. Then give the students a ¼ teaspoon. Tell them that this ¼ teaspoon is *really* a small spoon. Ask them if they can guess how many of the small ¼ teaspoons can fit in the 1 teaspoon. Have one member of the pair scoop up ¼ teaspoon of rice and put it in the 1 teaspoon. See if the students can put four ¼ teaspoons into the 1 teaspoon. They will have to have *level* one-fourth teaspoons to fill it exactly. Empty the teaspoon.

5. Have the other member of the pair scoop up 2/4 teaspoons of rice and put it in the bowl. He should fill the 1/4 teaspoon two times. Empty the teaspoons. Then the first person should put in 3/4 teaspoon by filling the spoon 3 times and putting the rice in the bowl. Reverse the partners and repeat.
6. Give each student three pieces of white paper which are labeled 1/4, 2/4, and 3/4. Direct each student to put 1/4 teaspoon of rice on the piece of paper labeled 1/4, 2/4 teaspoon on the paper labeled 2/4, and 3/4 teaspoon on the paper labeled 3/4. The teacher should be able to check these answers visually.
7. Ask the students which is the largest pile of rice. Point out that 3/4 teaspoon of rice is larger than 1/4 teaspoon of rice. Write the fractions 1/4, 2/4, and 3/4 on the board or pad of paper. Show them that if several fractions have the same bottom number, you can tell which fraction is larger by looking at the top number of the fraction. Put all the rice back in the central bowl.
8. Have the students repeat the procedure in steps 4 and 5 using a 1/4 *cup* measure and three clear glasses or bottles. Use a funnel, if it helps. Ask the student which glass has 1/4, 2/4, or 3/4 of a cup of rice. He should be able to tell you by looking at the glasses.

GENERALIZATION INTO DAILY ACTIVITIES: One way that you can add interest to visualizing fractions is to use the "Hershey Method." A chocolate Hershey bar has indentions dividing it into twelve parts (4 columns of 3 pieces). The parts are rectangular and look like the fractions the student has been making by tearing or cutting up pieces of construction paper. The parts can easily be broken to show fractions of the whole bar. The teacher needs to lead the group (or individual) in discovering 1/2's, 1/4's, and 1/8's. It is also possible to see equivalents such as 2/4's and 4/8's.

A book by Jerry Pallotta called *The Hershey's Milk Chocolate Bar Fractions Book* can be used to introduce the concept. It's currently out of print, but you can find copies in online and used bookstores and in the library. Of course, the students will enjoy the method more if they get to eat the results when the lesson is over.

Baking to Teach Fractions



It is important that students who are hands-on learners learn about fractions in a meaningful way, and baking is a good way for them to learn. I have included some simple recipes using the first few fractions that are introduced. You do not have to use these recipes. You can add your own recipes or some that the students really enjoy. If you do not have an oven available, you can use one of the microwave recipes or one that uses a waffle iron. If you are unable to cook in any way in your school, you can try the recipes for smoothies.

I have avoided using the abbreviations for teaspoon and tablespoon, as well as some of the more difficult cooking terms such as "cream." Your students should be concentrating on the fractions, not

the cooking. Later, you may want to make other recipes using cooking terms and the standard cooking abbreviations. You will have to explain these terms if the students are to independently use standard recipes.

At first, have students do each step with close supervision. If you are working with a small group, use a different student for each step and have the rest watch. Later on you may be able to assign tasks and have them work without much supervision.

BAKING WITH FRACTIONS

Some ingredients are used in many recipes. I keep the following supplies on hand:

- white flour
- white sugar
- brown sugar
- eggs
- milk or water
- cooking oil
- soft margarine
- vanilla
- baking soda and powder
- cooking spray such as Pam

I put the flour and the white and brown sugar in refrigerator storage containers with sealable tops. Many students handle the ingredients better if they are not in large quantities such as five-pound bags of flour. Before each cooking session, I buy the special ingredients such as nuts, chocolate chips, blueberries, etc. that are needed for a particular recipe.

Students need to learn how to pour or spoon the ingredients into the measuring utensils. They also need to know how to level the ingredients in the measuring cups with a knife. At first I check every ingredient before they mix it with anything else.

The following utensils are needed for the recipes in this section:

- a large bowl for dry ingredients
- a medium-sized bowl for the liquid and fat ingredients
- a portable electric mixer or a hand-operated mixer
- a square baking pan
- a rectangular baking pan
- a set of measuring cups with the fractions labeled clearly on each cup (for each student, if possible). (You may be able to find measuring cups in which each cup in the set is a different color or has color-coded fractions.)
- a set of measuring spoons with the fractions labeled clearly (for each student, if possible) (The labels on teaspoons tend to be quite small, so it helps to have a set where the labels or spoons are different colors.)
- mixing spoons
- oven timer
- oven, microwave, or waffle iron

I also cover the table with newspaper and put used packages and trash into a small plastic bag to be disposed of later.

I review the different cup measures and the measuring spoons before each session. The older students read the ingredients out loud and show which measuring cups or spoons they need to use. With younger students, I initially go through each measurement and step as a group. Later we just check vocabulary before they began.

The students should have time to put away the ingredients, clean the table, and wash the cooking dishes after each session. Of course, they should be able to eat some of the results at the end of the class. The first recipes listed (for cookies or waffles) have a short baking time, so students can eat them at the end of each session. Some recipes listed later, such as for muffins and breads, have baking periods as long as forty-five minutes. At home, this will probably be no problem, but teachers at school may have to arrange to have the students come back later to sample the results.

Teachers have told me that they have used a microwave successfully for some of these recipes. The microwave is faster, and more affordable for schools. I have found that microwaves vary in temperature. Toward the end of the cooking time, check your baking goods frequently. If your microwave does not have a turntable, rotate your pan once in the middle of the cooking cycle.

Students always seem to enjoy baking sessions—and eating what they have created. Some older students may already know how to make simple lunches or dinners that don't require much measurement. Baking is often a new experience for them, however. Students I have worked with needed quite a bit of supervision in the beginning. Later, they went right to the tasks and did them correctly, with only an occasional lapse. They often asked to do a certain recipe a second time and could usually do the task better than the first time. There was never any question about why fractions were needed because the students knew why from practical experience.

If you are working with younger students, you may have to do some of the preparations yourself. The purpose of baking is to show why fractions are useful and get students to associate fractions with a pleasant experience. If the baking itself is too difficult for your students, you will defeat the purpose of the activity. Have the students measure the ingredients each time. You may have to crack the eggs, do the final mixing, or handle the oven, but they will still learn about fractions and have fun eating the results. Students often work best in pairs, with one student evaluating the other's work and then switching roles.

The following recipes emphasize various fractions of teaspoons and cups. Be sure to give each student a chance to physically do the actions of measuring and mixing.

Food	Measurement
Pineapple Pound Cake	½ cup
Moist Sugar Cookies	½ cup, teaspoons
Moist Oatmeal Cookies	½ and ¼ cups
Microwave Applesauce Cake	½ and ¼ cups, teaspoons
Chewy Chocolate Brownies	⅓ cups, teaspoons
Microwave Brownies	⅓ cups, fourths
Velvet Waffles	All cups and teaspoons

BAKING USING ½ CUP MEASURES

PINEAPPLE POUND CAKE

OBJECTIVE: The student will be able to measure ½ cup of ingredients to use in baking.

INGREDIENTS:

- 1 package yellow cake mix
- 1 package vanilla pudding mix (instant)
- 4 large eggs
- 1 can pineapple slices (or crushed pineapple) drained
- ½ stick of butter
- ½ cup packed brown sugar
- ½ cup pineapple juice, drained from the canned pineapple
- ½ cup water
- ½ cup oil
- Angel food or bundt cake pan or two pie-sized small cake pans
- Cooking spray

DIRECTIONS:

1. In large bowl, combine the following ingredients:
    a. 1 package yellow cake mix
    b. 1 package vanilla pudding
    c. 4 eggs
    d. ½ cup oil
    e. ½ cup water
    f. ½ cup pineapple juice
2. Beat for 2 minutes with electric mixer.
3. Melt ½ stick butter in microwave. (Be sure to have a student cut the stick in half.)
4. Stir in ½ cup brown sugar into butter.
5. Spray pan thoroughly with cooking spray or shortening.
6. Arrange pineapple slices or crushed pineapple on the bottom of pan.
7. Put brown sugar and butter mixture in small pieces over the pineapple.
8. Pour cake batter over pineapple slices.
9. Bake at 350 degrees for 50-60 minutes.
10. Let cool and then invert the pan over cake plate and serve.

MOIST SUGAR COOKIES

Now that the students have used the ½-cup measures in the previous recipe, you may want to introduce ½-teaspoon measures. Show the students the 1 cup measuring cup compared to the ½-cup measuring cup. Then show the 1 teaspoon as compared to the ½ teaspoon. Have the student use two ½-teaspoons of rice to fill the 1-teaspoon measure. If you feel that the student understands the use of ½ cup versus ½ teaspoon, have him bake a recipe that has both ½ cups and ½ teaspoons in it, as follows:

OBJECTIVE: The student will be able to use ½-cup and ½-teaspoon measures in baking.

INGREDIENTS:

- ½ cup butter or margarine, softened
- ½ cup sugar
- 1 egg
- ½ cup cooking oil
- ½ cup powdered sugar
- ½ teaspoon cream of tarter
- ½ teaspoon baking soda
- 2 cups flour

DIRECTIONS:

1. In a large bowl, mix the ½ cup butter and ½ cup regular sugar. Stir until smooth.
2. Add 1 egg and ½ cup cooking oil and stir.
3. In a medium bowl, mix together:
   a. ½ cup powdered sugar,
   b. ½ teaspoon cream of tartar,
   c. ½ teaspoon baking soda. Stir.
4. Add dry ingredients to butter, egg, and oil mixture and stir well.
5. Drop mixture by teaspoonfuls onto a cookie sheet.
6. Press the cookies down with the bottom of a glass.
7. Bake at 350 degrees for 7 minutes. Do not over bake.

Scott measured the dry ingredients for waffles into a bowl. He started to pour salt into a measuring cup. An assistant called to him, "Scott, you are going to have awfully salty waffles if you use that much salt." Scott replied, "But the recipe says to use one-fourth cup salt." "Look again, Scott," said the assistant, "the recipe says one-fourth TEASPOON salt." The assistant then had Scott read aloud the complete fraction, including the name of the unit (teaspoon or cup), before actually measuring the ingredients.

BAKING WITH HALVES AND FOURTHS

MOIST OATMEAL COOKIES

OBJECTIVE: The student will be able to use halves and fourths of both cups and teaspoons in simple baking recipes.

INGREDIENTS:

- ¾ cup shortening
- 1 cup brown sugar
- ½ cup sugar
- ¼ cup water
- 1 egg
- 1 teaspoon vanilla
- 1 cup flour
- ½ teaspoon baking soda
- 1 teaspoon salt
- 3 cups quick-cooking oatmeal
- 1 cup chopped nuts
- 1 cup raisins

DIRECTIONS:

1. In a large bowl, mix together the following ingredients until fluffy and smooth:
   - ¾ cup shortening
   - 1 cup brown sugar
   - ½ cup white sugar

- ¼ cup water
- 1 egg
- 1 teaspoon vanilla

2. In a medium bowl, mix together
   - 1 cup flour,
   - ½ teaspoon baking soda
   - 1 teaspoon salt
3. Add these dry ingredients to the mixture in the large bowl.
4. Stir in 3 cups oatmeal until well blended.
5. Add nuts and raisins (optional) to the mixture.
6. Drop by teaspoonfuls onto greased cookie sheets.
7. Bake at 350 degrees for 12-15 minutes or until lightly golden.

MICROWAVE APPLESAUCE CAKE

The following recipe using halves and fourths can be baked in a microwave:

INGREDIENTS:

- 2 cups flour
- ¾ teaspoon baking soda
- 1 teaspoon cinnamon
- ½ cup raisins
- ¼ teaspoon salt
- 1 cup sweetened applesauce (can be chunky)
- ¼ cup butter or margarine
- 1 cup brown sugar, packed
- ½ teaspoon cloves
- ¼ teaspoon nutmeg
- Canned frosting, optional
- Cooking spray

DIRECTIONS:

1. Spray an 8" microwavable (glass, silicone, or heavy plastic) pan with nonstick spray.
2. Put butter in microwave in a large microwavable bowl for 40 seconds until melted. (Again, remember to have a student cut off ¼ of the stick.)
3. Add 1 cup brown sugar and stir.
4. Add 1 cup applesauce and blend well by hand.
5. In another bowl, mix:
   a. 2 cups flour
   b. ¾ teaspoon baking soda (show the students how to fill the ¼ tsp 3 times)
   c. 1 teaspoon cinnamon
   d. ½ teaspoon cloves
   e. ¼ teaspoon nutmeg
   f. ¼ teaspoon salt
6. Add the dry ingredients to the sugar, butter, and applesauce mixture.
7. Stir thoroughly—perhaps with electric beaters.
8. Add ½ cup raisins, stirring with a spoon.
9. Spread mixture in microwavable pan.
10. Microwave on high for 6 minutes. Rotate and cook for 2 more minutes, checking to see if done after each minute.
11. Cool in pan and frost if desired.

Note: *microwave ovens vary in temperature. Check the cake with a toothpick after the first 6 minutes.*

THIRDS

The other commonly used fractions are thirds. Recipes often call for ⅓ and ⅔ cups. Thirds, however, cannot be easily converted into halves, fourths, or eighths. Teach thirds as separate fractions without reference to the halves and fourths that the students have already learned.

INTRODUCING THIRDS

OBJECTIVE: The student will be able to read the fractions ⅓ and ⅔ and demonstrate their meanings.

MATERIALS:

- Strips of construction paper divided by lines into thirds
- ⅓ measuring cup
- 1 cup of rice
- Clear bowl
- Slate, whiteboard, or pads of paper
- Marker or chalk

PROCEDURE:

1. Using the construction paper strips divided into thirds, teach the students about thirds in the same manner as you taught about fourths above.
2. Have the students make ⅓ and ⅔ out of their construction paper strips by tearing or cutting them and test each other.
3. Have one student measure ⅓ cup of rice with the measuring cup and pour it into a clear bowl. Then have another student pour a second ⅓ cup of rice into the bowl. Ask the students to tell how much rice is now in the bowl (⅔ cup). Have them measure with the measuring cup to check.
4. Have the students write the fractions ⅓ and ⅔ on a slate, whiteboard, or pad of paper. Repeat procedures until you think the students are secure. Also review ½, ¼, and ¾ cups using the rice and cups.

BAKING WITH THIRDS

CHEWY CHOCOLATE BROWNIES

OBJECTIVE: The student will be able to use thirds in baking.

INGREDIENTS:

- 1 large package regular chocolate pudding mix (Do not use instant pudding)
- 3 eggs
- 2 teaspoons vanilla
- 1 cup chopped nuts (optional)
- ⅓ cup sugar*
- 1 cup flour
- ½ teaspoon baking powder
- ⅔ cup butter or margarine
- 1 cup sugar*

**The recipe actually calls for 1⅓ cup sugar, but we haven't introduced mixed numbers yet so the sugar is presented as two separate ingredients.*

DIRECTIONS:

1. In a large bowl, stir together until blended:
    a. pudding mix
    b. 1 cup flour
    c. ½ teaspoon baking powder
2. Melt ⅔ cup butter (in microwave or on the stovetop) and put in another bowl.
3. Stir in 1 cup sugar. Then stir in ⅓ cup sugar.
4. Beat in 3 eggs, one at a time.
5. Blend the dry ingredients with the butter, sugar, and egg mixture.
6. Add 2 teaspoons vanilla to the mixture and stir.
7. Stir in nuts (if desired).
8. Spread in a greased 9 x 13 pan.
9. Bake at 350 degrees for 20-25 minutes. Do not over bake.

MICROWAVE BROWNIES

OBJECTIVE: The following recipe uses thirds and can be made in a microwave.

INGREDIENTS:

- 1 cup flour
- ⅓ cup unsweetened cocoa
- ¼ teaspoon baking powder
- ¼ teaspoon salt
- ¾ cup chopped nuts (optional)
- cooking spray
- ⅔ cup butter or margarine
- ⅔ cup white sugar
- ⅓ cup brown sugar, packed down
- 2 large eggs
- 1 teaspoon vanilla extract

DIRECTIONS:

1. Coat an 8 or 9 inch microwavable square or pie pan with nonstick spray.
2. In a small bowl, microwave butter 20 seconds or until softened and pour into a large bowl.
3. Measure ⅔ cup white sugar and ⅓ cup brown sugar. Stir into the butter.
4. Add 2 eggs and 1 teaspoon vanilla and beat until well blended.
5. In another bowl, mix:
    a. 1 cup of flour,
    b. ⅓ cup cocoa powder,
    c. ¼ teaspoon baking powder
    d. ¼ teaspoon salt.
6. Add the dry ingredients slowly to the egg and sugar mixture. The dough will be very thick.
7. Stir and then add nuts (optional).
8. Spread the brownies evenly in the microwavable pan.
9. Microwave at 100% power for 5 to 7 minutes. After 5 minutes, check frequently (and rotate if needed).
10. Take the brownies out of the microwave. Set pan on a rack or a small bowl overturned to cool.
11. Sprinkle with powdered sugar or frost lightly if desired.

VELVET WAFFLES

OBJECTIVE: The following recipe uses thirds and can be made in a waffle iron.

INGREDIENTS:

- 1 cup flour
- 1 tablespoon baking powder
- ½ teaspoon salt
- 1 tablespoon sugar
- 2 eggs
- ⅔ cup milk
- ⅓ cup oil
- Cooking spray

Note: This is a favorite recipe of the young adults I have worked with. It requires a waffle iron and a little guidance during the baking process. They especially like it with strawberries and a little whipped cream.

DIRECTIONS:

1. Heat waffle iron and spray with cooking spray.
2. Mix dry ingredients together in large bowl.
   a. 1 cup flour
   b. 1 tablespoon baking powder
   c. ½ teaspoon salt
   d. 1 tablespoon sugar
3. Crack 2 eggs into small bowl and beat them with a fork.
4. Add ⅓ cup oil and ⅔ cup milk to the eggs in the small bowl.
5. Mix the liquids into the dry ingredients in the large bowl and stir or beat with electric beaters until smooth.
6. Use a half-cup measure to pour waffle batter onto waffle iron.
7. Cook the waffles, without opening the lid, until no steam is visible.

Makes 4-5 waffles.

The students I worked with seemed to learn very much about fractions and baking skills while making these recipes. They enjoyed the cooking (and the eating) so much that we did all the recipes in this chapter.

Other Common Fractions

The fifths and sixths are not that commonly encountered in everyday life, but by naming them (as well as other fractions such as eighths and sixteenths) the student can show that he understands the pattern used in naming all fractions. Note that fractions such as ⅜ that can be reduced to smaller terms are not taught here.

INTRODUCING FIFTHS, SIXTHS, AND OTHERS

OBJECTIVE: The student will be able to read the common fractions (½, ⅓, ¼, ⅕, ⅙, ⅔, ¾, ⅖, ⅗, ⅘, and ⅚) correctly.

MATERIALS:

- Chalk and slate or whiteboard and marker

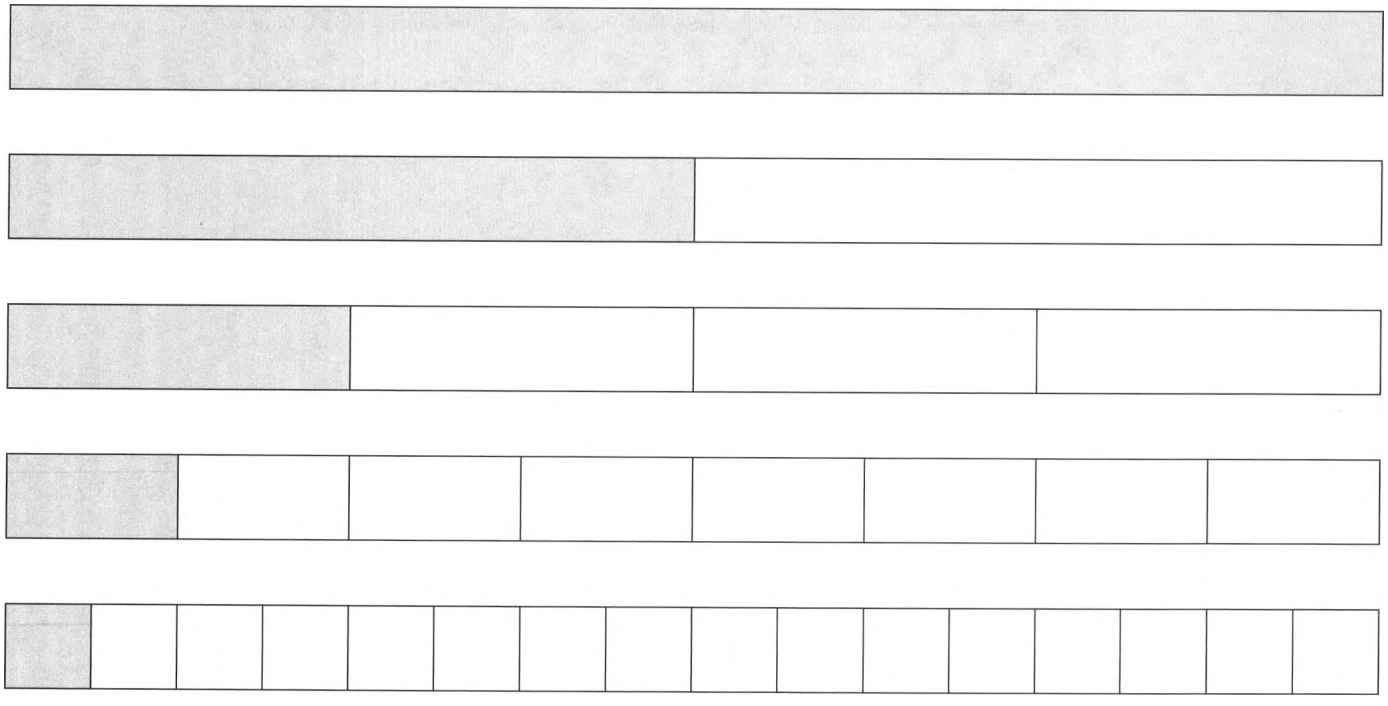
- Two copies of the Fraction Strips sheet (from Appendix B-43). (If possible, one sheet should be copied onto cardstock and laminated before cutting for further use. The second sheet is for the student to look at when he is working with the fraction strips.)
- Pen, pencil, or fine-line marker
- 6 different small objects such as a pencil, eraser, etc.
- Egg carton cut in half—leaving 6 cups
- 6 poker chips or magnetic bingo chips

SUCCESS STEP: Have the students arrange six objects in line. Say, "The pencil is first in line, the eraser is second. What position is the tennis ball in? (third).

PROCEDURE:

1. Tell the students that when naming fractions, you use the same words that you use for telling a position in line—third, fourth, fifth, sixth, seventh, eighth, etc.
2. Write fractions using 1 on the top and 2 through 6 on the bottom. Help the students to read them correctly. Tell them that ½ is the naughty one. He is not called 1 twoth or 1 second, but one half (maybe twoth sounds too much like tooth).
3. Review naming fractions for a few minutes at the beginning of several sessions. When the students are secure with the naming of fractions with 1 on the top, introduce fractions such as ¾ with other small numbers on top.
4. Show the egg carton with the 6 cups. Have the students put 1, and then 2, etc., poker chips in the cups. Give each person a turn and have everyone call out the name of the fraction that is formed; e.g., ⅙, ⅔, etc.
5. Have the students cut apart the top half (1—1/16's) of the fraction strip sheet from the appendix.

Fraction Strips



6. Have them label the 1, ½'s, ¼'s, ⅛'s, and the 1/16's with a pen or marker. Have them color each line a different color.
7. Cut out first the 1 whole, then the halves, etc., with you cutting your own sheet as the students cut theirs. Have them put fractions (each a different color) in separate piles in front of them.
8. Practice with the cutout pieces, showing how 2 halves make 1 whole and 4 fourths also make one whole, etc. Lay the cut-out pieces on the 1 whole to help the students see this relationship.
9. Call out fractions such as one whole, one-half, three-fourths, two-eighths, or three-sixteenths. Have the student show you that fraction with his fraction strips. Practice on other occasions.
10. When the student is confident with the fractions from the top half of the worksheet, continue as above using the tenths and the fifths.
11. Then proceed as above using the thirds, sixths, and twelfths. These fractions are probably the most commonly used ones in daily life.

Teaching the Parts of a Fraction

It may be important for the students to know the names of the parts of the fraction, especially if they are working in the general education classroom. The numerator is the number on the top of the fraction or the one that is written first. The denominator is the number on the bottom or the one that is written last. If appropriate, you can tell them that if you take the first two letters of the numerator and the first two letters of the denominator, you have NU DE (NUDE!). Using this memory aid, you can tell which is the first number written (NU) and the second number written (DE).

FRACTIONS AS WHOLE NUMBERS

OBJECTIVE: The student will demonstrate that when a fraction has the same number on the top and the bottom of the fraction, the fraction is really the whole number 1.

MATERIALS:

- ¼ cup, ½ cup, ⅓ cup, and 1 cup measuring cups
- 2 cups rice
- 2+ cups of water (colored with a little food coloring)
- Slate, whiteboard, or pad of paper with marker or chalk
- Fraction strips from previous activity

SUCCESS STEP: Ask the student to name the ¼-cup and ½-cup measures. Praise him for the correct answers or point out the correct cups and have him repeat the words.

PROCEDURE:

1. Have the students put 4 one-fourth cups of rice in the 1-cup measure. Write the fraction 4/4 on the board. Show them that whenever the fraction has the same top and bottom number it means that one whole unit is there. Write 4/4 = 1 on the board.

2. Then write 2/2 on the board. Ask each student to put 2 one-half cups of rice in the cup measure and identify it as 1 whole unit.
3. If you have 1/3 cup measures, ask the students how many thirds of a cup make one whole (3).
4. Ask them to chant:

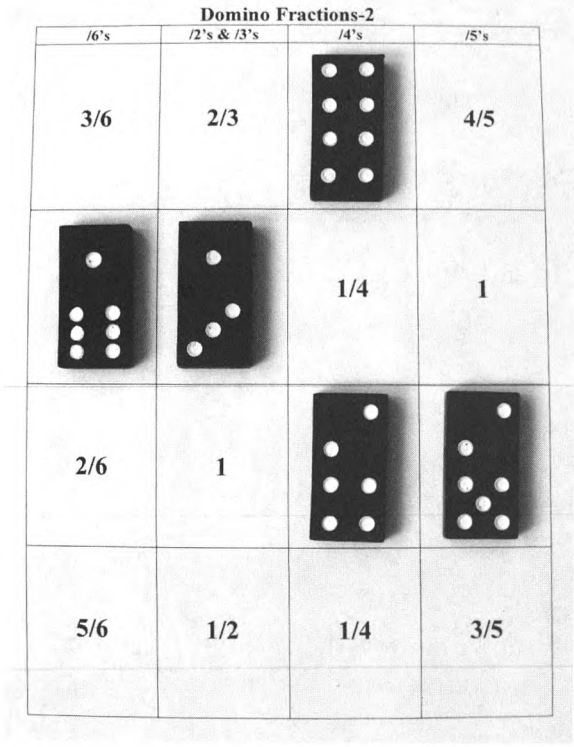
Fractions with the same
bottom and top
Are really number 1
so you can stop.

5. Have the students repeat steps 1 and 2 using water instead of rice. It is easier for them to see the quantity if you color the water slightly (and more fun also).
6. Using the fraction strips from the previous activity, have the student show you 2 twos, 3 threes, 4 fours, 5 fives, etc. Compare them to the 1 whole strip. Emphasize that these fractions are equal to one whole.

GENERALIZATION INTO DAILY ACTIVITIES: The next time you make a cake, brownies, etc., cut the baked goods into sixths, eighths, or another fraction. Ask the student(s) to tell you what fraction you've cut it into (e.g., sixths). Before you give out any of the pieces, point out that the whole has now been cut into 6/6 or 8/8. Ask them if that is the same as one. Do the same when you are cutting sandwiches or pizza at home.

DOMINO FRACTION GAME

OBJECTIVE: The student will be able to demonstrate by playing the game that he can read fractions correctly, including fractions which are equal to one.



MATERIALS:

- Dominos or domino fraction cards (Appendix B-41) (If using real dominos, use only the dominos with two sets of dots on them; put away the dominos with only one set of dots.)
- Individual domino fraction boards for each student, as found in Appendix B-42.

PROCEDURE:

1. Show some of the dominoes to the students, holding them so the half with the most dots is on the bottom. Tell them that the dots on the top of the domino represent the top number of the fraction, the line in the middle of the domino represents the line/slash of the fraction and the bottom section of the dots is the bottom of the fraction for this game. Turn all the dominos over on the table so the dots do not show.
2. One student turns over a domino, making sure that the smallest number of dots is on the top. He then names the fraction that it represents. If he has that fraction on his

game board, he puts the domino on top of it. If he doesn't have the fraction, he turns it over again on the table and another person takes a turn.

3. If a student turns over a fraction that is equal to 1, such as 3/3, he may put it on the number 1 on the game board.
4. The first student to cover one line on the game board—up or across—is the winner. Alert the students to watch where the dominos are that have not been used.
5. The important part of this game is to make sure the students are naming the fractions correctly. An adult or older student should play with a beginner to make sure he names fractions correctly. The game boards do not have any fractions that have the same number on the top as on the bottom. If the student picks up any domino that has the same number on the top and on the bottom, he should recognize that it is equal to 1. There are several 1s on each fraction board, so he can use that domino to cover any of those 1s.
6. The fractions are arranged on the fraction boards with one column showing sixths, another showing halves and thirds, another fourths, and another with fifths. Alert the students to this pattern so they can search their cards more efficiently.
7. The Domino Fraction game is played quite quickly like a combination of tic-tac-toe and Bingo. Students should be able to play several games at a time and keep track of their winnings.

Note: If you think that the students are having trouble with the procedure of counting the dots and then reading the numerals of the fractions, make a set of cards with the numerals instead of the dots and have the students read the fractions.

Lowest Terms

It is not necessary to teach your student(s) how to reduce *all* fractions to their lowest terms at this point. For now, teach them that 2/4 and 4/8 are the same as one half. This will familiarize them with the idea that some fractions are equivalent to each other. On a practical level, it will also help them see that it really doesn't make any difference if they use two 1/4-cup measure or one 1/2-cup measure while baking—they will get the same results. Just show them that the amounts are the same so some recipes will not confuse them.

INTRODUCING FRACTIONS EQUIVALENT TO ONE HALF

OBJECTIVE: The student will be able to show that 1/2, 2/4 (and 4/8) are the same amounts.

MATERIALS:

- 1/4-cup and 1/2-cup measuring cups for each student
- (1/8-cup measuring cup, if you have one)
- Approximately one cup uncooked rice or sand for each student
- Slate or whiteboard and chalk or marker
- Newspapers to catch spills

SUCCESS STEP: Have the student identify the 1/4-cup and the 1/2-cup measures. Praise him if correct or show him what each one is and then ask him to identify them again. He may look at the labels on the measures.

PROCEDURE:

1. Ask the student which of the two measuring cups is larger (the half-cup measure). Then ask the student to fill up the ¼-cup measure and dump the rice into the ½-cup measure. Then ask him to put another ¼ cup of rice into the ½-cup measure. If he is careful, the ½-cup measure should now be full.
2. Write on a slate or whiteboard the sentence: ¼ cup rice and ¼ cup rice = ¾ cup rice, which is the same as ½ cup rice. Then write directly below that sentence:
   ¼ + ¼ = ¾ or ½.
3. Have each student repeat step 1 himself and experience that two fourths equals one half.
4. If you have ⅛-cup measuring cups, repeat steps 1 and 2, pouring four ⅛ cups into the half-cup measure and writing:
   ⅛ cup + ⅛ cup + ⅛ cup + ⅛ cup rice = ⅝ cups rice or ½ cup rice.
5. Review the above activities several times at different math sessions. (You may wish to change the materials to cereal or small dried beans so that the students can generalize to other materials.)
6. Ask each student, "Which is larger, ¾ cup of rice or ½ cup of rice?" They should say that they are the same.

Mixed Numbers

Students often have difficulty reading and writing mixed numbers—numbers that have a whole number and a fraction. Perhaps the small difference in spacing between the whole number and the numerals in the fractions makes it difficult to see which is the large number. When the fraction is written as numeral / (slash) numeral (e.g., 5/6), there is no difference in size to use as a cue. Sometimes the students see the whole number as part of the fraction, such as reading 2 5/6 as 25 sixths. When first introducing mixed numbers, I put two or three spaces between the whole number and the fraction to make the difference more obvious (2 5/6). Later, the extra space can be faded so the students can read mixed numbers in commercial textbooks, workbooks, etc.

INTRODUCING THE CONCEPT

OBJECTIVE: The student will be able to correctly read mixed numbers (1- to 2-digit whole numbers and simple fractions) and demonstrate understanding with real objects.

MATERIALS:

- Construction paper with 5 to 6 pieces of the same color for each student
- Teacher-made sets of whole and fractional sheets of construction paper to represent the mixed numbers: 1½, 2½, 3½, 1¼, 1¾, 2¾, 3¾. Make each set using the same color of paper (e.g., represent 1½ by putting a whole sheet and a half sheet of blue construction paper together.)
- Various fruits (apples, oranges, pears, etc.) as used above, one fruit being cut in half
- Candy or granola bars

- Knife for teacher
- Slate, whiteboard, or pad of paper and marker or chalk

SUCCESS STEP: Ask the student to give you one-half of one of the fruits. Praise him for the effort.

PROCEDURE:

1. Hold up the half fruit and then a whole one of the same fruit. Ask the students how many of the fruit you have in your hands. Help them to see that you have one whole fruit and one half fruit.
2. Ask, "How do you think we can write this number?" Using the board or pad, write the number 1½. Point out which is the whole number and which is the fraction.
3. Give each student 1 and ½ pieces of the same color construction paper. Ask them how much construction paper they now have. They should say, "One and one-half pieces of paper." Point out that you say the word *and* between the whole number and the fraction. If they say, "One big piece and one little piece," say, "This is a one WHOLE piece and this is HALF of a piece."
4. Give the students one more piece of the same color construction paper. Have them tell you that they have two and one-half pieces. Let each student write *2 and ½* on a paper for himself. Then have him write 2 ½ as is normally done (but with a little extra space between the whole number and fraction).
5. Write mixed numbers on the board and see if the students can name and make them with construction paper. Use mixed numbers such as 3½, 1½, 2¼, 2¾, 3¼ with small whole numbers and only the fractions that the students already know.
6. Have the students make their own mixed numbers to show to you or the rest of the class. Remind them that they can fold and tear the pieces of construction paper into halves and fourths. Have one student show the pieces of construction paper that make up his mixed number and see if the others can read it. If you have only one student, you will have to read the student's mixed number yourself.

Fraction Equivalents

Commercial materials often use pies or pizzas to demonstrate fractions. I have found that dividing pies works well for halves, thirds, and fourths. However, when the circles are divided into fifths, sixths, or more pieces, it is difficult to see how the slices are different, especially if the fractions are close in value. You may want to use commercial learning games such as *Pizza Party* and *Pizza to Go* (Ideal) or *Pie in the Sky* (Learning Resources) for their interest value, but do not be discouraged if the student has difficulty with the smaller fractions. (See Resources for sources of commercial games.)

It might be important for your student to learn to do the typical type of worksheet often given in schools for fractions. The Fraction Strips chart (Appendix B-40) shows one whole (8½ inches long) divided into halves, fourths, eights, sixteenths, and also thirds, fifths, sixths, tenths, and twelfths. This worksheet is semi-concrete, however. On this worksheet, the fractions, similar to the construction paper pieces the students have been using, are now turned into strips that make it a little easier to see equivalents when they are divided into many small fractions.

FRACTION STRIPS EQUIVALENTS

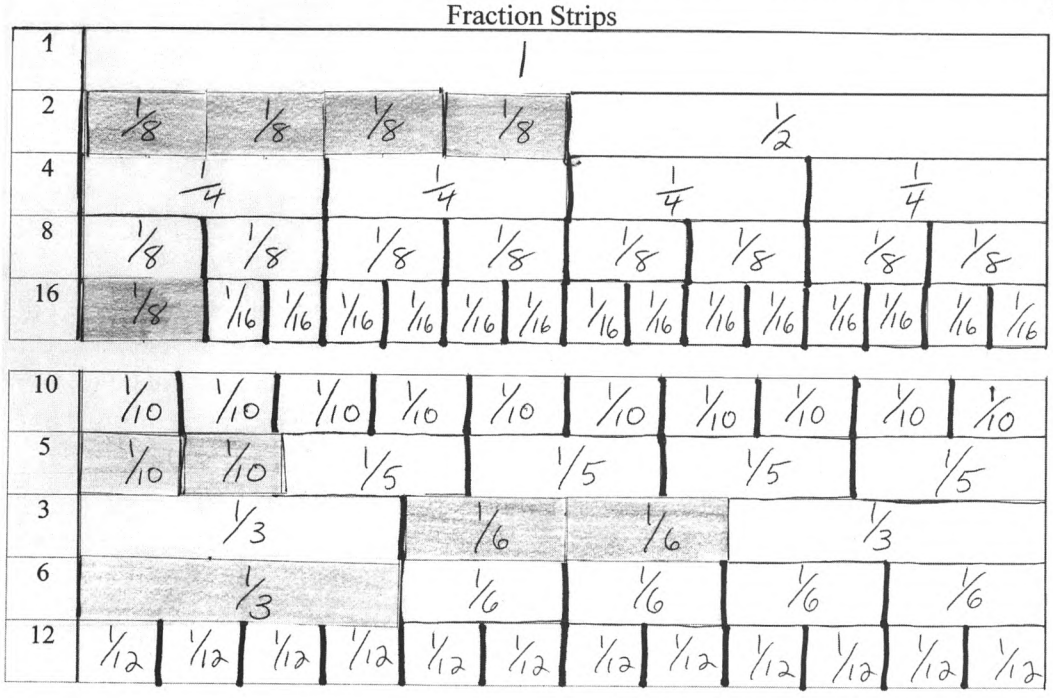
OBJECTIVE: The student will be able to demonstrate with fraction strips what fractions are equivalent to each other.

MATERIALS

- Fraction strips used in the section "Other Common Fractions" on page 208
- Uncut Fraction Strip sheet (Appendix B-40)
- Fraction Strip Worksheet (Appendix B-43)
- Chalkboard or slate, chalk
- Equivalent Fractions Worksheet (Appendix B-44)

PROCEDURE:

1. Review the Fraction Strips chart (B-40). Go through the first five fraction strips (1, 2, 4, 8, and 16) and point out what the equivalents are. Have the students label each division with black pen or markers (e.g., label each half as ½).
2. Have the students put the cut fraction pieces on the uncut Fraction Strip sheet to show that 4 eighths are equal to 1 half, 2 fourths are equal to 4 eighths or 1 half, etc.
3. Then do the same with the tenths and fifths.
4. Later, do the fraction strips that show the equivalents of thirds, sixths, and twelfths.
5. Using the slate or chalkboard, show the students that four of the sixteenths can be written 4/16.
6. Call out some of the fractions on the top part of the fraction chart and have the students point to them on their strips. When you think that they have the idea, do two problems on the Fraction Strip Worksheet (B-43) with them and check their answers.
7. Have the students independently do the Fraction Strip Worksheet. They need only recognize the divisions of the fractions to do this worksheet. Repeat if necessary.
8. For additional practice in fraction equivalents, try the Equivalent Fractions Worksheet (B-44) with the students. Show them how to figure out the first two problems, and then have them independently do the rest.



Adding Simple Fractions

In survival math, there is not much need to add or subtract fractions, especially fractions with different denominators. True, adults who are doing some or all of their menu planning or shopping independently may sometimes encounter situations where being able to add fractions might be helpful. However, there are usually practical ways to get around adding the fractions. For example, an adult might want to make two batches of a muffin recipe. If he knew how to add fractions, he would realize that if one batch calls for ⅔ cups of sugar, two batches must call for ⅔ or 1⅓ cups. If he cannot add the fractions, however, he can measure out ⅔ cups of sugar twice and get the proper amount. For another example, he may be planning to make recipes during the week that call for ½ cup of milk, 2¼ cups of milk, and 1½ cups of milk. Some of us might add up the amounts and make sure we have at least 4¼ cups of milk on hand. But others of us (even those of us who can add fractions) might just buy a half gallon or gallon and assume that will be plenty.

However, especially if the students are participating in the general math curriculum, it is important to learn how to add fractions with the same denominator (with the same last names). It is also important to understand that you cannot add fractions together if their denominators are *not* the same (for instance, adding ½ and ¼ to get ⅔).

INTRODUCING THE CONCEPT

OBJECTIVE: The student will be able to add fourths together. Later he will be able to add thirds.

MATERIALS:

- Slate or whiteboard and chalk or marker
- Juice, water, or other beverage
- ¼- and ⅓-cup measuring cups
- Clear drinking glasses

SUCCESS STEP: Ask the student to pour the juice or other beverage into the ¼ measuring cup. Assist him, if necessary, and praise.

PROCEDURE:

1. Ask the students to pretend that they have been running and are very thirsty. "Would this much juice (water) be enough for you to drink?" (No)
2. Tell the students you want them to show you how much they would want to drink when they were thirsty, but that you want to measure the total amount they pour into the glass. How could you measure the amount if you only have a ¼ measuring cup? They should say to use the ¼ measuring cup several times.
3. Let one student pour the ¼ cup of juice into the clear drinking glass. Then ask them how many ¼ cups to pour in the drinking glass and do the pouring.
4. Write the fractions (¼) on the board according to the number of quarter cups that have been poured in the drinking glass. Show the students how to add the top

Common Denominators

Sometimes the student may be included in general math classes or need to add or subtract fractions with different denominators. Math classes usually spend a lot of time learning to find the lowest common denominator so fractions can be added or subtracted. Most of the time the fractions involved have one-digit denominators. Why not use the easiest common denominator for two fractions—the product (answer in multiplication) of the denominators of each fraction? For example:

$$1/6 + 1/4 = ____.$$

Conventional math requires you to find the lowest common denominator, which is 12. Then you have to figure out how many 12ths are the same as 1/6 and 1/4. Then you add $2/12 + 3/12 = 5/12$.

The easiest common denominator would be $6 \times 4 = 24$ (multiply denominators by each other).

It is easier to convert 1/6 to 24s by multiplying by the other denominator (4). $1/6 = 4/24$. Likewise $1/4 = 6/24$. $6/24 + 4/24 = 10/24$, which is the same as $5/12$. (You will probably have to teach the student to reduce to lowest terms. Again, a fraction calculator can be handy for these kinds of operations.)

numbers together, e.g., 1/4 + 1/4 + 1/4 = 3/4. Don't let them pour more than 4 of the 1/4 cups into the drinking glass.

5. Tell the students, "You can only add the top numbers if the bottom numbers are the same.... If they have the same last names (fourths)."
6. Have one student pour a beverage by 1/4 cups into the drinking glasses while the teacher or another student writes the number sentence on the slate.
7. Let the students drink the beverage.
8. Later have the students repeat the procedure using the 1/3 cup measure.

Note: *You may want to show the students what happens if the bottom numbers (denominators) are not the same. Show them that 1/2 + 1/2 cups of drink fills the whole cup. BUT if you add 1/2 + 1/2 WRONG by adding the top numbers and the bottom numbers you get the fraction 2/4 which is a lot smaller than what the real drink shows.*

GENERALIZATION INTO DAILY ACTIVITIES:

- Practice folding a letter into thirds so it will fit in a business-size envelope.
- Find a recipe for potpourri that uses fractional amounts of ingredients. Help your child make it as a present for someone.
- Make homemade play dough using a recipe that uses fractional amounts. You can find many interesting recipes for play dough on the Internet such as Peppermint Play Dough, Jell-O Play Dough, and Edible Play Dough. For examples of recipes, see www.recipegoldmine.com.

- Make a special mixture of birdseed for your feeder using fractional amounts of different kinds of seeds (e.g., 1½ cups of sunflower seed, ¾ cups of millet seed, 1⅓ cups corn kernels).
- Make your own trail mix, designing the recipe so your student uses the fractions he needs to practice (for example, 1½ cups of cereal, ¼ cup of M&M's or chocolate chips, ⅔ cup of raisins, ⅓ cup peanuts).
- When you are trying out a new cereal (or other food item such as chips, peanuts, raisins, or ice cream measured in cups), check the packaging to see what a serving size is and then measure out that amount.
- If you are watching your cat's or dog's weight, let your child measure out the right amount of dry food for your pet.
- Search the Internet or cookbooks for appealing recipes that include fractions and that your child or student can help you make or can make independently.

Commercial Games to Buy or Try

- Play **One! The Fraction Solution** (card game, available from educationallearninggames.com). There are several games to play using cards with visual representations of the most common fractions. The games are designed to teach about fractions that equal one, as well as equivalent fractions.

- Play **Pie in the Sky** (www.learningresources.com). You spin to see what fraction of a pie you earn and place the pieces on pies on a board, trying to build a whole pie.

CHAPTER 15

Money

Pre- /Post Checklist

Can the student:

1. ____ Identify 1, 5, 10, and 20 dollar bills.
2. ____ Skip count $5s, $10s, and $20s to $100 in currency.
3. ____ Count pennies to 20, labeling the answer as cents.
4. ____ Read prices that have both dollars and cents.
5. ____ Round off money amounts to the next highest dollar.
6. ____ Use the next-highest-dollar strategy for prices that have both dollars and cents.
7. ____ Use money to buy one item, then several items.
8. ____ Identify commonly used coins
9. ____ Identify the value of commonly used coins.

The area of money handling is, perhaps, the most important use of mathematics for all of us—including for students who are concrete thinkers. The ability to handle money is certainly a survival skill in modern life. Students with disabilities can learn to handle money with a calculator and support from home or from the community.

In my opinion, one of the barriers that keeps students with disabilities from succeeding with money is that we spend so much time in their early years teaching them to identify coins, learn their worth, combine them to make needed amounts,

and count them, count them, and count them. Coins are not the simple first choice for learning about money. Coins are often difficult for students for the following reasons:

- Their size does not reflect the amount they are worth (nickel bigger than dime, etc.).
- We show coin value in two confusing ways—using a slashed "c" or as numbers after a decimal point.
- Naming the numbers to the right of the decimal point does not follow the same pattern as naming the numbers to the left of the decimal point.
- Most calculators delete the 0 at the end of a number so that $.80 becomes .8—causing confusion for children who have little or no understanding of decimals.

In addition, it often does not make sense to spend a great deal of time teaching how to count coins for these reasons:

- Many students have already figured out that they can avoid counting coins by paying with a dollar bill and letting the cashier figure out the change.
- Many vending machines display how much money you have inserted, so you don't have to figure out the amount yourself.
- Many cash registers display the amount of money you should get back in change, or even automatically release your change down a chute, so the cashier doesn't even have to count it out.

I think that we have traditionally started teaching with coins first because we figured that young students would have more experience with coins. That is not necessarily true in our inflated economy. Students are deluged with ads and pictures about items costing more than just change.

In this book, we begin by teaching about currency because it is much easier, and because we can teach strategies to pay for most things with currency. This book does teach what the coins are and what they represent, but there are no activities related to counting change or finding how many ways you can use coins to get $0.85.

One group of older students told me that they don't use coins for anything but vending machines. Even with vending machines they usually use quarters, only dealing with nickels or dimes as they receive them in change. The students also said, "The other coins don't always go in the vending machine right. Quarters are heavy, and we can hear them plunk into the machine, and then we get our Cokes."

The following topics are covered in this chapter:

- **Currency**
  - ▼ Currency identification (sorting)
  - ▼ Beginning functional use of money
  - ▼ Skip counting with money
  - ▼ Concept of cents

- Reading dollars and cents prices
- Next-highest-dollar strategy
- Purchasing items
- Using the calculator for money handling

- **Coins**
  - Recognition
  - Values of coins

American Currency

American currency is rather difficult to identify compared to the various colored currencies of other countries. There is a different picture of a person on each bill, but the coloring is the same, and one must look at the details to distinguish one person from another. Students may have difficulty seeing the difference between the one- and the ten-dollar bill because the numerals look similar against the dark, detailed background. The United States government has started adding subtle colors to the traditional green background of the five-, ten-, twenty-, fifty-, and hundred-dollar bills, but the bills are all still predominately green in color. In addition, the traditional all-green bills will still be legal and used for quite a long period of time. (You can find information about upcoming changes to the designs of U.S. currency at www.newmoney.gov.)

MATCHING AND RECOGNIZING CURRENCY

OBJECTIVE: The student will be able to identify a one-, five-, ten-, and twenty-dollar bill.

MATERIALS:

- A set of realistic one-, five-, ten-, and twenty-dollar bills, such as the following:
  - *Hands on Money* from www.AttainmentCompany.com (800-327-4269), around $30
  - other less expensive realistic play money
  - real money if you are teaching just one student or your own child
  - one-side printed play currency from Appendix B-45

SUCCESS STEP: Show the student a one- and a five-dollar bill. See if she can identify either bill. If not, tell her what they are. Then have her handle the bills and repeat their value. Praise her money skills.

PROCEDURE:

1. Now see if the student can already name the ten- and twenty-dollar bills (as well as the one and five.) Show her each bill and see if she can tell you what it is. If she can, skip this identification procedure and skip down to step 6.
2. *Matching:* Put a one- and a five-dollar bill in front of the student. Then show her another one-dollar bill and see if she can match that bill to the one-dollar bill in front of her. If she cannot, explain the features of the one-dollar bill and have her point to them on the bill in front of her.
3. If she matches the one-dollar bills, repeat the matching with the five-dollar bills using the same one- and five-dollar bills in front of her.
4. Repeat the matching procedure with the ten- and twenty-dollar bills. If you are using real currency, do not mix newer colored tens and twenties with the old bills.
5. Lay all four denominations of bills in front of the student and ask her to match a bill that you hold from the four choices. Repeat until she seems secure in the procedure.
6. *Recognizing:* Now ask her to listen to you and point to the bill you are talking about. This task is more difficult because she does not have a visual cue and because she has to decide from a brief spoken phrase. At random, say clearly the name of one of the four bill choices. Praise her for the correct choice. If she is incorrect, pull the incorrect bills away from her, leaving the correct bill close to her hand and repeat the request. Practice recognizing all four bills using the above procedure.
7. To accomplish the objective, the student only has to be able to recognize all four types of bills correctly. However, you may want to see if she can *name* each bill correctly.

BEGINNING FUNCTIONAL USE OF CURRENCY

OBJECTIVE: The student will indicate that she has some idea of what the four denominations of bills ($1, $5, $10, and $20) can buy by matching pictures or words with the appropriate amount of currency.

MATERIALS:

- Realistic play money
- 12 index cards with pictures of items (such as from sales fliers) that cost roughly $1, $5, $10, or $20 glued to them. (Find 3 pictures for each denomination—e.g., a small bag of French fries, cheap pen, or small ice cream cone for $1; a birthday cake, playground ball, or fast food combo for $5; a movie ticket, a music CD, or action figure for $10; or a movie DVD, baseball cap, or disposable camera for $20)

SUCCESS STEP: Have the student help you set up the index cards showing the one-dollar and the ten-dollar items. Put them up against glasses or mugs so that they are standing up like you would see them in the store. Praise the student for her help.

PROCEDURE:

1. Give the student a one- and a ten-dollar bill. Pretend you are a store clerk, and she is the customer.

2. Show her several items that are for "sale." Ask her to choose one. Then ask her whether she is going to pay with a one- or a ten-dollar bill. If she is correct, exchange the money for the appropriate index card. If she is incorrect, laugh and say, "Try again." If she needs assistance after several trials, you can say, "That's too much" or "That's not enough." Repeat with the rest of the one- and ten-dollar items.
3. Repeat the procedure at another session with the five- and twenty-dollar items.

This exercise is more of an awareness exercise than a specific training. We want her to learn that some items cost more than others and that some denominations of currency are worth more than others, but she is not required to learn all the names of the various bills to be reminded of the usefulness of money while working on all the activities in this chapter.

Skip Counting with Money

Although this book emphasizes the importance of using a calculator, counting currency is one skill that is worthwhile for students with math difficulties to learn to do without a calculator. Using a calculator at the same time you are handling money can be cumbersome, especially if you are actually buying something at a store. Your students may be able to count their money accurately without using a calculator if they are able to skip count by fives, tens, and twenties. Skip counting is done when the student names only some of the numerals in a consistent pattern. For example, we can count by fives by saying "5, 10, 15, 20." The skill is useful in counting money and in understanding multiplication.

COUNTING BY TENS

OBJECTIVE: The student will be able to count by $10s to $100.

MATERIALS:

- 5 bundles (with 10 each) of straws or sticks for each student.
- 5 ten-dollar bills in play money for each student.
- 10 one-dollar bills in play money

SUCCESS STEP: Ask a student to bring you one bundle of straws or sticks (with ten items in it). Praise her.

PROCEDURE:

1. Give each of the students five bundles of straws (ten straws in each bundle). Count them out as you put each bundle down: "10, 20, 30, 40, 50."
2. Ask them to recount to see if you counted right. Have them *chant* as they put down each bundle, "10...20...30...40...50." Repeat as necessary.
3. If your students do not understand that a ten-dollar bill is worth as much as 10 one-dollar bills, demonstrate that for them now. Explain that the ten-dollar bill is like 10 one-dollar bills bound together as they have done with straws. Point out all the 10s in the corners of the ten-dollar bill and show them where it says "ten dollars." You may want to put 10 one-dollar bills together with a paperclip and exchange it for a ten-dollar bill. Repeat as necessary.
4. Now model counting ten-dollar bills to $50. Have the students repeat with their ten-dollar bills. As you are practicing counting ten-dollar bills to $50, occasionally stop and check to see if the student remembers that 1 ten-dollar bill is the same as 10 one-dollar bills. Trade the student a bundle of 10 one-dollar bills for a ten-dollar bill every once in a while and see if she can still count by tens to $50.
5. Have the students work in pairs and put their five bundles with the other person's five bundles so they both can count to 100 by tens.

Varying Manipulatives

Sometimes students get tired of using manipulatives for math. I have found that changing the type of counters you are using can reawaken their interest. For instance, try using straws, sticks, toothpicks, licorice, raisins, and stick pretzels. For items that cannot be bundled together like sticks, put ten items into a small plastic bag that zips closed.

Once I ended up doing a small group math lesson at McDonald's. As you might have guessed, we counted French fries.

COUNTING BY TWENTIES

OBJECTIVE: The student will be able to count by $20's to $100.

MATERIALS:

- 5 twenty-dollar bills in play money for each student

PROCEDURE:

1. If your student(s) can understand that a ten-dollar bill is the same as 10 one-dollar bills, she should also be able to grasp that a twenty-dollar bill is the same as 20 one-dollar bills. Check to make sure. Show her a twenty-dollar bill and ask her, "How much is this worth?" If she answers incorrectly, show her the 20s on the bill. You may also give her several bundles of 10 one-dollar bills and ask her to show you how much the $20 bill is worth.
2. If you really want to make sure she understands the relative value of currency, show her a twenty-dollar bill and a stack of 10 one-dollar bills. Ask, "Which would you

rather have?" Exchange the twenty-dollar bill for 20 one-dollar bills if she answers incorrectly, so she can see that you get an even bigger stack of ones for a twenty-dollar bill.

3. When you are sure the student understands how much the twenty-dollar bill is worth, tell her you will show her how to count twenty-dollar bills. Count out five twenty-dollar bills for each student: "20, 40, 60, 80, 100."
4. See if the students can chant the numbers back when they recount the money themselves. If they can't remember the sequence, try showing them how counting by twenties is similar to counting by twos. Write 2, 4, 6, 8, 10 on a chalkboard or paper and show them how you just add a zero after each number to get 20, 40, 60, 80, 100. If they know the cheer, "2, 4, 6, 8, who do we appreciate?" you could try teaching them the response, "20, 40, 60, 80, we appreciate that lady!"
5. If the student can count by tens but is having difficulty with counting by twenties, you may want to have her use her skill with counting tens to count twice on the $20 dollar bill. For example, show her how to make two touch points on the twenty-dollar bill. Have her touch Jackson's left eye and say "10" and touch his right eye and say "20." On the next bill, she can touch one eye and say "30" and touch the second eye and say "40," etc. The student only has to count by twenties to 100 at this time.

COUNTING BY FIVES

OBJECTIVE: The student will be able to count by $5s to $50.

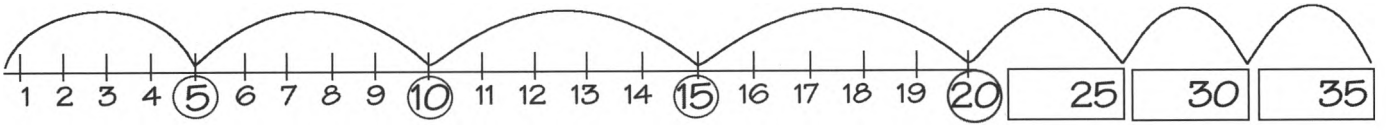
MATERIALS:

- 10 five-dollar bills in play money for each student.
- A number line from 1 to 20 with numbers one inch apart.

PROCEDURE:

1. First, try to teach counting five-dollar bills by having the student chant 5, 10, 15, etc. as she touches or moves each bill. If she can count by fives correctly, you can just practice in this manner. Frequently, however, students have more difficulty with counting by fives than counting by tens because there are more numbers to remember and because of the alternation between numbers ending in 5 and then 0.
2. Using a number line (from 1 to 20 with multiples of 5 highlighted) and a small object, show the student how the object jumps from 5 to 10 in one jump and repeat this step for 10 to 15. Ask the student if she can guess where the object will jump next. Praise her if correct.

3. Repeat jumping the object from 5 to 20, counting by fives.
4. Using pairs of students (or teacher and student), have one student make the object hop by fives as in step 2 above while the other student puts down 1 five-dollar bill for each hop. Then reverse the tasks.
5. Have each student put down 4 five-dollar bills while slowly counting "5, 10, 15, 20" without using the number line. Write the numbers as the student says them so the students can see the numbers written. Repeat as necessary. Let one student write the numbers on large paper or the chalkboard, if possible.
6. Model having the object jump beyond the number line, saying "25, 30, 35, 40, 45, 50." Put down cards with the numerals 25, 30, 35, 40, 45, and 50 to extend the number line beyond 20. Show the student how to make the object jump on the number line by fives (from 5 to 20), and then jump the object on the numeral cards to 50.



Counting by fives may be difficult, but it is very useful for counting money. Practice as a quick exercise for subsequent sessions to maintain the skill.

Count the Jumps

Remember to have the student count the jumps, not the squares or spaces, on the number line. Children often have difficulty remembering whether to count the square they are on at the beginning or the square that they land on. If they get in the habit of counting the jumps, they will not be confused.

Skip Counting with the Calculator

Some students may not be able to reliably use skip counting with money. Other students may be able to skip count the money needed for most real-life transactions in stores, but may find it helpful to count larger sums (such as from their banks) with a calculator. Teach your students that when they use their calculators for adding money, they should write down the value of each bill on paper before doing the addition so the amounts can be checked. They may also want to cross out each number once they have added it, so they can recover if they lose their place when punching in a long series of 5s or 10s.

Practice punching out the 5s, the 10s, and the 20s on the calculator. For example, have the students punch in the number 5 on their calculators. Then have them punch in the addition sign (+) and look at the window to see the answer. Continue adding 5s until the answer in the window shows 100. Repeat with adding 10s and then 20s in the same manner.

GENERALIZATION INTO DAILY ACTIVITIES: Now the student should be able to apply skip counting to real money or realistic play money. Often, having the student put bills in a wallet or purse will make the situation seem more realistic. The teacher says, "Here's your wallet. How much money do you have in your wallet?" The student skip counts the bills out loud so the adult can check the accuracy. Students using the realistic play money usually enjoy handling and counting big bills like tens and twenties.

For now, do not mix the denominations of bills (e.g., two twenty-dollar bills and one five-dollar bill). Just use all tens or twenties, etc.

- Parents, let your child count the currency in your wallet at various times during the week (under supervision, of course.) You may want to have her just count the different denonimations separately. Counting real money (currency) is a great motivator for many individuals.
- Of course, getting your own money is an even greater motivator than counting other people's money. Give your student an allowance in currency, have her do jobs to earn currency, or do both.

Practice in Money Handling Situations

Now is the time to regularly play the *Earn and Pay* game from Appendix A. This game provides a great deal of practice in skills that will translate into actual money-handling skills for older children, adolescents, and young adults. I recommend playing the game daily while you are working on the skills in this chapter. The entire game, including the final money counting, should take less than fifteen minutes, once the students have thoroughly learned how to play.

The game uses the following skills:

1. sorting
2. skip counting by fives and tens
3. simple money concepts of paying and receiving money
4. next-highest-dollar strategy
5. simple subtraction to find change needed (with a calculator)
6. simple addition (with a calculator)

Refer back to Chapter 4, page 30 for game instructions.

Reading Money Amounts with Dollars and Cents

Before learning to round off to the next highest dollar, the student needs to be able to understand the difference between dollars and cents. She also needs to be able to recognize money amounts written as dollars and cents using the decimal point.

UNDERSTANDING "CENTS"

OBJECTIVE: The student will be able to count up to 20 pennies, using one-to-one correspondence and labeling the amounts as cents.

MATERIALS:

- 20 or more pennies for each student
- Several one-dollar bills (real or realistic play money)
- Jar
- Pencil and paper for the teacher to write numerals on
- Index cards with numbers 1 to 20 on them, if needed.

SUCCESS STEP: Hold up a penny and ask the student what it is. Praise her if she says "a penny" or "one cent."

PROCEDURE:

1. Have the student count 10 pennies and drop them one by one into a jar or some other container that will make a noise as the pennies are dropped.
2. Tell the student that she now has 10 *cents* in the jar. "Ten pennies make up 10 *cents*."
3. Empty the jar and have her drop in 13 pennies. Then ask, "How many cents do you have in the jar?" Repeat the activity using different quantities of pennies, emphasizing the word *cents* each time.
4. Reverse the activity and have the student count the pennies as you or another student drops the pennies in the jar. Have the student tell you how many cents are in the jar. If the student is nonverbal, have her point to cards that have the numerals from 1 to 20 on them. Even if the student is verbal, showing her the numerals will strengthen the visual learning capacity.
5. To make sure she knows the difference between cents and dollars, give her a couple dollars and a couple pennies and see if she can tell you how many dollars and how many cents she has.

READING PRICES UNDER $10

OBJECTIVE: The student will be able to read prices under $10 that have dollars and cents, reading the decimal point as "and," reading the tenths and hundredths as a two-digit number, and ending with the label "cents."

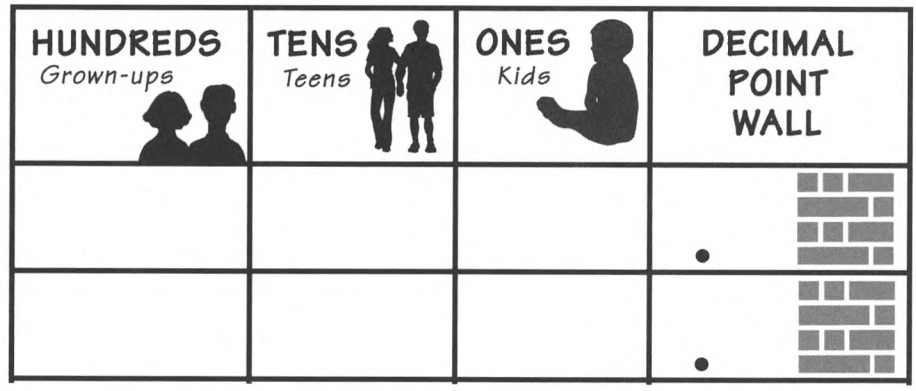
MATERIALS:

- Two matching sets of 10 index cards with prices (in dollars and cents) written on them such as $2.48 and $1.44. Do not make prices that have less than 10 cents. Make the decimal point twice as large as normally written.
- Two matching sets of 5 index cards that have prices (in dollars and cents) with cents amounts from 1 to 9, such as $4.03 and $9.06.
- Place value chart for dollars and cents from Appendix B-11

SUCCESS STEP: Put 5 index cards with the prices out on the table or desk and ask the student to point to one card that has both dollars and cents on it. Any one that she points to will be correct.

PROCEDURE:

1. Show the student the features of the price on a card. Tell her that the big dark circle or point is like the brick wall on Number Street (see "Kids, Teens, and Grown-Ups" in Chapter 8). It is the fence between the whole numbers or dollars that we have been working with and the pennies or cents. After the big decimal point, we start counting the cents, and there will only be two digits (e.g., .23) that tell us how many cents there are. Use the place value chart that shows dollars and cents to illustrate visually.
2. Model reading several prices for the student. Do not use any prices with less than 10 cents at this time.
3. Show the student one index card at a time from the set with cents amounts over 10 cents. Ask her to point to a card on the table that is just like the one you have shown her. She can take the card in her hand while she is looking for the match. Then you model the number that should be spoken. The important part of this matching exercise is to have the student pay attention to the details of the dollars and cents on the cards.
4. Repeat the procedure with the other 5 price index cards. If possible, have the student match with all 10 cards resting in front of her.



Notating Cents

At this stage, I do not recommend teaching the alternate way of showing cents with the slashed **c**. The slashed **c** is used less frequently than decimals in daily life, and it can be very confusing for students to encounter two entirely different ways of expressing cents during early money learning activities.

5. Model reading the prices on all the price index cards that she has matched. See if she can read those prices correctly after hearing your model.
6. If the student has trouble reading prices, try covering the decimal point and cents with an index card. For example, if the price is $5.78, cover all but the $5 with the card. After she says "five dollars," slide the card over so the decimal point is revealed, and prompt her to say "and" when she sees it. Finally, reveal the 78 and prompt her to say "cents" after she reads the numbers.
7. Next use the index cards that show prices of several dollars and 1 to 9 cents. Explain that there always have to be two places to the right of the decimal point (or wall) when we are working with money. If there are no tenths, you have to put a 0 in the

tens place. This is usually a difficult concept and can best be learned by showing a variety of examples.

8. After the student is secure in reading prices, show her some prices that are written incorrectly, such as \$2.5 or \$5.6 and ask her what is wrong with them. See if she can tell you that there must be *two* numbers after the decimal point. If not, tell her you need two numbers, and show her how you can't tell whether \$2.5 means \$2.50 or \$2.05, because you can't tell where the 0 should go.
9. If she is successful with correcting the prices above that need another digit, show her some prices with three digits after the decimal point, and see if she can tell you what is wrong. Help her understand that you can only have 99 cents.

Katie counted up the currency and coins in her bank and then informed her mother that she had "3 dollars and 150 cents." Although she knew the values of the coins and currency, she hadn't learned that you can only have 99 cents; one penny more and you have a dollar.

PLACE VALUE PRICES (ACTIVITY SHEET)

MATERIALS:

- Place Value Prices from Appendix B-46
- Chalkboard and chalk or paper and marker

SUCCESS STEP: Have the student write a dollar sign on the chalkboard (or point to a dollar sign that you have written within a group of other symbols or letters).

PROCEDURE:

1. Write some prices with dollars and cents (under \$9.99) on the chalkboard and model reading the prices. Review the following steps with the students:
   - Read the dollars first.
   - Then read the decimal point as "and."
   - Finish reading the cents as a two-digit number.
   - End with the label "cents."

   \$9.24 is read **9 dollars...and** (for decimal point or wall) **24** (for the two-digit number on the right side of the decimal point or wall) **cents** (as the label).

2. Have the students repeat each part of the price after you have modeled it. Then see if they can read some of the prices on their own activity sheets. Praise successful efforts. This is not an easy task. Repeat this activity using other prices you have made up. Spend extra time on the prices that have dollars and cents that are 9 and under. Remind them that they have to have the zero as a placeholder or the amount of money is wrong. Do not go beyond \$9.99 for the prices at this time.

3. Have the students punch in the prices on their activity sheets on the **calculator.** Show them how to put in the decimal point. Emphasize the importance of putting

in the decimal point accurately. Check the windows on their calculators to see they are correct each time.

4. When the students are reading prices under $10 accurately, introduce higher prices (under $50) to see if they can extend their learning to larger numbers.

Writing Money Amounts

For now, I recommend writing dollars alone (without any cents). For example, write $5, not $5.00. It makes the learning simpler and less confusing. Traditionally, five dollars is written as $5.00 to make lining up the dollars and cents easier when doing paper and pencil addition or subtraction. When using the calculator, we don't have to worry about lining up the numbers underneath each other as long as we are accurate in entering the numbers in the calculator. Later, you can teach the student that $5.00 is another way of showing that there are 5 dollars and no cents, but that we enter just $5 on the calculator because it is faster and easier.

GENERALIZATION INTO DAILY ACTIVITIES: Parents, whenever you are with your child in a store, restaurant, etc. that has items for sale, look for meaningful ways to engage your child in reading prices. For example:

- If your child asks you to buy a certain brand of cereal, cookies, etc. at the grocery store, say, "Maybe. How much does it cost?" (Then show her where to read the price.) Or, perhaps say, "No way! Look how much it costs!"
- If your child asks you to buy her something that you aren't inclined to buy her at the moment, show her the price and suggest she save up for it. "See, it's $14.95. You can buy it for yourself if you save a little more money."

At school, your child can visit the cafeteria or school store (if there is one) and practice reading prices for desired items there.

Next-Highest-Dollar Strategy

Although this book advocates working with currency before working with coins, there is no escaping decimals in dealing with money in the real world. Students will see many ads and commercials that give prices with cents—for example, $1.25 or $19.95. An important strategy to teach when working with money is that of rounding numbers to the next highest dollar. If students understand the next-highest-dollar strategy, they can pay without having to count out coins.

Many people use the next-highest-dollar strategy without being aware of it. I drive up to the window at Burger King and order a Whopper and fries. The clerk tells me that my order costs $3.89 and to drive to the next window. Do I get out my change purse and fumble to get $.89 cents in change? No! I hand the clerk four dollars and

let her give me back the change. When time is short, many of us use the next-highest-dollar strategy.

Although you probably know the rules for rounding a number to the *closest* dollar, only the *highest* dollar strategy will be taught here. Students who are concrete thinkers often have trouble when several options are given for different situations. Rounding down to the closest dollar might result in the student trying to buy something and not having enough money to pay for it—a very embarrassing situation for any of us.

Scott has figured out that most things that he buys cost under $20. Therefore, he keeps just one twenty-dollar bill in his wallet. Then if he wants to buy anything, he just hands the clerk his $20. He says that it keeps him from being embarrassed by not having enough cash to pay for an item.

ROUNDING UP TO THE CLOSEST DOLLAR

OBJECTIVE: The student will be able to round up to the next highest dollar for prices under $49.99.

MATERIALS:

- A large teacher number line (1 to 20) or a straight line on the chalkboard that is marked off in 20 equal parts.
- A number line (1 to 20) for each student (not a ruler)
- Six prices under $20 written on index cards or on a chalkboard ($1.11, $9.08, $12.10, $17. 62, $6.10, $18.12)

SUCCESS STEP: Have the student read the price on one of the index cards. Model the correct price if she is incorrect and have her repeat. Praise success.

PROCEDURE:

1. Hold up the first index card and read off the price ($1.11). Ask the students to pretend they only have dollar bills and no coins. Then ask how many dollars they would need to buy something that costs $1.11. If they say "one," say, "OK, one dollar is enough to pay for the one dollar part of the price" (and cover the $1 with your hand). But what about the eleven cents left over?" Then tell them you are going to show them how to always know how many dollars they need to buy something.
2. Using the teacher's number line, go through each of the six prices and point to approximately where the price is on the number line. Have students look for the next higher number on the line and point to it on the teacher's number line. Tell them this is the next highest dollar. Explain that you can give the next highest dollar to the clerk in the store, have enough money to pay the price of the item, and maybe get back change. Write the next-highest-dollar amount next to the exact price on the chalkboard.
3. Ask the students if they can see a pattern in how we figure the next highest dollar. You may need to help them see that the next highest dollar is always one more

dollar than the dollar amount in the price. You may also show then that they just need to look at the numbers to the left of the decimal, say that number, and then count up one to find the next highest dollar.

4. Give a number line to each student. Go over where the *x*'s (the real price) will be on their number lines for every price. Then let them find the next highest dollar for each price and write it on the paper (or circle it, if writing is a problem).
5. On another occasion, have them locate the same prices on the number line themselves and then determine the next highest dollar.
6. Repeat as often as necessary for them to internalize the rule for next highest dollar.
7. You may want to laminate the number lines. Then the students can use a washable marker and mark the prices and next-highest-dollar amounts of all the problems on their worksheets. They then can wash it off to use another day.

COUNTING MONEY AND ROUNDING UP

OBJECTIVE: The student will be able to count how much money she has and determine the next highest dollar.

MATERIALS:

- Six or more pieces of light-colored cardstock or construction paper
- Play money dollars and pennies to tape to the cardstock. Use amounts under $20 and use a manageable number of pennies (no more than 12).
- Next-Highest-Dollar practice sheets from Appendix B-47

PROCEDURE:

1. Put the pieces of paper and money in front of the student at a table or at several learning stations around the room.
2. Give each student an answer sheet. Tell them to count the money on the cards and write the exact amount in the first column. Then write the next-highest-dollar amount in the second column.
3. Repeat this activity using different amounts until students are secure in finding the price and the next-highest-dollar amount.

Season wrote the numbers quite confidently until she got to a card that had 2 dollars and only 4 pennies. At first she wrote it as $2.4, but decided that looked strange. Then she wrote the amount as $2.40 so she had two places in the cents.

Her teacher asked her to read the number and she read 2 dollars and 40 cents. "But I don't really have 40 cents or pennies," she said. The teacher then showed her how to write 4 cents using decimals with a 0 in the tenths (dimes) place—$2.04.

Forgetting the 0 when there are no tenths is a very common error when writing money amounts.

SHOPPING ACTIVITY

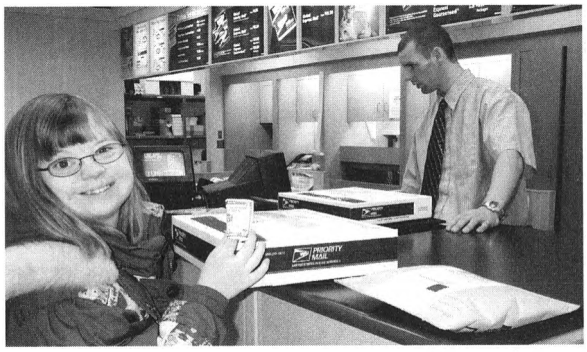
OBJECTIVE: The student will be able to figure out the next highest dollar using a pretend shopping scenario.

MATERIALS:

- Advertisements that show dollars and cents in the prices
- Garage sale money labels, adhesive printer mailing labels, or Post-It notes

PROCEDURE:

1. Tell the students that you are going to show them how to pay at stores using the next-highest-dollar strategy.
2. Show them some ads for products that cost several dollars and up to $.99 cents in addition to dollars.
3. Tell them that some advertisers want us to think their product is cheaper by putting a lower dollar amount and then adding 99 cents to the dollars. Ask them how close 99 cents is to the next highest dollar.
4. If they don't know, tell them there are 100 cents in a dollar. Then have them count with you from 90 to 100, pausing after 99 (you don't say the 100) to make sure they know 100 comes after 99. Then ask them again how close 99 is to 100 cents (or a dollar).
5. Explain that no one can sneak a 99-cent trick on them anymore. They have caught on to the trick.
6. Let the students change some prices in the ads to the next highest dollar. Have them write the new price on a blank printer label, Post-It note, or garage sale sticker and paste it over the old price.
7. Divide students into pairs and give them some realistic play currency ($1, $5, $10). Have them look at the ads with the whole dollar amounts and take turns "buying" items with the play money.
8. Repeat the pretend shopping with different ads for items under $49.



GENERALIZATION INTO DAILY ACTIVITIES:

- At this stage, the student should be able to purchase one item by giving the cashier the next highest dollar. Students should be able to buy one item at the school store or one item of food without direct assistance from an adult. The adult does need to plan with the student and check the amount of money she is carrying. Frequent experience with purchasing items using the next-highest-dollar strategy is necessary to maintain the concept.
- Do not expect the student to be able to buy more than one item when she accompanies you as you do your regular shopping. Make sure that she has many successful experiences with purchasing one item before she has to purchase several items. At first, you can ensure success by giving your child exactly the number of dollars she needs to use the next-highest-dollar strategy. For example, if she is buying a loaf of bread

that costs $2.30, you would give her $3 and, if necessary, prompt her to give all three bills to the cashier when told the price. Later, you can give her an extra dollar or two and watch to see whether she hands over the whole amount to the cashier or counts out the bills correctly.

Sales Tax

Students at this stage of math learning are not ready to do the multistep process needed to figure out sales tax. In addition, some states do not have state sales tax, some states do not charge on essentials such as food, and other states have various rates of state sales tax, which further complicates figuring sales tax. Generally, I just explain to students that we pay taxes (a little extra money) to the governor on each item we buy. Therefore, to be sure they have enough money, they always have to have $1 or more over the price of the item.

Using the Next-Highest-Dollar Strategy to Purchase More Than One Item

If you have been regularly playing the *Earn and Pay* game, you have already been teaching a multistep process similar to what is needed for buying several items. At the end of the game, the student has been skip counting the amount of money in each denomination of bills—e.g., $5, $10, $15 in five-dollar bills. When each denomination has been counted, the student has been adding the subtotals to get the total amount of money. Both a skip-counting process and an addition process are needed to get the total.

To purchase several items, your student needs to go through a similar multistep process. She needs to add the exact prices of the items she wants to buy (with a calculator) to find a total amount needed. Then she needs to count the money that she has. After figuring both totals, she decides whether she has enough money to buy the items.

SHOPPING ACTIVITY FOR TWO (OR MORE) ITEMS

OBJECTIVE: The student will be able to add (on the calculator) the prices of two items that she wants to purchase, count her available money, and determine whether she has enough money to buy the items.

MATERIALS:

- Newspaper ads, flyers, or catalogs
- Index cards and a marker
- Realistic play currency—2 or 3 ten-dollar bills (adjust the amount of money depending on the prices in the flyers or catalog)

SUCCESS STEP: Let the student look through the ads or catalog and find two things that she would like to purchase. Praise her for finding two items. Note: Specific items to

buy have not been listed in the Materials section. Having items that the student would really like to buy will help her learn the skill faster and make the sessions more fun.

PROCEDURE:

1. Give each student 2 or 3 ten-dollar bills (or the amount you have decided is enough).
2. Have the students mark or cut out the items she desires from the catalog or ads. Write each price on a separate index card.
3. Have her add the amounts (on the calculator) to get a total.
4. Have her count the money you have given her.
5. Then have her tell you which amount is bigger—the total price of the items she wants or the amount of money that she now has. Let her pretend to buy the items. Expand upon the activity by making other similar situations. At this time, make up only situations where she has enough money.
6. When she is secure in handling situations in which she has enough money, introduce a situation where she does not have enough money. (You may need to take back some of her play money.) If she does not have enough money, she can't purchase the items. Do three situations or more in which she does not have enough money. Hopefully, she will get a little upset at not being able to buy what she wants. Then you can ask her, "How much more money do you need to save so that you can buy what you want?" If she does not know that this situation calls for subtraction, show her on paper how you figure the answer.
7. Determining how much more money is needed is one of the most difficult uses for subtraction that students learn. Have the students practice many of these how-much-more-money-do-I-need situations until, at least when dealing with money, they automatically subtract the smaller number from the larger to find the answer.

GENERALIZATION INTO DAILY ACTIVITIES: The preceding lessons have taught the basic skills for using money to purchase one or more items. Since this is such a crucial skill for daily living, students need both regular practice in using the calculator to solve these kinds of problems and also real-life situations in which they pay for real things. Home and school need to work together to give ample practice so the skill becomes part of the student's long-term memory. Also essential are real occasions to buy items and thus motivate the student to learn this vital skill. Do not have her buy more than two or three items at this stage.

Other books and workbooks are probably going to be needed for the student to get the practice she needs. However, many of the current texts and workbooks are crammed with coin exercises, and students who are concrete learners would be better served by learning how to make purchases with currency. You therefore need to carefully preview any workbooks before you buy them or else make up your own problems involving buying one or two items.

At first do the problems together, being careful to show the actual numbers to the student in clear, rather large numerals. Explain each problem at first, simplifying the language and directions. When the student seems secure with adding multiple items and finding how much more money is needed to buy an item, you may try to give her a commercially designed worksheet and see if she can read the word problems and work out the answers with little assistance.

Introducing Coins

At this stage, students will be taught the name for each coin, how to recognize the coins, and the value of each coin. They will not be asked to learn the equivalents of one coin to another (for example, five nickels are the same as one quarter). This skill is best taught in simulated or related real-world situations. While your students are learning about coins, they should continue to practice the next-highest-dollar strategy for purchasing items.

COIN RECOGNITION

OBJECTIVE: The student will be able to recognize pennies, nickels, dimes, and quarters (perhaps also half-dollars, if they are used in your area), giving the proper name and value for each coin.

MATERIALS:

- A light-colored mat or piece of construction paper to put the money on
- Pennies, nickels, dimes, and quarters, plus $1 in currency
- Index cards and a marker

SUCCESS STEP: Throw some pennies on the mat. Ask the student what they are. Praise her for the answers pennies, cents, or money.

PROCEDURE:

1. Talk to the student about the pennies. Show her the pictures on each side of the coin. Point out that Lincoln, the man on the coin, has a short beard. Talk about the color of pennies. Tell her that no other coin has this reddish brown color.
2. Then show her some nickels—both the old and the new style. Use the name *nickel* frequently when describing the nickel. The building on the nickel has a round dome top. The man on the other side either has a ponytail hanging low on his head or he has short bangs and longish hair. Talk about the nickel's silver color and the smooth edges.
3. Give the student some pennies and nickels. Put a penny and a nickel at the opposite ends of the math mat. Cover one of the coins with your hand and ask the student to put a coin just like the coin that's still showing on the mat next to your coin. Reverse the coin that you are covering and have her match the other coin. Repeat as often as needed.
4. Put out four coins (just pennies and nickels) widely separated on the mat and have the student put the matching coins right beside your coins.
5. Practice with just the pennies and nickels until the student is secure in that understanding.
6. Now show the student a dime and a quarter and point out their features. Be sure to point out how small and thin the dime is and how it usually has ridges on the side. Roosevelt doesn't have a beard or a pony tail. Describe the quarter as a heavier and bigger coin than the others. The markings on the quarter can be tricky because Washington has the same hairstyle as Jefferson does on old-style nickels, and the

designs on the "tails" side of the commemorative state and national park quarters vary quite a bit.

7. Repeat the matching procedure from steps 3 and 4 with dimes and quarters on the mat.
8. Mix up all four coins on the mat and have the student match them again.
9. The three commonly used silver coins can be difficult for students to distinguish. If your student is having trouble telling them apart, you may want to back off and only talk about the quarter, the dime, and the penny for now. The penny is a different color, so students usually recognize it first. The quarter is large and heavy, while the dime is small and thin. Practice the distinctions between the quarter and the dime in a variety of ways:
    - Glue a real quarter onto an index card and do the same with a dime. Have the students line their quarters on the table beneath the index card with the quarter and do similarly with the dimes.
    - Blindfold the student and see if she can tell the difference between the quarter and the dime by feeling each of them.
10. Once the students can readily distinguish the quarter and the dime, glue a real nickel to an index card and have the student match the three different silver coins to the three coins glued to index cards. Point out that only the nickel is a solid silver color and is smooth on the edges. The quarter and dime have ridges on the edges (and also have two colors on the edges, which students may or may not be able to pick out).

Coin Values

As mentioned at the beginning of the chapter, learning coin values can be very difficult for all children, in part because their sizes do not match up with their values. Why *is* the dime smaller than the penny and the nickel if it is worth more? And, unlike currency, where dollar amounts are marked prominently in the corners, coins do not all spell out their value in plain language. Your student(s) may take quite a while (months or years) to be secure in understanding coin values. Gradually, though, as they have practice using real money and come to equate certain coins with certain functions (such as using a quarter to get a gumball out of a machine), they will likely master this knowledge.

Adding up a handful of change, especially mixed change, can be an even larger stumbling block for students who are concrete learners.

INTRODUCING COIN VALUES

OBJECTIVE: The student will be able to tell (or point out) how many cents each of the four common coins is worth.

MATERIALS:

- Index cards
- Penny, nickel, dime, quarter (plus a half dollar, if used in your area)

SUCCESS STEP: Show the student a penny and ask her how much it is worth (or ask, "How many cents is this?").

PROCEDURE:

1. Using the actual coins, teach the student the value ($.25, $.05, etc.) for each coin. As you are talking about the value, write the number (use decimals) on an index card. Put the card down where the student can see and put the proper coin next to the number.
2. Remove the coins from the index cards and see if the student can put them back on the appropriate index cards.
3. Repeat several times. If the student can match the coins with their values, try putting the index cards down in a different order to make sure she is not just remembering what order to put the coins down in (rather than recognizing value). If the student cannot match the coins with their values, try working on just two coins' values for a time or teach her some of the mnemonics below.

Coin Mnemonics

The poems below are widely used to teach coin values to young people. As far as I know, the author is unknown (but I will be happy to give credit to the author in the next printing, if I am incorrect). It may help some students to say these as chants.

Penny, penny
Easily spent.
Copper brown
And worth one cent.

Nickel, nickel
Thick and fat.
You're worth five cents,
I know that.

Dime, dime
Little and thin.
I remember
You're worth ten.

Quarter, quarter
Big and bold.
You're worth 25,
I am told.

The Dime Song
(To the tune of *My Darling Clementine)*
One, two, three, four
Five, six, sev-en
Eight, nine, ten cents in a dime

One, two, three, four
Five, six, sev-en
Eight, nine, ten cents in a dime.

The Quarter Song
(To the tune of the *Farmer in the Dell)*
A quarter's 25
A quarter's 25,
Hi, ho, the derry-o
A quarter's 25.

GENERALIZATION INTO DAILY ACTIVITIES:

- If your child wants to buy a gumball from a machine, tell her she needs a quarter. Hold out several coins in your hand and let her pick the right one. Likewise, if she is buying something from a vending machine, you could say, "You need three quarters" and see if she can pick them out of your hand. (Again, you are not asking her to figure out that three quarters equal 75 cents.)
- Start a collection of state and national park quarters as a family activity. Let your child help look for the different designs in your change and put them into the folder so she gets used to the variety of designs on the backs of the quarters.
- Borrow or buy the computer program *Counting Coins* (Attainment), which has various levels of basic coin recognition and values (about $80).
- Try some coin counting apps such as the following:
  - Jungle Coins on the Apps for Children with Special Needs website http://a4cwsn.com
  - Coin Math http://www.recessionapps.com/Coin_Math.html

CHAPTER 16

Standards-Based Mathematics Learning

In the United States of America, all states have developed standards for reading and mathematics. These academic content standards provide a clear indication of what students should know at the various grade levels. Accountability for student learning according to these standards has been accomplished by testing all students, except for a very few students with severe cognitive disabilities who use an alternate assessment. Students' learning is organized around these standards, even if students have disabilities or are still concrete learners.

Many states originally based their math standards on the recommendations made by the National Council of Teachers of Mathematics (NCTM) in 1989 or the more current revisions of the NCTM standards. Recently, however, many states have set new standards based on *Common Core State Standards (*National Governors Association Center for Best Practices).

The Common Core is a program that was developed by a committee of teachers and education experts with an eye to standardizing the content and sequence of what is taught in math and English and language arts in each grade level across the country. The evaluation component of the Common Core is intended to give parents and schools a way to evaluate all students' progress in math and other academic standards on a grade by grade basis. States that opt in to the Common Core standards receive federal grants. Note that as this book goes to press, support for the Common Core has weakened somewhat, and it is possible that more states may opt out.

Regardless of whether your state follows the Common Core standards for math or has adopted other standards, standards and accountability are not going away for American schools. If your child attends a public school in the United States, he will be expected to learn certain specified skills, to the extent possible, and the school will be held accountable for what he learns. As described below, this can sometimes make it difficult for students with learning challenges to work on the foundational skills they need to learn at school.

What Do These Standards Mean for Students Who Are Concrete Learners?

Typical students learn most of the computation skills (addition, subtraction, multiplication, and division) by the end of fourth grade. Using the calculator, many students with Down syndrome and other concrete thinkers can learn to use the basic principles of computation in elementary school. Of course, this does not mean that they will have learned the math facts. If we spend the time that is usually devoted to having students memorize all of the math facts to instead teach them how to use the calculator and how to set up problems, concrete learners might become quite competent in math.

Public schools do not always teach about the functional uses of math in the general curriculum. That is, although they have traditionally taught students *how* to add, subtract, multiply, and divide, they do not usually spend much time teaching them *when* to use these operations in daily life. In fact, for states that follow the Common Core, one of the primary changes is that students are to work on developing independent math problem-solving skills—to discover on their own how to solve various types of problems—which may or may not be helpful for student(s) with math difficulties.

Now that schools are required to show that most students with disabilities are making progress in the general curriculum, it may be even harder to find time to work on everyday math skills in inclusive classrooms. Parents and teachers will have to make sure that students are able to apply the math skills they learn in school to functional uses at home and in the community.

Where Do the Objectives in This Book Come From?

The objectives used in this book come from a variety of books about teaching mathematics. Some of the most useful included the following:

Bird, Gillian, and Sue Buckley. *Number Skills for Individuals with Down Syndrome (four modules)*. Southsea, Hampshire, UK: Down Syndrome Educational Trust, 2001.

Fosnot, Catherine, and Maarten Dolk. *Young Mathematicians at Work: Constructing Number Sense, Addition, and Subtraction.* Portsmouth, NH: Heinemann, 2001.

National Research Council. *Adding It Up: Helping Children Learn Mathematics.* Washington, DC: National Academies Press, 2001.

Reys, Robert E., Mary M. Lindquist, Diana V. Laubdin, Marilyn N. Suydam, and Nancy J. Smith. *Helping Children Learn Mathematics.* 11th ed. Hoboken, NJ: Wiley, 2014.

Smith, Susan Sperry. *Early Childhood Mathematics.* 5th ed. Needham Heights, MA: Allyn & Bacon, 2012.

Thornton, Carol A. *Teaching Mathematics to Children with Special Needs.* Upper Saddle River, NJ: Prentice Hall, 2000.

As you can see, the majority of books I used are about mathematics instruction for typically developing children. Ohio mathematics standards were developed during the time that I was writing the book that this volume is based on—*Teaching Math to People with Down Syndrome and Other Hands-On Learners* (Horstmeier, 2008)—so I was also able to compare the curriculum presented here with the Ohio standards.

The objectives in this book are quite similar to the computation objectives or indicators of the Ohio standards. The techniques for teaching them are different, however, because I needed to slow up the procedures somewhat. For example, the Ohio standards had the first grade student counting forward from 1 to 100, backward from 100 to 1, and counting forward or backward from any number between 1 and 100 (Ohio Department of Education, 2002). *Teaching Math to People with Down Syndrome and Other Hands-On Learners* has the student count forward to 30 at first, only later progressing to 100. This book does not have the student count backwards from 100 at all, choosing only 20 as the starting point. Our students with math difficulties usually learn things more slowly and are more easily confused with large numbers.

When looking at the details of a standard, I used the "so what" criteria to determine the emphasis. That is, I asked myself, "If the student learns that concept, so what? How useful will it be in his life? Does he need this skill in order to learn more difficult math content?"

Because addition and subtraction are so commonly taught together, it is suggested that students participating in the regular classroom follow the progression that is followed in that classroom. That way, when the students are tested, they will have had the same exposure to addition and subtraction as their typical peers. However, if the student is older or has not caught up with students working on grade level, it may be important to teach addition thoroughly and then go on to subtraction, to teach multiplication and division separately, etc.

The Importance of Individualizing Math Instruction

Although the school climate in the United States is strongly focused on achieving academic content standards, we need to remember that some students with disabilities have the right to an ***individualized*** education program. While the achievement of academic content standards is very important as a part of schooling, our students with math difficulties need many functional math skills for daily living that are not taught directly in most academic curricula. Therefore, functional skills such as money handling as used in shopping should be taught in upper elementary grades and in secondary classrooms with frequent practice to maintain them. Parents and teachers may need to advocate strongly for goals related to these skills to be included in the IEP (if the student has one), especially in school districts where there is a great deal of emphasis on "teaching to the test." If some schools do not provide enough instruction on functional math skills, parents can teach the skills in real-life situations that will really show their importance.

Writing Individualized Math Goals

Parents and teachers can base student goals on the objectives in this book. However, the objectives given in this book are not complete *instructional* objectives as written in IEPs. You must add the ***conditions*** under which the objective will be learned. Also crucial is designating the ***criteria*** that can be measured and will indicate whether the student has accomplished the objective.

For example, an objective in this book is:
The student will be able to read prices under $10 that have dollars and cents.

An instructional objective would be the following:
*In the school general classroom and cafeteria (**conditions**), Scott will accurately read prices having dollars and cents, using the word "and" for the decimal point for 9 out of 10 prices on three separate occasions (**criteria**).*

Another objective is:
The student will listen to numerals 11 to 19 given orally and match them with the correct written numerals.

As an instructional objective for the IEP, it could be expressed like this:
*In the general classroom and at home, Scott will be able to point (**conditions**) to the named teen numbers (from 11 to 19) correctly for 3 out of 4 consecutive math sessions and at least once at home (**criteria**).*

In general, some parents of students with documented disabilities feel it is very important for their child to take the regular grade-level assessments, in order to make the school more accountable for how much progress he or she makes each year. They may have experienced, or heard about, situations where children with disabilities were allowed to make little or no academic progress year after year, with no repercussions for the school district. Other parents of students with disabilities feel that it may be too frustrating for their children to take the regular assessments and that if their children score very low each time they take the test, it will not provide a good indication of what they are learning and what needs to be emphasized in future instruction.

Students who are progressing in the general classroom are supposed to be assessed by the standardized test that is keyed to the knowledge taught in that general classroom. If your student is to be assessed with the standardized test, he should have accommodations such as the use of a calculator, an alternative or augmentative communication device, extended time, an answer sheet with more space for answers, or instructions read aloud to him, as part of his assessment. Those accommodations need to be part of the student's IEP.

Conclusion

Since learning math skills can be more difficult for students who are concrete learners, it is crucial to optimize the learning environment for them so they can make as much progress as possible. The ideal situation for these students would be to be given access to the general education math curriculum with additional support in the form of tutoring or one-on-one instruction tailored to their individual needs. In addition, they would be involved in hands-on, interesting math learning activities at school. Also, the school would encourage parents to be active partners in their children's learning, and supportive families would help their children learn at home with games and concrete activities such as those in this book. As a team, parents and teachers would ensure that students with Down syndrome or other math difficulties had early and frequent success with math concepts and activities so that they felt like competent math learners. There is no telling how much a student might learn, if only parents and teachers would work together to help him learn in ways that are truly meaningful and motivating to him.

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Resources

Counting & Number Recognition Books

You can find a counting book to match almost any child's interests. This section includes only a very small sample of books available. Books listed here are included because they are either counting books with a twist (for example, backwards counting), because they may be especially appropriate for children with Down syndrome, or because they may be more appropriate for older children and young adults who could benefit from looking at counting books.

Arena, Jen. *100 Snowmen.* Las Vegas, NV: Amazon Publishing, 2013.

A book with humorous illustrations of increasing numbers of snowmen for readers to count. Besides giving children the opportunity to count 100 snowmen, the book also includes simple addition statements such as 5 + 6 = 11 snowmen.

Baker, Keith. *Big Fat Hen.* Orlando, FL: Harcourt Brace, 1999.

A simple counting book in which bright-colored, humorous pictures illustrate the rhyme, "One, two, buckle my shoe" (only up to "Nine, Ten, a big fat hen"). May be a useful rhyme for teaching rote counting.

Bang, Molly. *Ten, Nine, Eight.* New York, NY: Greenwillow, 1996.

A bedtime book in which a father and daughter count backwards from 10 to 1.

Brown, Ruth. *Ten Seeds.* Minneapolis: Andersen Press (Learner Books), 2013.

Big, bold pictures show what happens to 10 seeds after they are planted. They are carried away one by one by an ant, a pigeon, a slug, etc. Excellent for counting backwards from ten.

Carter, David. *600 Black Spots: A Pop-Up Book for Children of All Ages.* New York: Little Simon, 2007.

For older students who can handle pop-up books gently, this book would offer hours of fun counting practice. There are 600 spots total spread across ten colorful paper sculptures, some with pull tabs.

Falwell, Cathryn. *Feast for Ten.* New York: Houghton Mifflin, 1995.

A book with simple, clear rhyming text that gives readers a chance to count up to ten twice in the context of a family shopping trip and preparation of a holiday meal.

Johnson, Stephen T. *City by Numbers.* New York: Puffin, 2003.

A good book for older learners working on number recognition, this book features realistic paintings in which numbers are formed by objects in city scenes (e.g., the 8 is made from two round trash cans as seen from above).

Keller, Laurie. *Grandpa Gazillion's Number Yard.* New York: Henry Holt, 2005.

A visually interesting picture book that emphasizes the shapes of numbers and points out what else they resemble (for example, a seven can be used as a walking stick).

Marzollo, Jean & Wick, Walter. *I Spy* series. New York: Cartwheel Books, various dates.

Books in the *I Spy* series feature "picture riddles"—full-color spreads of photos of many different objects with rhyming instructions to spy things such as 5 clothespins or buttons or pencils. For students who are not overwhelmed by busy pages, the books can also be helpful in teaching about categorizing (for instance, finding the specified number of Christmas ornaments, when none of the ornaments look alike). The *I Spy Little* series of books are simplified board book editions with fewer objects per page and easier instructions. *I Spy Little Numbers* might be especially useful for students learning to recognize numbers.

Moerbeek, Kees. *Count 1 to 10: A Pop-Up Book*. Harry Abrams, 2011.

This brightly illustrated pop-up book showcases the numbers themselves as counting objects. For example, there are five number fives to count. The specified number of ladybugs also meander across each page.

Schwartz, Stephen. *How Much Is a Million?* New York: Harper Collins, 2004.

A good book to help a child visualize how much a million actually is.

Sloat, Teri. *From One to One Hundred.* Upper Saddle River, NJ: Scott Foresman, 1995.

This book offers many interesting objects to count on each page, from 1 to 10 and then by tens to 100. Numbers are shown as numerals and also written out in words.

Sources of Teaching Materials

Attainment Company
P.O. Box 930160
Verona, WI 53593-0160
800-327-4269
www.attainmentcompany.com

Realistic play money; money-related board games for older students; software for teaching about time and money.

City Creek Press
P.O. Box 8415
Minneapolis, MN 55408
800-585-6059
www.citycreek.com

Publishers of *Addition the Fun Way* and *Times Tables the Fun Way*.

Different Roads to Learning
121 W. 27th St., Ste. 1003B
New York, NY 10011
800-853-1057; 212-604-9637
www.difflearn.com

Products for helping children with autism learn, including some math games that would be useful for children with Down syndrome and other disabilities, as well as children with autism.

Down Syndrome Education USA
1451 Quail St., Ste. 110
Newport Beach, CA 92660-2747
949-757-1877
info@dseusa.org

Offers Numicon materials for teaching numeracy, as well as many monographs on math skills and other issues in people with Down syndrome. (Numicon is a system of manipulatives and activities that uses patterns to help children visualize numbers and their relationships; has been used successfully to teach basic math concepts to children with Down syndrome.)

EAI Education
118 Bauer Dr.
P.O. Box 7046
Oakland, NJ 07436-7047
800-770-8010; 201-891-9466
www.eaieducation.com

This educational supply company has a wide variety of rulers, counters, calculators, play coins and currency, teaching clocks, educational games, and algebra manipulatives.

Gander Publishing
412 Higuera St., Suite 200
San Luis Obispo, CA 93401
800-554-1819; 805-541-5523
www.ganderpublishing.com

Publishers of the "On Cloud Nine" math program designed for students with learning disabilities, and also sells math workbooks.

Handwriting Without Tears
8001 MacArthur Blvd.
Cabin John, MD 20818
301-263-2700
info@hwtears.com
www.hwtears.com

Workbooks, slates, and other teaching materials used in the HWT system for teaching letter and number formation.

Innovative Learning Concepts
5445 Mark Dabling Blvd.
Colorado Springs, CO 80918
800-888-9191
customerservice@touchmath.com
www.touchmath.com

Makers of the TouchMath series of teaching products, including manipulatives and workbooks for teaching about addition, subtraction, multiplication, division, fractions, time, and money.

Lakeshore Learning
2695 E. Dominguez St.
Carson, CA 90895
800-778-4456
www.leakeshorelearning.com

Offers a variety of educational products, including manipulatives, and games, and free downloadable worksheets, lesson plans, and flashcards.

Learning Resources
380 N. Fairway Dr.
Vernon Hills, IL 60061
800-333-8281
www.learningresources.com

Many math games, calculators, manipulatives, blank dice, workbooks, and other teaching materials. They also offer products such as an Elapsed Time Line and a Time Equation Solver that may be useful for adults with disabilities who need help with tasks such as figuring out how much time to allow to get ready for an event.

Memory Joggers
24 Nuevo
Irvine, CA 92612
888-854-9400; 948-854-9300
www.memoryjoggers.com

This company offers materials for teaching the addition and multiplication facts using visual and verbal mnemonics.

Remedia Publications
15887 N. 76th St., Ste. 120
Scottsdale, AZ 85260
800-826-4740
www.rempub.com
Workbooks with a real-world focus designed for special education students.

Rhymes 'n' Times
Turner Educational Products
650 E. State Highway 121, Ste. 107
Lewisville, TX 75028
888-684-6376
www.rhymesntimes.com
Offers separate multisensory programs for teaching the addition, subtraction, multiplication, and division facts using rhymes and manipulatives.

See and Learn
http://www.seeandlearn.org/en-us/
A research-based program specifically designed by Down Syndrome Education International to teach children with Down syndrome counting, early number concepts, and addition up to ten.

Semple Math
26 Semple Village Rd.
Attleboro, MA 02703
888-868-6284
info@semplemath.com
www.semplemath.com
Publishers of the Semple Math program, which uses mnemonic devices to help students master basic math concepts.

Silver Lining Multimedia
P.O. Box 544
Peterborough, NH 03458
888-777-0876
www.silverliningmm.com
Offers a "sliding number line" and teaching clocks and timers, along with other products for hands-on learners.

SmallToys.com
Novelty House
50720 Corporate Dr.
Shelby Township, MI 48315
800-633-4477
www.smalltoys.com
A source for magnetic bingo wands and chips, as well as inexpensive small toys that can be used as manipulatives.

Teachers Pay Teachers
www.teacherspayteachers.com

An online marketplace where teachers advertise and sell products they have developed in order to help other teachers teach specific concepts. Many math games, workbooks, flashcards, charts, etc., are available.

Texas Instruments
http://education.ti.com/

Descriptions of calculators appropriate for hands-on learners may be found under "Basic and Elementary Calculators." For example, the TI-10 shows the whole problem as entered (e.g., 5 + 7 = 12), and division quotients can be displayed with remainders instead of decimals.

Time Timer, LLC
7707 Camargo Rd.
Cincinnati, OH 45243
877-771-TIME; 513-561-4199
www.timetimer.com

Many versions and sizes of the Time Timer, as well as apps for iPhone and Android.

U.S. Bingo
3438 E. Ellsworth Rd.
Ann Arbor, MI 48108
800-254-0773
www.us-bingo.com
A source for magnetic bingo wands and chips.

Math Games

Many of the games referenced in this book are easily found in most toy stores. The companies listed below carry math games that may not be as widely available in stores. Many of the companies and organizations in the section above also carry educational games.

Conceptual Math Media
5728 Lonetree Blvd.
Rocklin, CA 95765
888-433-2224; 916-435-2810
www.conceptualmathmedia.com

Makers of the Equate game (similar to Scrabble, only players form equations rather than words), as well as the Conceptual Bingo series of games (versions cover telling time, decimals, fractions, money, number concepts).

Discovery Toys
www.discoverytoysinc.com

Educational toys and software.

Educational Insights
800-995-4436
www.edin.com
Math games and Hot Dots learning cards.

Educational Learning Games
727-786-4850
info@ELGames.com
www.educationallearninggames.com
Thousands of educational games, including a vast array of games for teaching and reinforcing computation, telling time, handling money, using fractions, and solving words problems.

Gamewright
www.gamewright.com
Many of Gamewright's card games use math skills in a fun way. For example, Rat a Tat Cat involves trying to collect four cards that have the lowest total possible; Maya Madness involves addition of small positive and negative numbers; Zeus on the Loose involves adding numbers to a total of 100 and recognizing multiples of 10.

International Playthings
75D Lackawanna Ave.
Parsippany, NJ 07054
973-316-5883 (fax)
www.intplay.com
This company makes a number of board games, card games, and puzzles that use early math skills. For example, Leaping Frogs and Bean Bag Toss are both active games that use addition skills.

Jax Ltd.
141 Cheshire Lane
Minneapolis, MN 55441
763-449-9699
www.jaxgames.com
Has a variety of educational math games, including Sequence Numbers, Over and Out, and Link-O.

Learning Resources
380 N. Fairway Dr.
Vernon Hills, IL 60061
800-333-8281
www.learningresources.com
Learning Resources has games that help children learn a variety of math skills, including addition, subtraction, multiplication, and division; money skills; time telling; fractions.

MindWare
P.O. Box 45307
Omaha, NE 68145
800-274-6123
www.mindwareonline.com

This company sells educational games and books, including many designed to help with learning addition, subtraction, multiplication, and division facts.

Ravensburger
1 Puzzle Lane
Newton, NH 03858
603-382-3377
www.ravensburger.com/us

This German toy company produces high quality educational games, some of which incorporate early math concepts. They include My First Clock and Funny Bunny.

Talicor
901 Lincoln Parkway
Plainwell, MI 49080
800-433-GAME; 269-685-2345
www.talicor.com

Offers many educational games, including *MathAnimals* (reinforces addition, subtraction, multiplication), *Moneywise Kids,* and *MathSmart* dominoes.

Helpful Math Websites

This is only a small sample of websites offering worksheets and/or interactive math games and activities on the Internet. Note that some of these sites require payment to access some or all of the activities.

ABCYa
www.ABCYa.com

APlusMath
www.APlusMath.com

Billy Bear's Playground
www.billybear4kids.com

Funbrain
www.funbrain.com

Improving Education Inc.
www.onlineworksheets.org

LearningPlanet
www.learningplanet.com

Math Fact Café
www.mathfactcafe.com

Math Forum @ Drexel
www.mathforum.com

Math Goodies
www.mathgoodies.com

Math Playground
www.mathplayground.com

MathStories.com
www.mathstories.com

Multiplication.com
www.multiplication.com

SchoolExpress
www.schoolexpress.com

SuperKids Math Worksheet Creator
www.superkids.com/aweb/tools/math

Math Apps

There are thousands of math apps—games, activities, tools—that might conceivably be useful for any given student. Here are a few sites (out of many) that gather together and review helpful educational apps.

Apps for Children with Special Needs
http://a4cwsn.com/tag/math/

Autism Speaks: Autism Apps
https://www.autismspeaks.org/autism-apps

BridgingApps
www.bridgingapps.org

Common Sense Media:
Math Apps, Games, and Websites
https://www.commonsensemedia.org/lists/math-apps-games-and-websites#

iPad Apps for Autism:
A Spreadsheet of Reviews and Recommendations
http://is.gd/12ygv8

Know What's Inside
https://knowwhatsinside.com/discover

Learning Works for Kids
www.learningworksforkids.com

The Math Learning Center
http://catalog.mathlearningcenter.org/apps

Special Needs Apps
http://www.friendshipcircle.org/apps/

Understood for Learning & Attention Issues: Tech Finder
https://www.understood.org/en/tools/tech-finder

Organizations

The Arc of the United States
1825 K St., NW, Ste. 1200
Washington, DC 20006
800-433-5255; 202-534-3700
www.thearc.org

Autism Society of America
4340 East-West Highway, Ste. 350
Bethesda, MD 20814
800-328-8476; 301-657-0881
info@autism-society.org
www.autism-society.org

Canadian Down Syndrome Society
2003 14th St. NW, Ste. 103
Calgary, AB
Canada T2M 3N4
800-883-5608
info@cdss.ca
www.cdss.ca

Down Syndrome Education International
info@dseinternational.org
www.dseinternational.org

Down Syndrome Education USA
1451 Quail St., Ste. 110
Newport Beach, CA 92660
949-757-1877
info@dseusa.org
http://www.dseusa.org/en-us/

Kansas Institute for Positive Behavior Support
Dole Human Development Center
1000 Sunnyside Ave., Room 1028
Lawrence, KS 66045
785-864-1284
kipbs@ku.edu
http://www.kipbs.org/new_kipbs/fsi/behavassess.html

Learning Disabilities Association of America
4156 Library Rd.
Pittsburgh, PA 15234
412-341-1515
info@LDAAmerica.org
www.ldanatl.org

Learning Disabilities Association of Canada
323 Chapel St
Ottawa, Ontario K1N 7Z2
613-238-5721; 613-245-5391 (fax)
www.ldac-acta.ca

Learning Disabilities Online
WETA Public Television
2775 S. Quincy St.
Arlington, VA 22206
703-998-2060 (fax)
www.ldonline.org

LuMind/Research Down Syndrome
225 Cedar Hill St., Ste. 200
Marlborough, MA 01752
508-630-2177
LuMind@lumindfoundation.org
www.lumindfoundation.org

National Council of Teachers of Mathematics
1906 Association Dr.
Reston, VA 20191
800-235-7566; 703-620-9840
www.nctm.org

National Down Syndrome Congress
30 Mansell Ct., Ste. 108
Roswell, GA 30076
770-604-9500; 800-232-6372
info@ndsccenter.org
www.ndsccenter.org

National Down Syndrome Society
666 Broadway, 8th Floor
New York, NY 10012
800-221-4602
info@ndss.org
www.ndss.org

Positive Behavioral Intervention & Supports
Technical Assistance Center
Behavioral Research and Training
5262 University of Oregon
Eugene, OR 97403
541-346-2505
support@pbis.org
www.pbis.org

Appendix A: Assessment Materials

1. Earn and Pay Game Board / 266
2. Earn and Pay Game Cards for Older Players / 268
3. Earn and Pay Game Cards for Younger Players / 272
4. Money Total Slip / 276

Missed the bus.
Go back 2 spaces.
SCHOOL BUS
EARN $
Sell stuff at a garage sale.
PAY $
Buy a video.
EARN $
PAY $
EARN $
Sell lemonade.
PAY $
Go to the movies.
Did a good deed.
Go ahead 2 spaces.
EARN $
Clean the basement.
EARN $
Get a prize for most cookie sales.
1
pizza.

PAY $ $ 10	PAY $ $ 15	PAY $ $ 25
PAY $ $ 13	PAY $ $ 26	PAY $ $ 35
PAY $ $ 40	PAY $ $ 30	PAY $ $ 8

PAY $ $ 15	PAY $ $ 10	PAY $ $ 21
PAY $ $ 20	PAY $ $ 10	PAY $ $ 11
PAY $ $ 8	PAY $ $ 20	PAY $ $ 22

EARN $ $ 10	EARN $ $ 15	EARN $ $ 20
EARN $ $ 13	EARN $ $ 16	EARN $ $ 30
EARN $ $ 21	EARN $ $ 18	EARN $ $ 8

EARN $ $ 12	EARN $ $ 10	EARN $ $ 14
EARN $ $ 20	EARN $ $ 7	EARN $ $ 11
EARN $ $ 8	EARN $ $ 20	EARN $ $ 9

PAY $ $ 10	PAY $ $ 5	PAY $ $ 8
PAY $ $ 11	PAY $ $ 2	PAY $ $ 7
PAY $ $ 9	PAY $ $ 10	PAY $ $ 8

PAY $ $ 4	PAY $ $ 10	PAY $ $ 11
PAY $ $ 7	PAY $ $ 9	PAY $ $ 11
PAY $ $ 8	PAY $ $ 4	PAY $ $ 3

EARN $ $ 10	EARN $ $ 5	EARN $ $ 2
EARN $ $ 4	EARN $ $ 6	EARN $ $ 7
EARN $ $ 9	EARN $ $ 3	EARN $ $ 6

EARN $ $ 6	EARN $ $ 10	EARN $ $ 5
EARN $ $ 4	EARN $ $ 7	EARN $ $ 11
EARN $ $ 3	EARN $ $ 2	EARN $ $ 9

Type	No. of bills	Amt. of money
$1		
$5		
$10		
$20		
$50		

Type	No. of bills	Amt. of money
$1		
$5		
$10		
$20		
$50		

Type	No. of bills	Amt. of money
$1		
$5		
$10		
$20		
$50		

Type	No. of bills	Amt. of money
$1		
$5		
$10		
$20		
$50		

Type	No. of bills	Amt. of money
$1		
$5		
$10		
$20		
$50		

Type	No. of bills	Amt. of money
$1		
$5		
$10		
$20		
$50		

Appendix B: Teaching Materials

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10. Counting by Fives / 298
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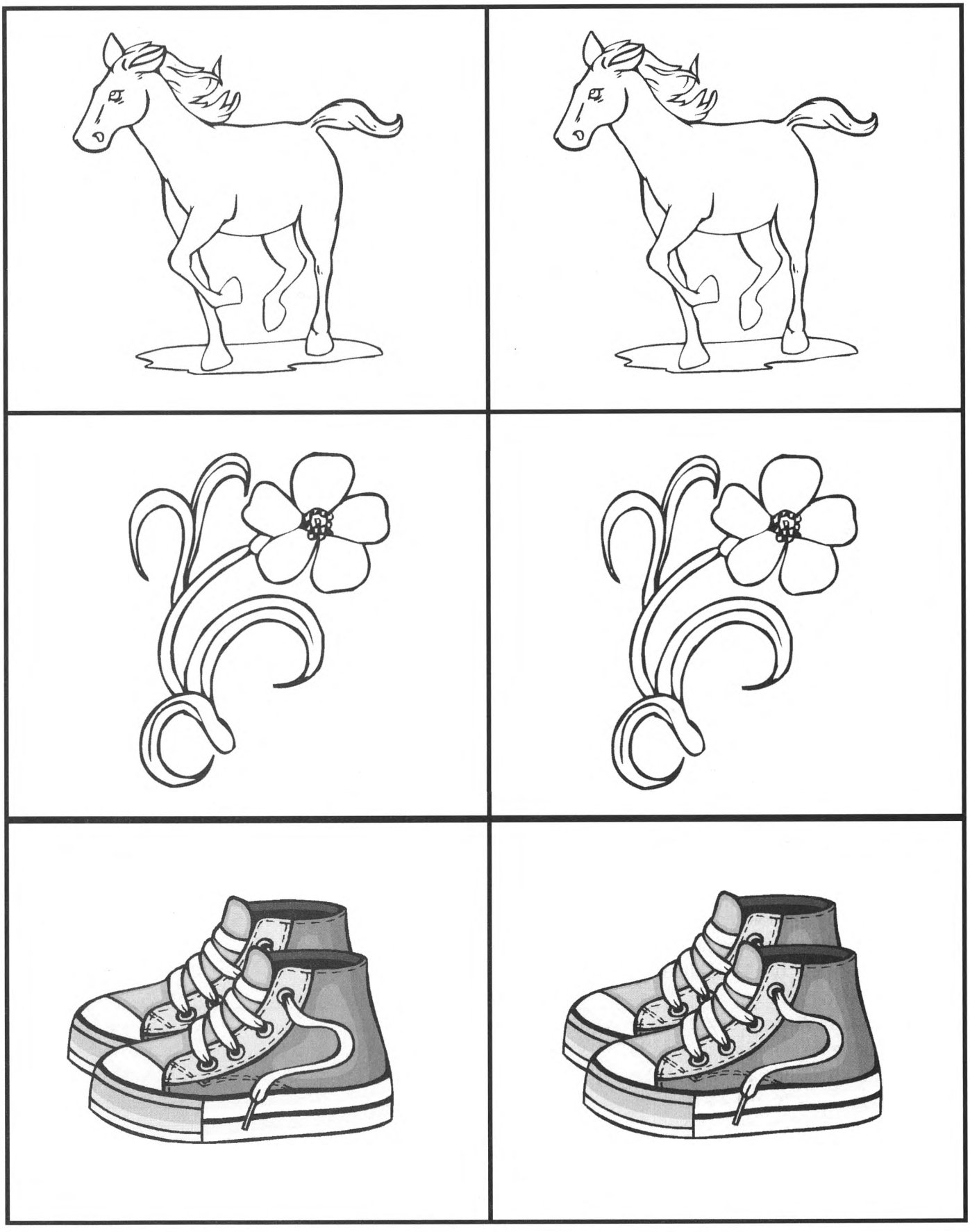
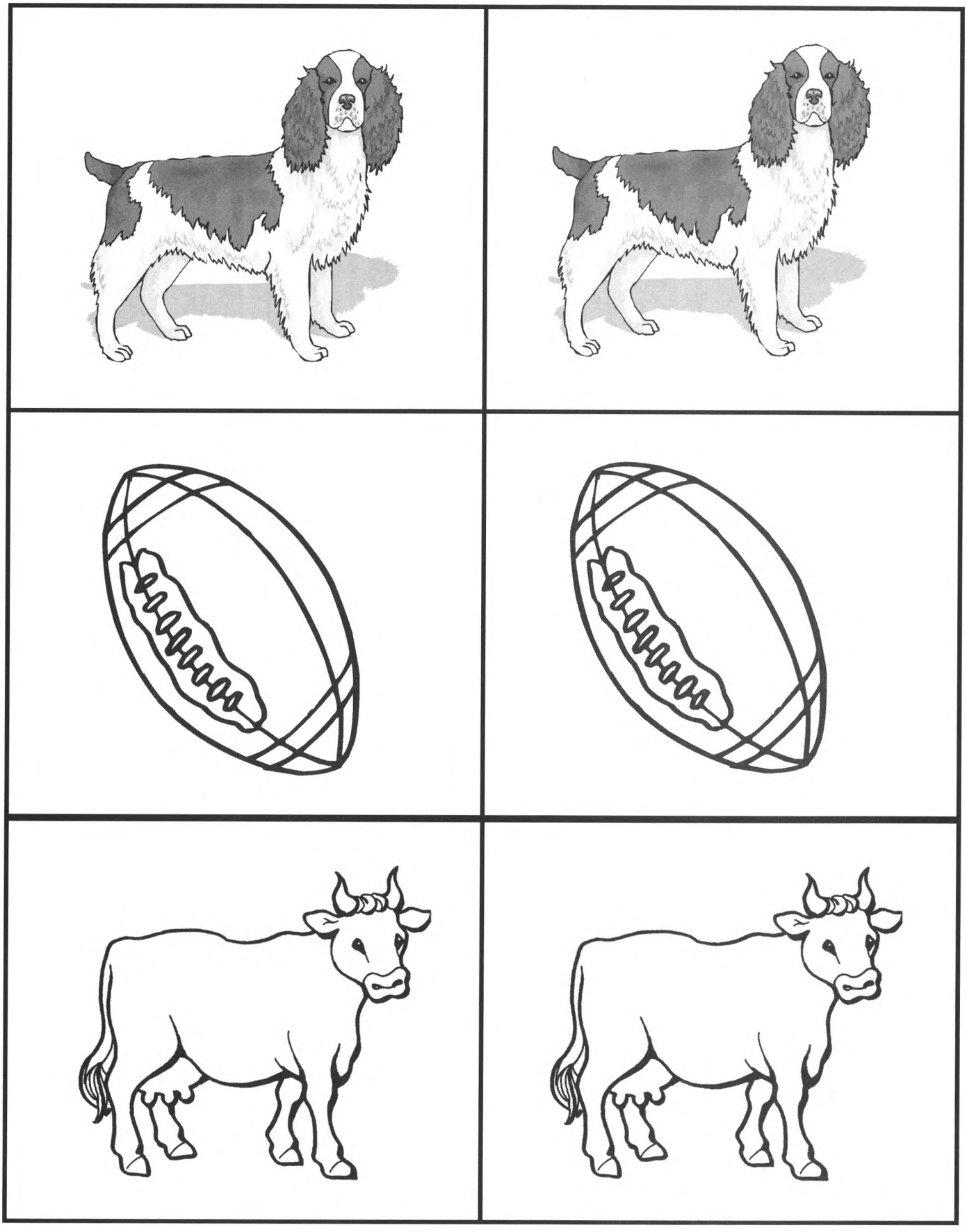
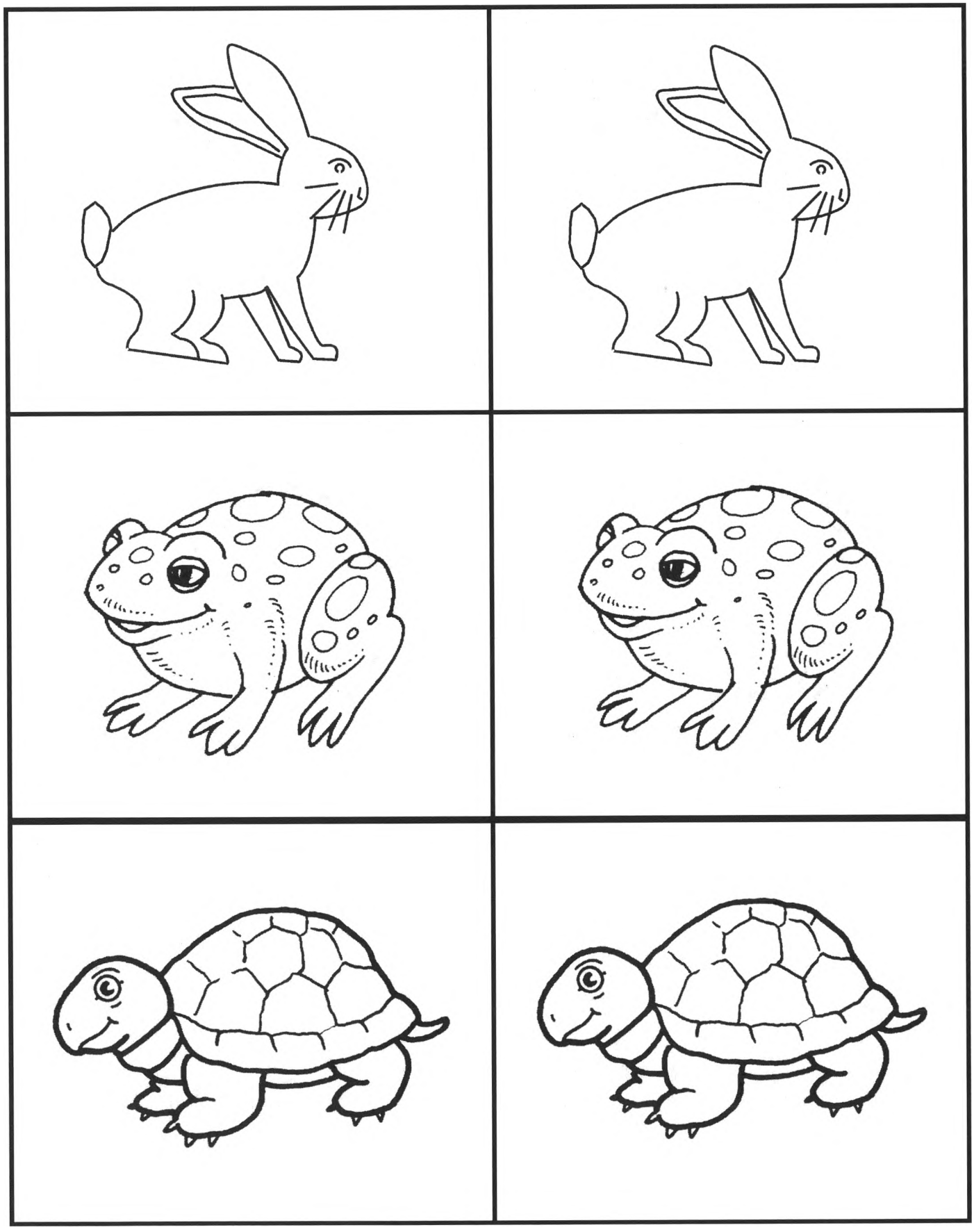
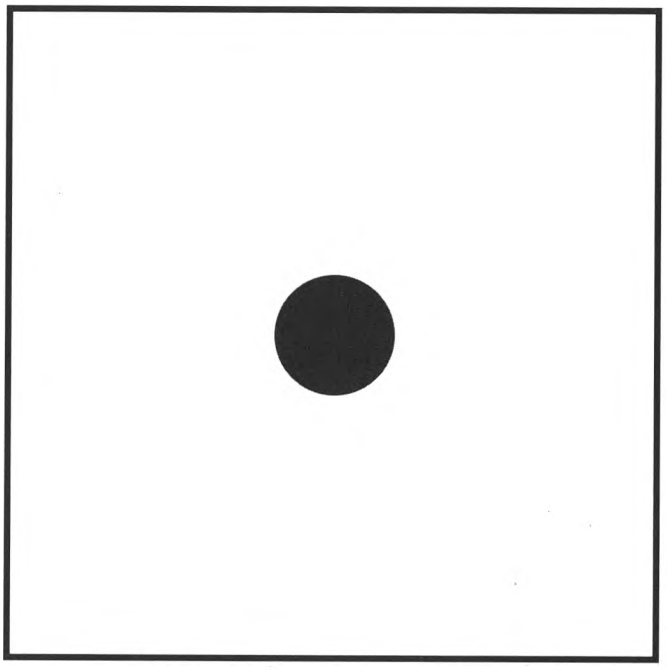
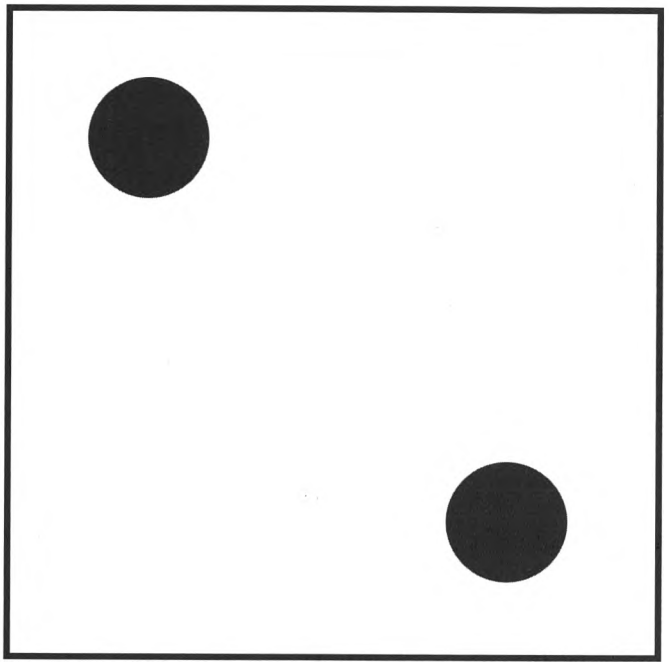
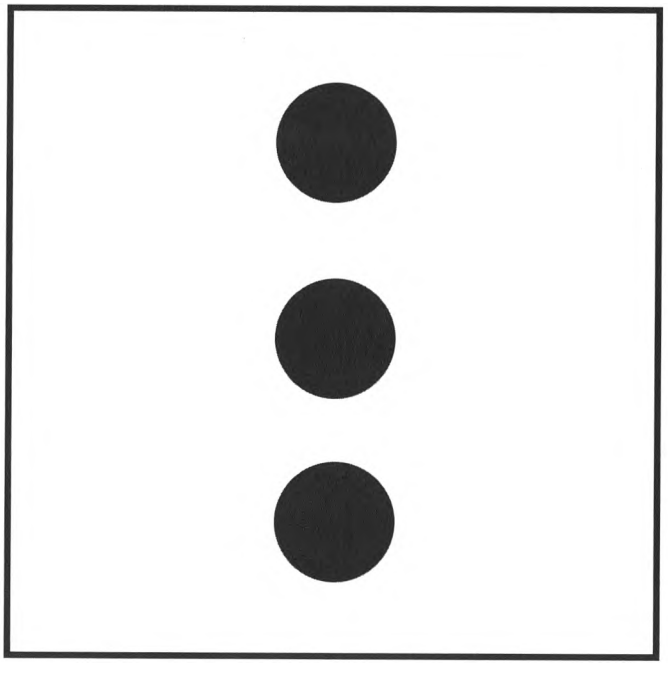


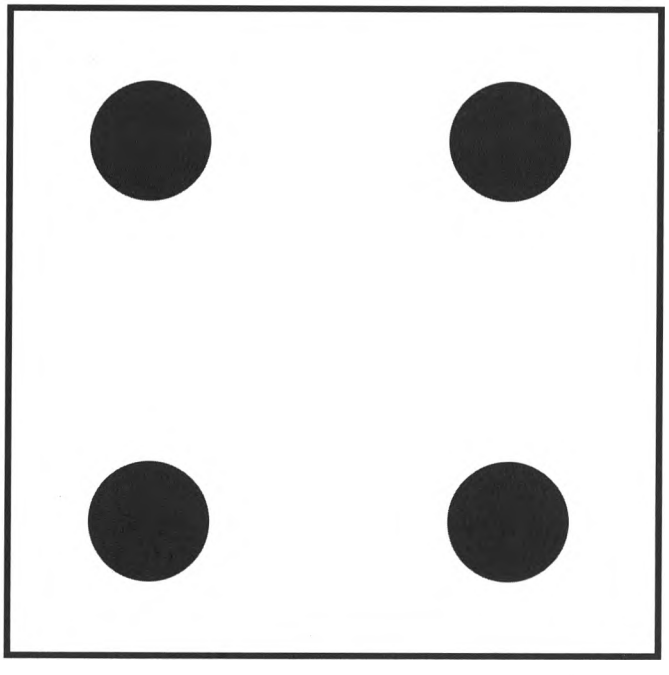












M	A	T	H
14	8	5	2
18	1	8	13
3	16	12	9
11	4	19	17
7	15	10	6

M	A	T	H
11	2	5	17
17	7	9	14
1	3	13	9
12	4	6	15
8	16	10	18

M	A	T	H
16	10	7	4
1	3	10	15
5	18	14	11
13	6	2	19
9	17	12	8

M	A	T	H
14	28	25	22
18	21	30	13
23	16	12	29
11	24	19	17
27	15	10	26

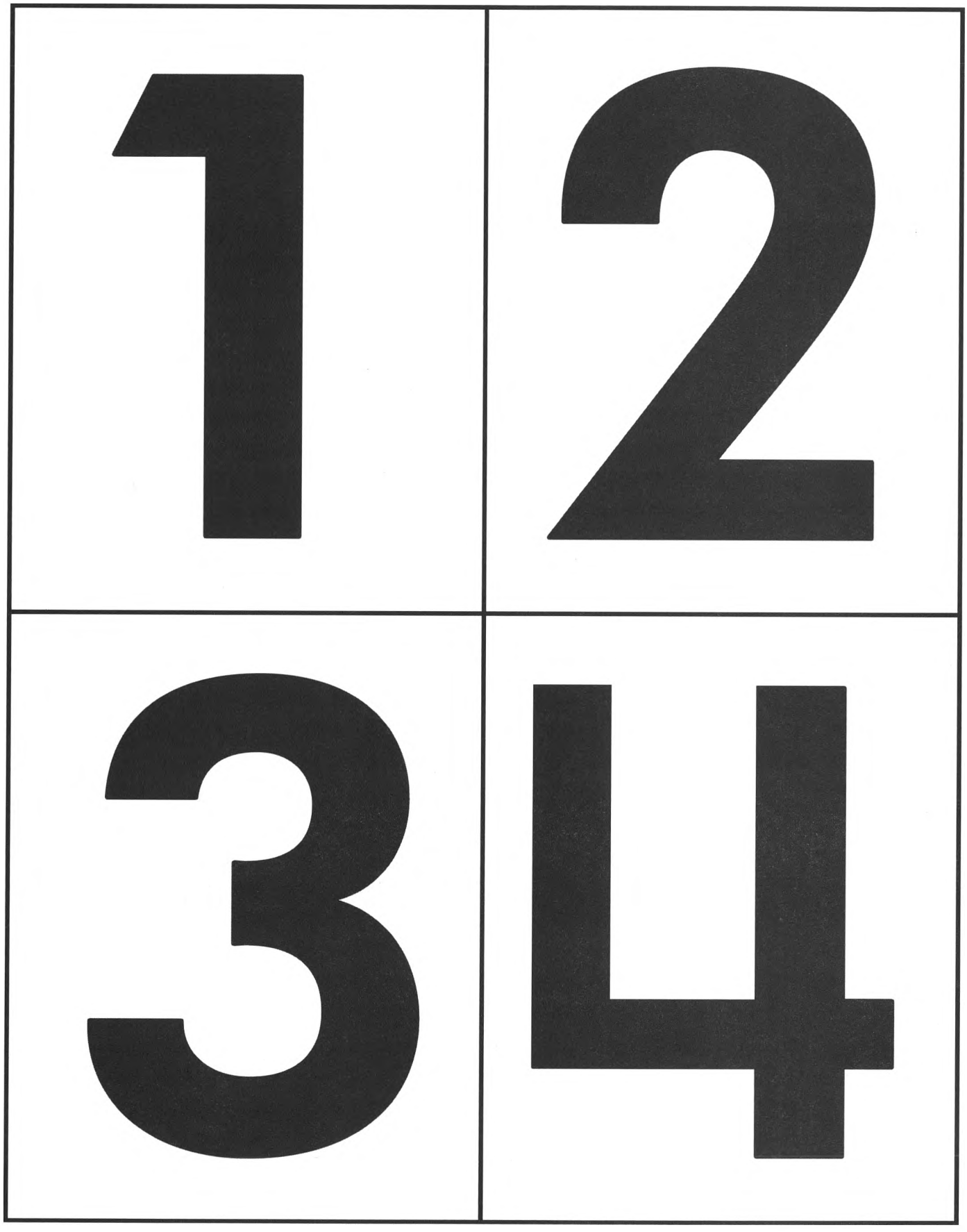
M	A	T	H
11	22	25	17
17	27	29	14
21	23	13	30
12	24	26	15
28	16	10	18

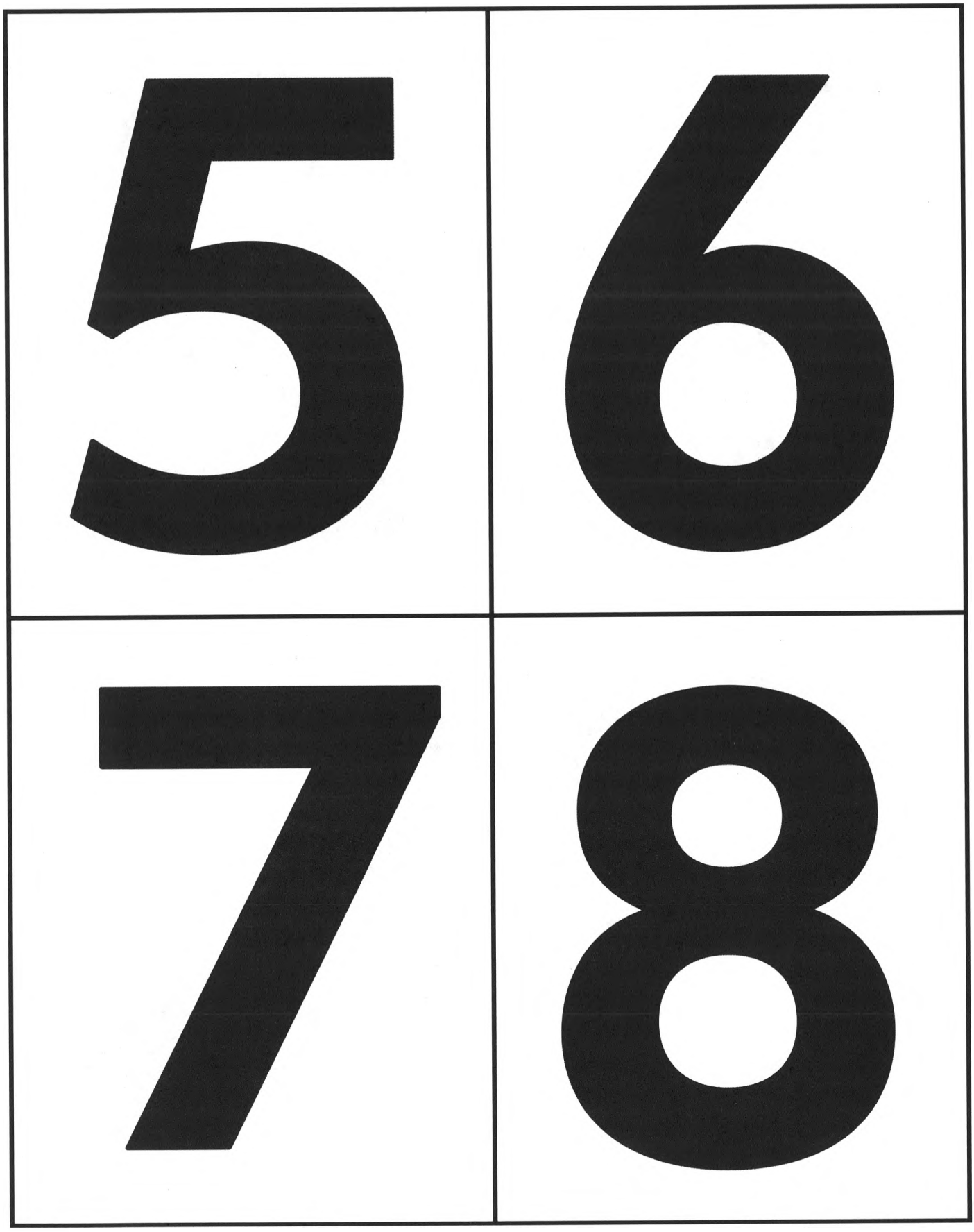
M	A	T	H
16	10	27	24
21	23	30	15
25	18	14	11
13	26	22	19
29	17	12	28

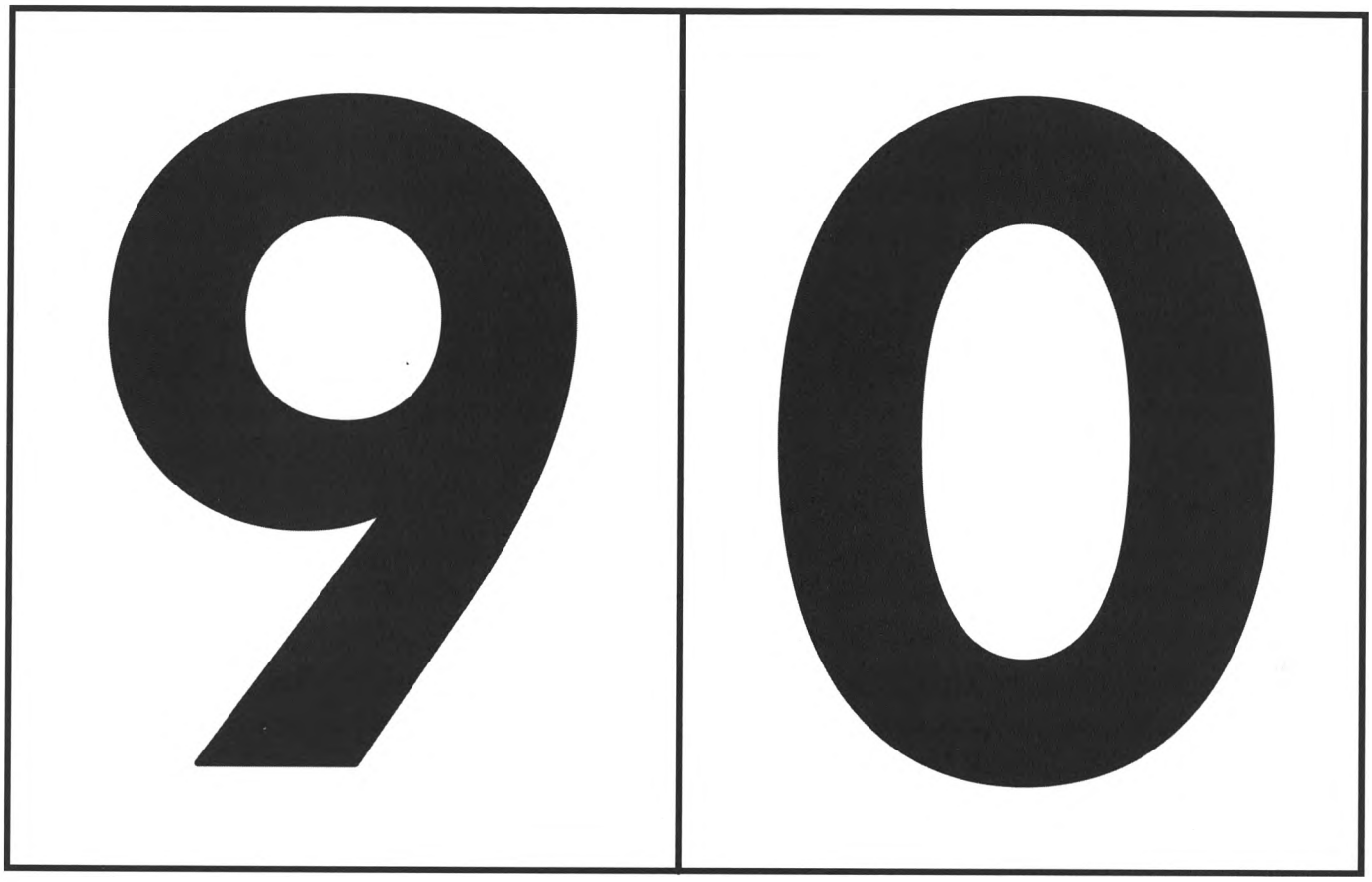
M	A	T	H
44	98	55	82
68	41	70	63
83	76	92	59
40	64	89	47
57	75	40	60

M	A	T	H
51	72	65	47
87	47	59	94
61	93	83	70
42	64	60	85
58	76	50	48

M	A	T	H
46	40	57	54
61	63	70	55
75	88	44	71
83	76	92	50
49	77	42	80


1
2
3
4


5
6
7
8


90

Number Lines

1. Cut each line of ten out separately. You will then have 5 strips of ten. Start using just the numbers from 1 - 10. As the student progresses, add another line for the numbers 1-20.

1	2	3	4	5	
11	12	13	14	15	
21	22	23	24	25	
31	32	33	34	35	
41	42	43	44	45	

2. Paperclip or Velcro the narrow cell which follows each of the *tens* to the next number sequence. You can then have a paper line from 1–40.

6	7	8	9	**10**	
16	17	18	19	**20**	
26	27	28	29	**30**	
36	37	38	39	**40**	
46	47	48	49	**50**	

Missing Number Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Counting by Tens

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Counting by Fives

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Counting by Tens to 100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

You may want to try and see if the students can handle all the detail of a 100's chart. Make sure that they are very secure with using 1 - 50 chart before you start to continue the pattern of counting by 10's from 51 - 100.

HUNDREDS *Grown-ups*	TENS *Teens*	ONES *Kids*	DECIMAL POINT WALL
			•
			•
			•
			•
			•
			•
			•
			•
			•
			•
			•
			•

Teens (Tens) and Kids (Ones)

Look at each number below. Draw a **red** circle around all the Teens (Tens). Draw a **blue** circle around the Kids (Ones). The student can just scribble the color over the number if it is difficult to draw a circle.

. 1 2	. 3 4
. 9 1	. 4 3
. 3 9	. 8 7
. 5 1	. 6 3
. 2 8	. 5 6
. 9 5	. 4 5
. 4 6	. 2 1
. 7 7	. 1 7
. 1 9	. 1 4

Cover Up Game Board

Cut apart columns. Tape #6 to #7 to form a line from 1-12.
(This page makes two boards.)

1	7	1	7
2	8	2	8
3	9	3	9
4	10	4	10
5	11	5	11
6	12	6	12

Shortcuts to Addition

Adding Zero

When you add 0 to any number, the answer is the same number. If you add nothing to 5, the answer is 5. If you add nothing to 8, the answer is still 8.

1 +0	2 +0	4 +0	6 +0	5 +0	3 +0	7 +0	9 +0	8 +0
0 +5	0 +3	0 +9	0 +6	0 +1	0 +8	0 +2	0 +7	0 +4

Adding 1 to Any Number

When you add 1 to any number, the answer is the next highest number you can count. If you add 1 to 7, the next number higher is 8. So, 7 + 1 = 8. If you add 1 to 2, the next higher number is 3. So, 2 + 1 = 3.

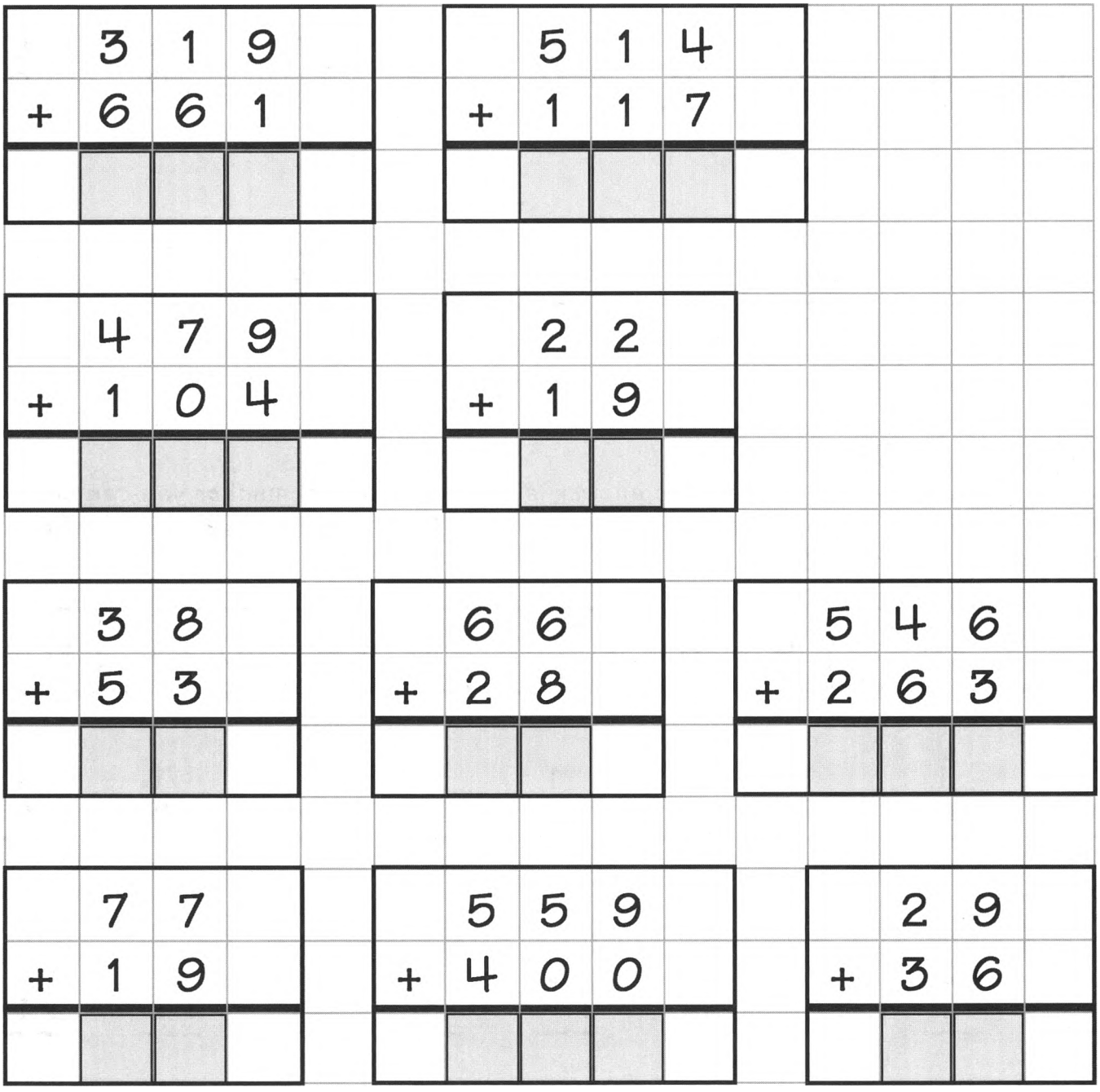
1	2	3	4	5	6	7	8	9	10	11	12

1 +1	1 +4	1 +7	1 +9	1 +5	1 +2	1 +8	1 +3	1 +6
4 +1	2 +1	7 +1	9 +1	5 +1	8 +1	6 +1	3 +1	1 +1

Name: _____

Date: _____

Addition with 2- and 3-Digit Numbers



Name: _____

Date: _____

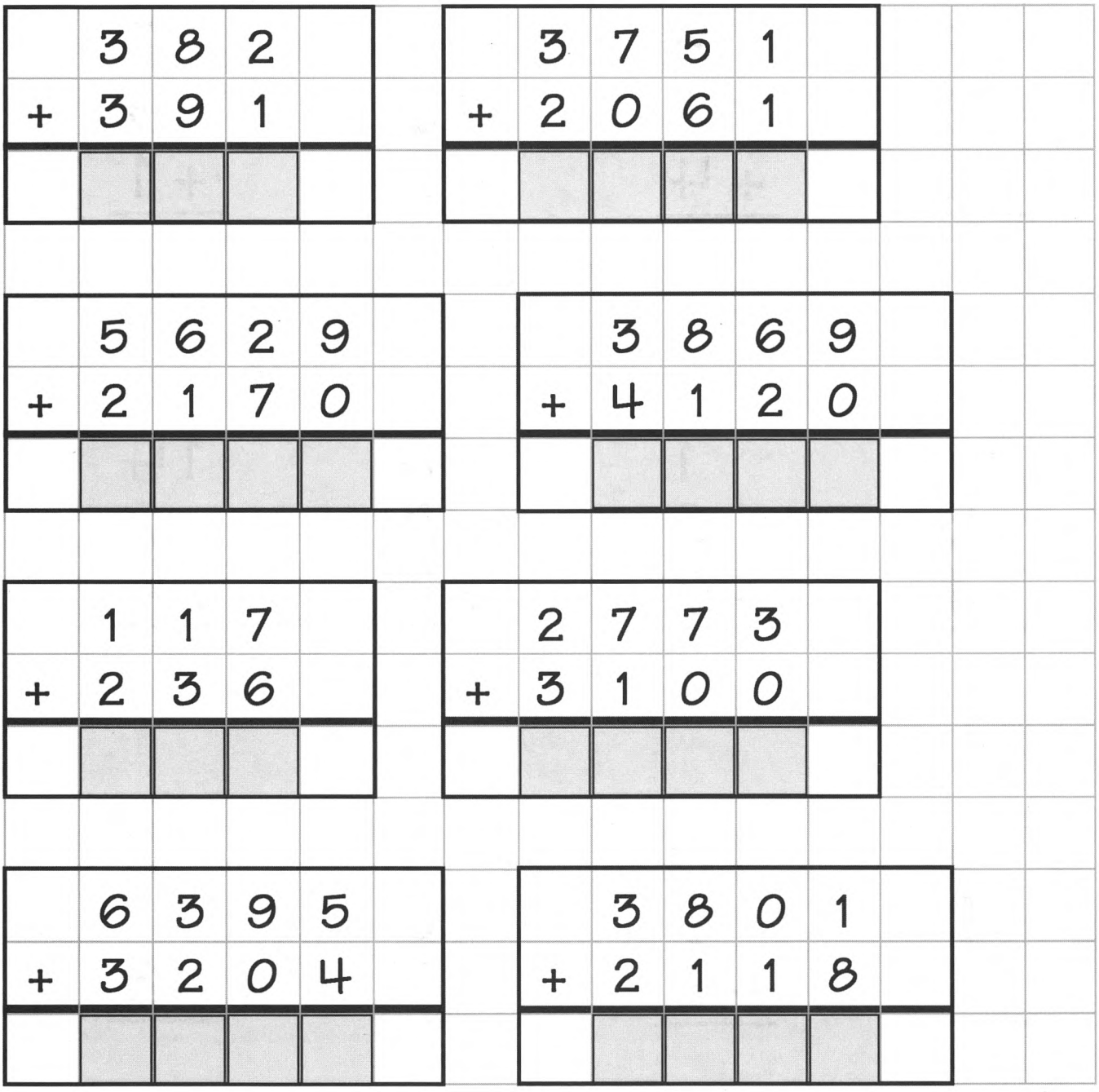
Addition of 3 Double- and Single-Digit Numbers

17 2 +1	3 11 +4	13 3 +2	9 9 +1
11 5 +1	1 6 +6	9 8 +3	14 2 +4
1 8 +11	10 0 +3	4 9 +7	2 1 +16

Name: _____

Date: _____

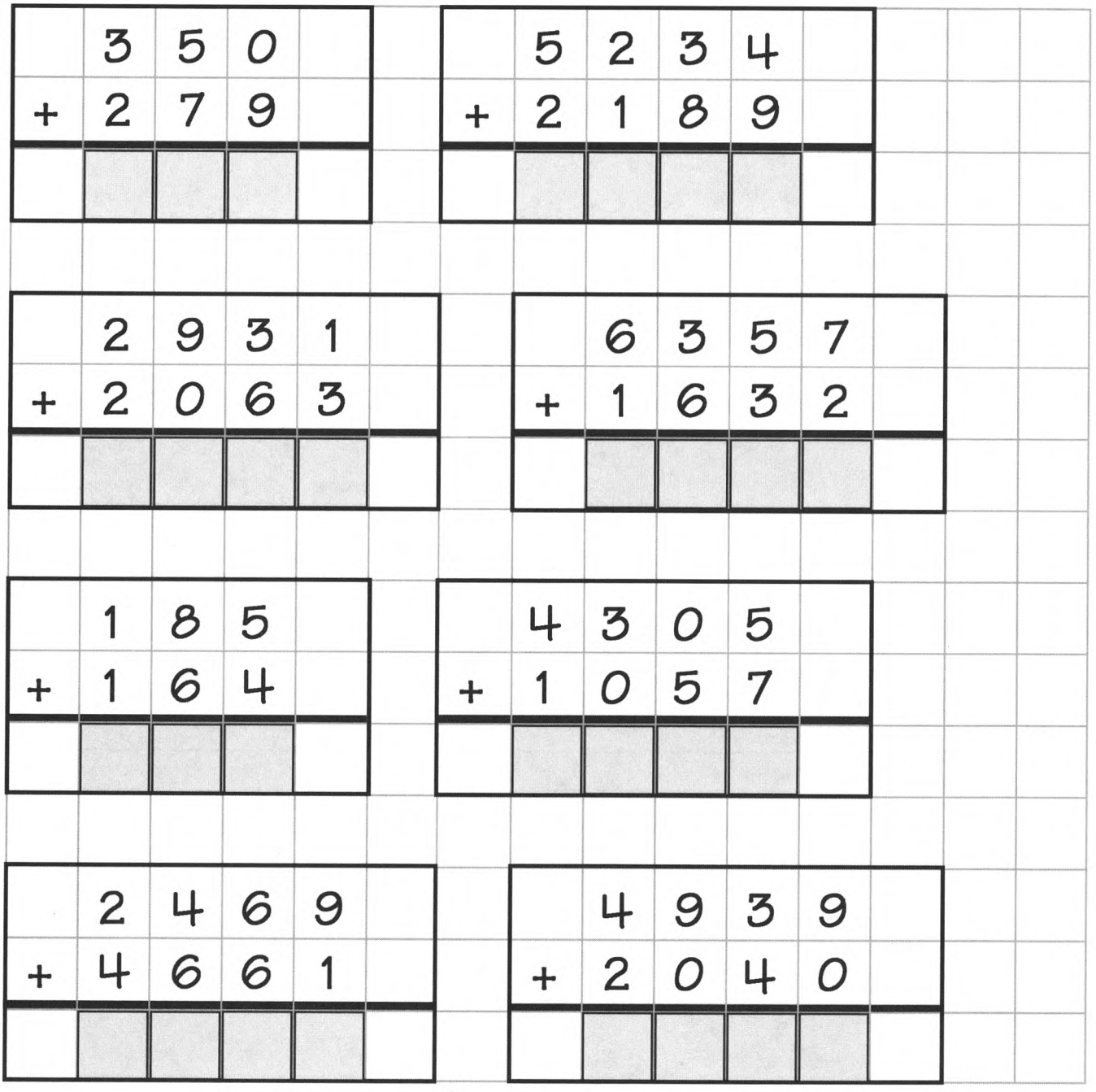
Addition with 3- and 4-Digit Numbers



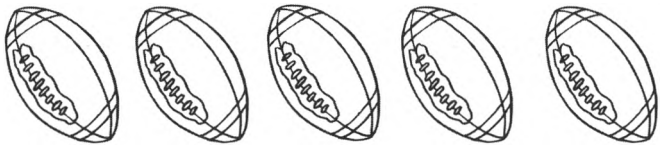
Name: _____

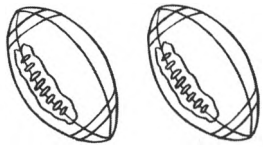
Date: _____

Addition with 3- and 4-Digit Numbers



Addition Story Problems — with Drawings

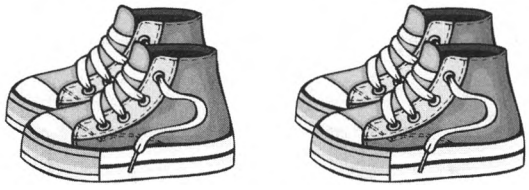
1. I have 5 footballs. 

   I found 2 more footballs in the garage. 

   Count the number of footballs. The total number of footballs is _____ .

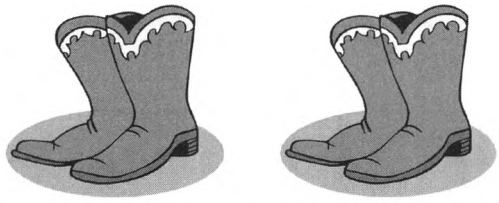
---

2. There are 6 gym shoes in my closet.

   I bring 4 more gym shoes to my closet. 

   I have (how many?) shoes in my closet _____ .

---

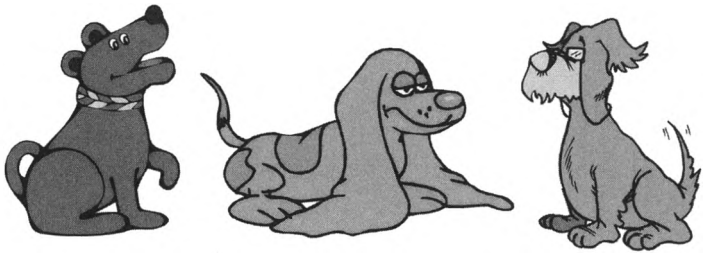
3. I have 4 boots to wear. 

   My dad bought me 2 new boots.

   Count all my boots. How many do I have now? _____

4. I have 1 dog.

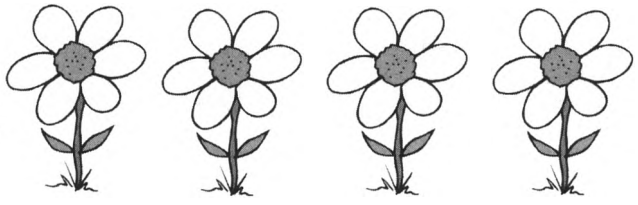
My best friend has 3 dogs.



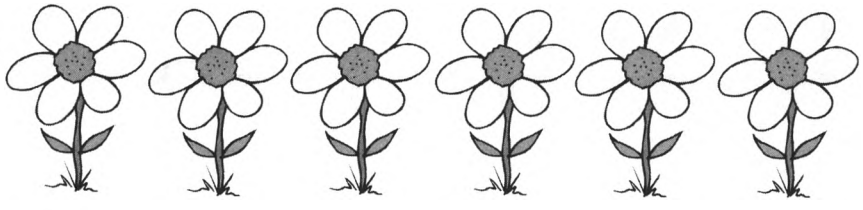
Together, we have _____ dogs.

---

5. I have picked 4 flowers.



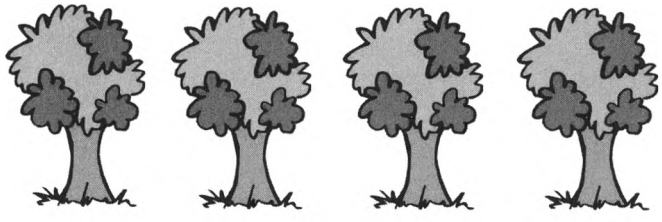
My mother picked 6 flowers.



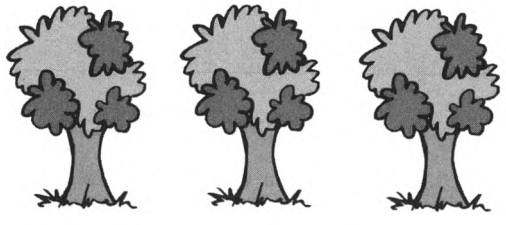
The total number of flowers is _____ .

---

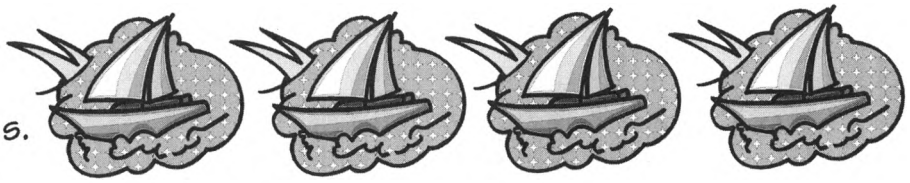
6. We have 4 trees in our backyard.



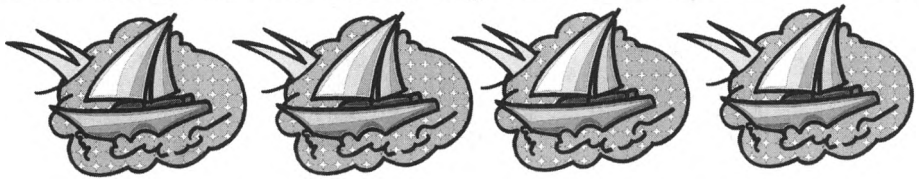
We have 3 trees in our front yard.



How many trees do we have in our front and backyard together? _____

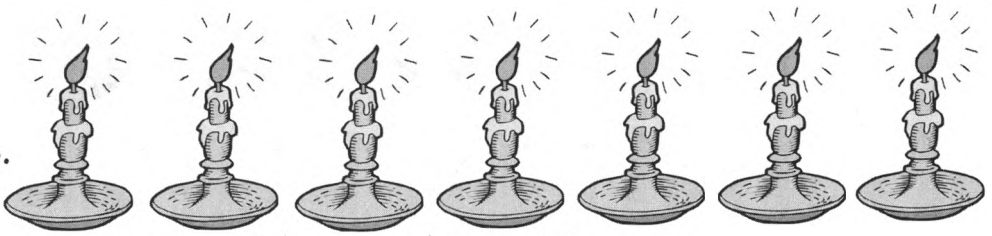
7. I have 4 small sailboats. 

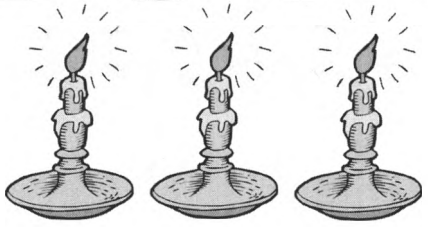
I race them in the creek. My best friend also has 4 boats.



We have (how many?) _____ boats altogether.

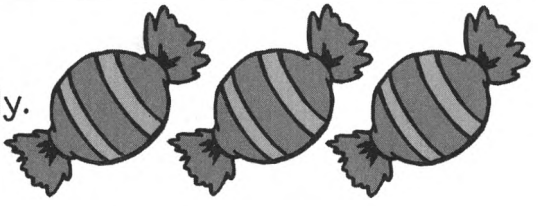
---

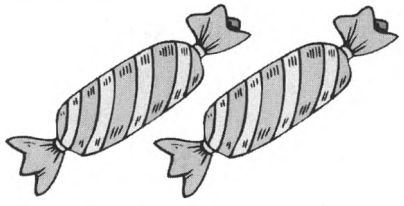
8. In our living room, we have 7 candles. 

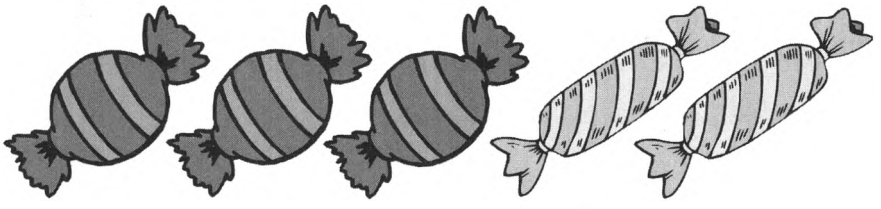
Mom bought 3 new candles. 

Now we have (how many?) _____ candles.

---

9. I have 3 pieces of candy. 

My sister has 2 pieces. 

My brother has 5 pieces. 

We have (how many?) _____ pieces of candy altogether?

Story Problems to Be Explained

Note: *The following problems should be explained one by one by the teacher. The student is not supposed to do the problems by reading this sheet. Read or tell one problem and demonstrate how to draw or stamp the number of items and then count to find the total.*

1. In the school baseball game, Ellen's team made 7 points. Tom's team made 5 points. Who won the game? How many total points were made in the game?

2. Doug's mother baked 36 chocolate chip cookies for the class party. Greg's older sister baked 17 sugar cookies and Rosie baked 24 peanut butter cookies for the party. The total number of cookies for the class party was _____.

3. Jake had 7 fish in his fish bowl. One of the fish gave birth to 20 baby fish. Jake now has _____fish in his fish bowl. He needs to get a bigger fish bowl!

4. Sandy and Nancy went on a picnic. They ate 4 sandwiches, 2 apples, and 3 brownies all together. How many things did they eat at the picnic? (Question format)

5. At the zoo I saw 20 monkeys climbing in the trees. Four monkeys were playing in the water. How many monkeys did I see altogether? (Question format)

6. On a summer campout, we went searching for red rocks. James found 23 red rocks, Jeff found 15 red rocks, and Stephanie found 16 red rocks. How many red rocks did James, Jeff, and Stephanie find altogether? (Question format)

7. Kathy found 15 white shells on the beach. She also found 7 colored shells. How many shells altogether did she find? (Question format)

8. Paul built a sand castle on the beach. He used 11 pails of wet sand to build his castle. He then used 2 pails of dry sand to make the walls of the castle look smooth. How many pails of sand did he use for the sand castle? (Question format)

9. Nate caught the football and ran 34 yards. Then he ran 23 yards for a touchdown. How many yards did he run altogether? (Question format)

10. In another game, Nate ran 13 yards with the football. The next play he handed off the football to Mark, and Mark ran for 21 yards. How many yards did Nate and Mark run altogether? (Question format)

11. Leslie ate 11 pancakes on Saturday morning. DeDe ate 3 pancakes. How many did they eat altogether? (Question format)

The word *question format* in parentheses indicates that the problem may be harder for the student to understand because the subject and verb order is changed by using the question format.

Name: _____

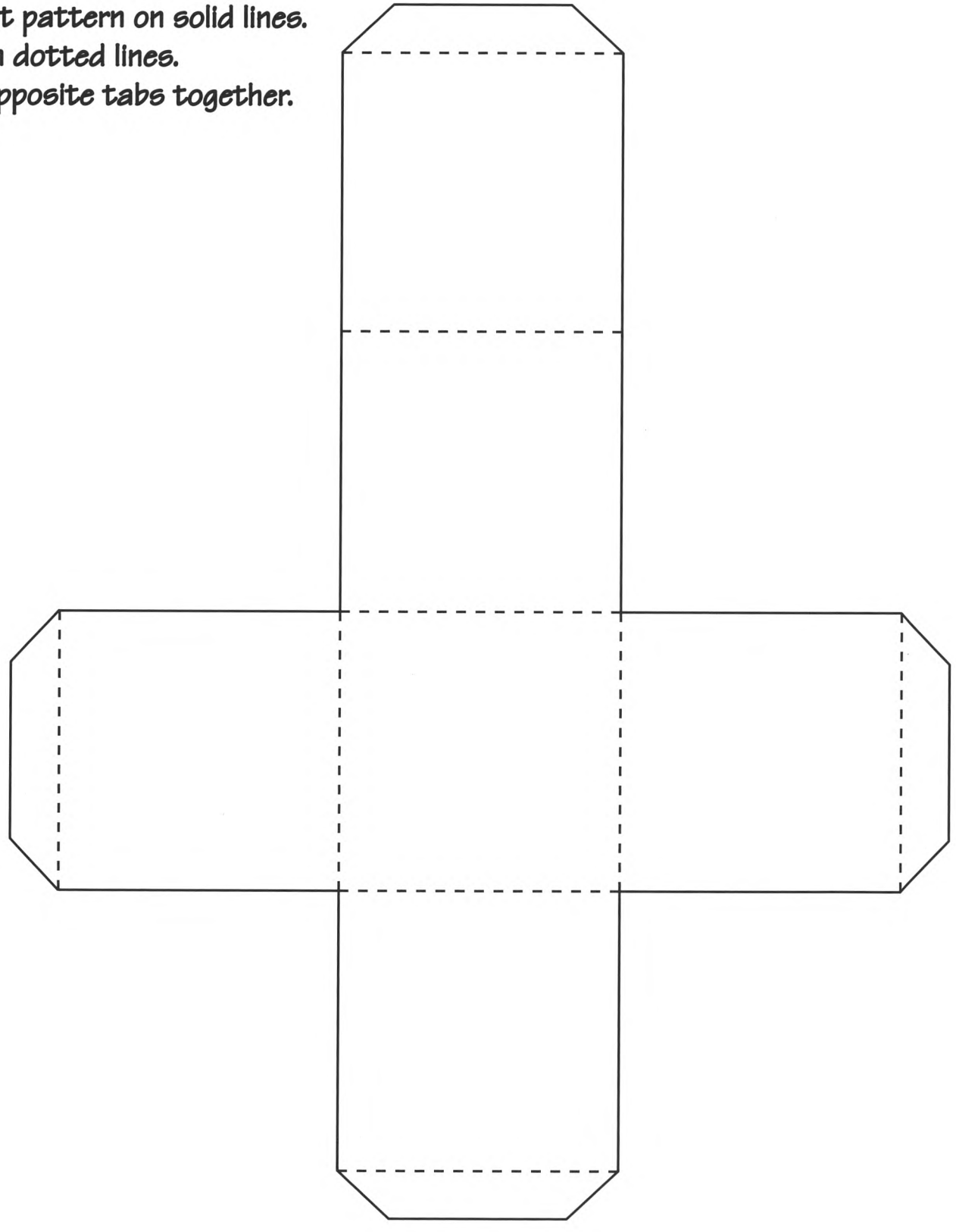
Date: _____

Subtraction Practice Worksheet

50 −11	81 −75	75 −69
10 −10	93 −25	34 −19
86 −61	33 −25	76 −76

Number Cube Pattern

Cut out pattern on solid lines.
Fold on dotted lines.
Glue opposite tabs together.



Turnarounds

Problem	Turnaround
EXAMPLE: 2 x 3 = 6	3 x 2 = 6
4 x 2 = ____	
5 x 2 = ____	
5 x 6 = ____	
3 x 3 = ____	
1 x 2 = ____	
1 x 5 = ____	
6 x 5 = ____	
4 x 3 = ____	
5 x 3 = ____	
4 x 4 = ____	
4 x 5 = ____	
6 x 4 = ____	
3 x 1 = ____	
4 x 3 = ____	
5 x 3 = ____	
2 x 6 = ____	
2 x 5 = ____	
3 x 6 = ____	
6 x 4 = ____	

Multiplying by Ten Worksheet

Fold this paper into halves, fourths, and then eighths so that you have only one problem showing at a time.

1) $5 \times 10 = ___$

2) $3 \times 10 = ___$

3) $2 \times 10 = ___$

4) $4 \times 10 = ___$

5) $6 \times 10 = ___$

6) $7 \times 10 = ___$

7) $4 \times 10 = ___$

8) $8 \times 10 = ___$

Calculator Multiplication Worksheet

1) $76 \times 0 =$ _____

2) $41 \times 0 =$ _____

3) $56 \times 1 =$ _____

4) $38 \times 5 =$ _____

5) $32 \times 4 =$ _____

6) $62 \times 3 =$ _____

7) $61 \times 1 =$ _____

8) $67 \times 1 =$ _____

9) $66 \times 7 =$ _____

10) $47 \times 9 =$ _____

11) $20 \times 2 =$ _____

12) $33 \times 0 =$ _____

Calculator Multiplication Worksheet

1) 380 × 0 = _____

2) 32 × 6 = _____

3) 980 × 4 = _____

4) 964 × 4 = _____

5) 54 × 7 = _____

6) 12 × 6 = _____

7) 455 × 1 = _____

8) 880 × 9 = _____

9) 131 × 1 = _____

10) 576 × 7 = _____

11) 933 × 6 = _____

12) 550 × 2 = _____

BATTER UP
GAME
Home
1
2
3
Pitcher

Batter Up Game Cards

Pitched ball hits you in the head. Walk to the first or next base.	Foul ball. Lose a turn.
Bunt. Move to first or next base.	Fielder caught your fly ball. You are out.
Fielder caught your fly ball. You are out.	Steal a base. Go ahead one base.
Steal a base. Go ahead one base.	You try to steal a base and are tagged out.
Steal a base. Go ahead one base.	Steal two bases.

Batter Up Game Cards

A 3 x 9 = or 12 x 33 =	A 6 x 7 = or 23 x 11 =
A 21 x 7 = or 8 x 9 =	A 48 x 7 = or 5 x 5 =
A 9 x 9 = or 45 x 22 =	A 7 x 7 = or 37 x 11 =
A 82 x 12 = or 4 x 9 =	A 5 x 9 = or 73 x 41 =
A 45 x 8 = or 8 x 3 =	A 22 x 5 = or 6 x 6 =

Batter Up Game Cards

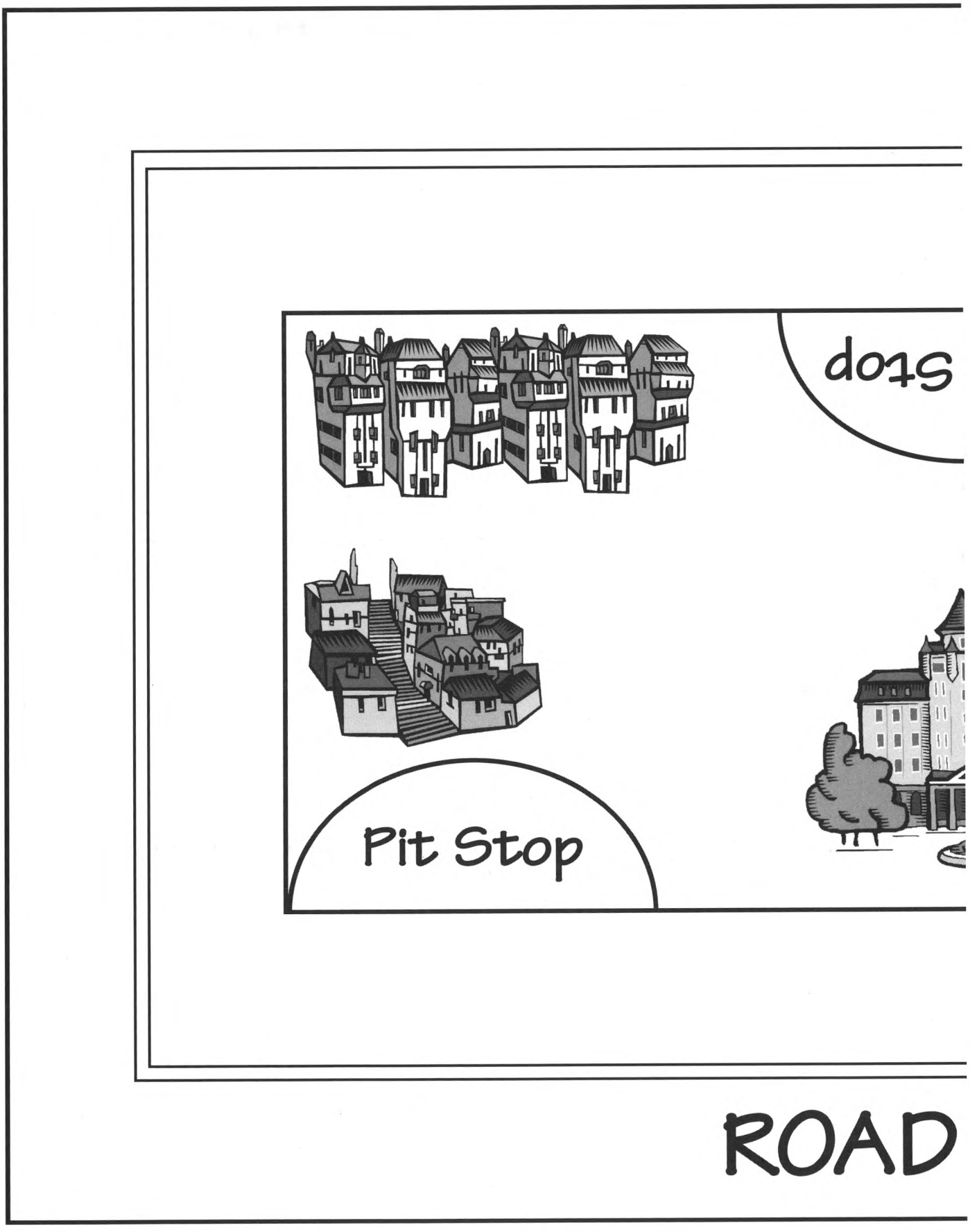
B 7 x 5 = or 22 x 43 =	B 4 x 7 = or 56 x 11 =
B 21 x 3 = or 8 x 3 =	B 8 x 7 = or 31 x 22 =
B 7 x 9 = or 61 x 12 =	B 7 x 7 = or 37 x 11 =
B 32 x 42 = or 4 x 6 =	B 54 x 9 = or 8 x 8 =
B 22 x 4 = or 6 x 8 =	B 51 x 2 = or 8 x 4 =

Batter Up Game Card Answers

A $3 \times 9 = 27$ or $12 \times 33 = 396$	A $6 \times 7 = 42$ or $23 \times 11 = 253$
A $21 \times 7 = 147$ or $8 \times 9 = 72$	A $48 \times 7 = 336$ or $5 \times 5 = 25$
A $9 \times 9 = 81$ or $45 \times 22 = 990$	A $7 \times 7 = 49$ or $37 \times 11 = 407$
A $82 \times 12 = 984$ or $4 \times 9 = 36$	A $5 \times 9 = 45$ or $73 \times 41 = 2993$
A $45 \times 8 = 360$ or $8 \times 3 = 24$	A $22 \times 5 = 110$ or $6 \times 6 = 36$

Batter Up Game Card Answers

B $7 \times 5 = 35$ or $22 \times 43 = 946$	B $4 \times 7 = 28$ or $56 \times 11 = 616$
B $21 \times 3 = 63$ or $8 \times 3 = 24$	B $8 \times 7 = 56$ or $31 \times 22 = 682$
B $7 \times 9 = 63$ or $61 \times 12 = 732$	B $7 \times 7 = 49$ or $37 \times 11 = 407$
B $32 \times 42 = 1344$ or $4 \times 6 = 24$	B $54 \times 9 = 486$ or $8 \times 8 = 64$
B $22 \times 4 = 88$ or $6 \times 8 = 48$	B $51 \times 2 = 102$ or $8 \times 4 = 32$


dots
Pit Stop
ROAD

Pit
Pit Stop
FINISH
START
Pit Stop
RACE

Forward 1 inch.	Forward 1 inch.
Forward 1 inch.	Forward 1 inch.
Forward 2 inches.	Forward 2 inches.
Forward 2 inches.	Forward 2 inches.
Forward 3 inches.	Forward 3 inches.
Forward 3 inches.	Forward 3 inches.

Forward 4 inches.	Forward 4 inches.
Forward 4 inches.	Forward 4 inches.
Go to next pit stop and miss one turn.	Go to next pit stop and miss one turn.
Go to next pit stop and miss one turn.	Go to next pit stop and miss one turn.
Oil on track. Slide back 1 inch.	Oil on track. Slide back 1 inch.
Oil on track. Slide back 1 inch.	Oil on track. Slide back 1 inch.

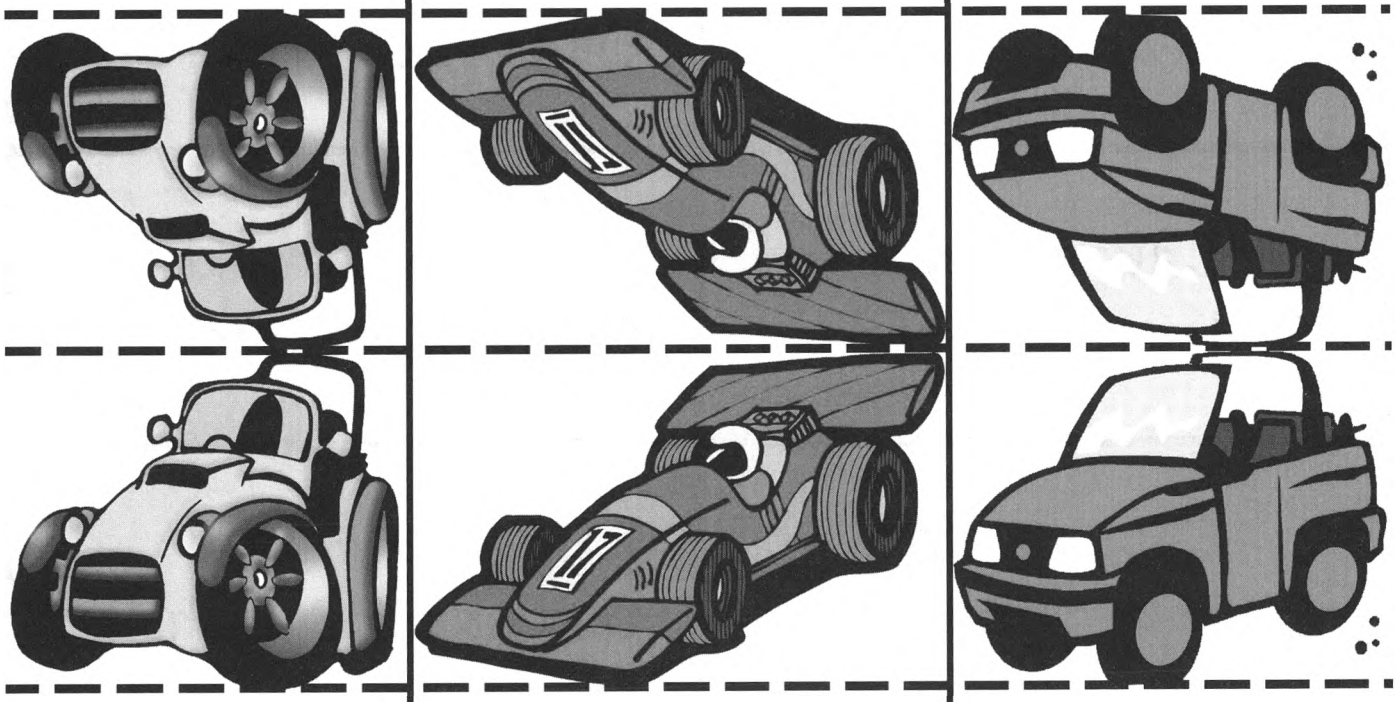
Forward 5 inches.	Forward 5 inches.
Forward 5 inches.	Forward 5 inches.
Go to next pit stop and miss one turn.	Go to next pit stop and miss one turn.
Go to next pit stop and miss one turn.	Go to next pit stop and miss one turn.
Oil on track. Slide back 2 inches.	Oil on track. Slide back 2 inches.
Oil on track. Slide back 2 inches.	Oil on track. Slide back 2 inches.

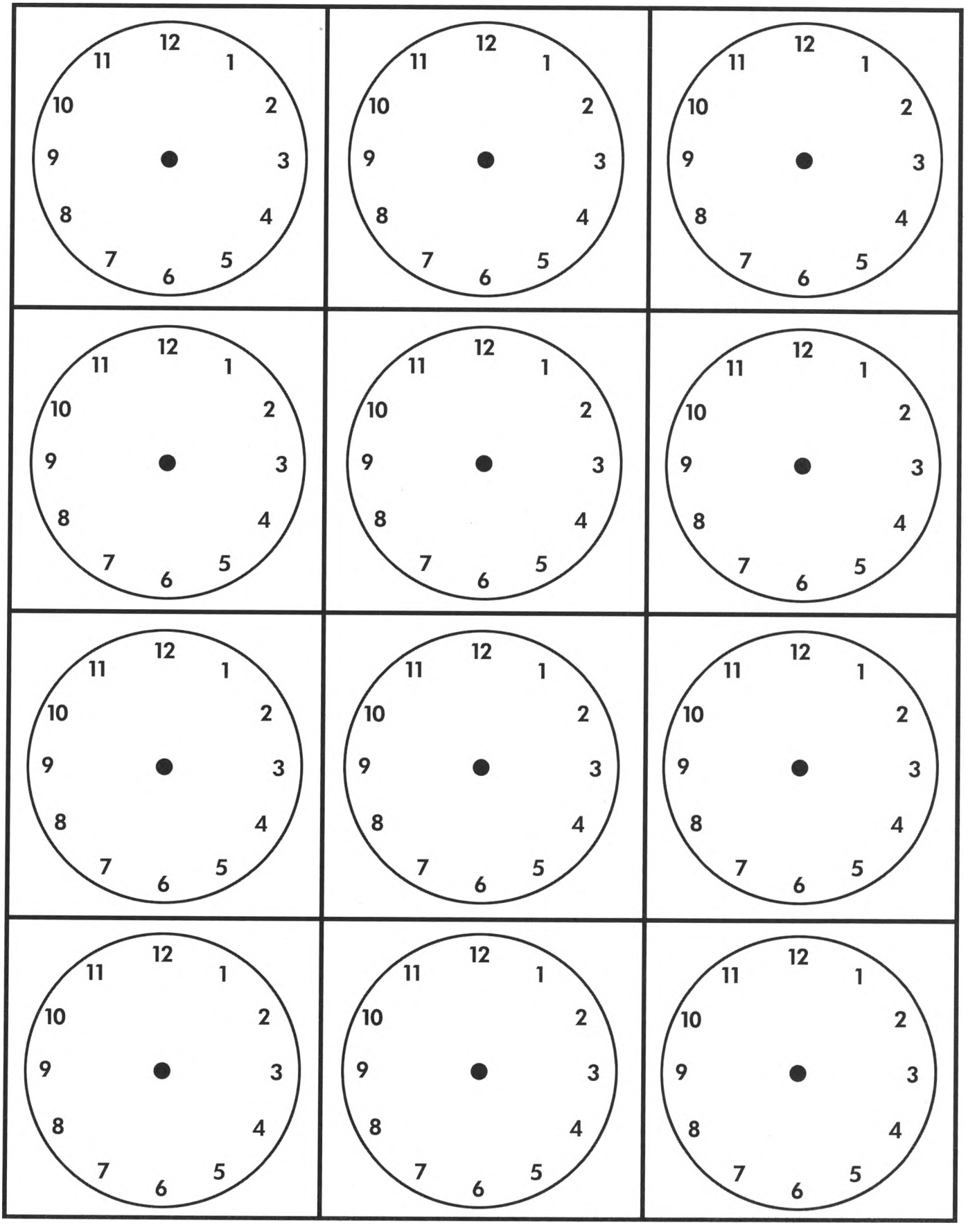
Avoid accident. Slide ahead 2 inches.	Avoid accident. Slide ahead 2 inches.
Avoid accident. Slide ahead 2 inches.	Avoid accident. Slide ahead 2 inches.
Cause wreck! Slide back 4 inches.	Cause wreck! Slide back 4 inches.
Cause wreck! Slide back 4 inches.	Cause wreck! Slide back 4 inches.
Cause wreck! Slide back 3 inches.	Cause wreck! Slide back 3 inches.
Cause wreck! Slide back 3 inches.	Cause wreck! Slide back 3 inches.

Road Race Car Pieces

Directions to make cars for the Race Car Game:

1. Photocopy and cut apart on the solid lines, making individual car pieces.
2. Fold on dotted lines.
3. Overlap bottom flaps to make a triangle-shaped stand.
4. Secure bottom with two paper clips.




12 1 2 3 4 5 6 7 8 9 10 11

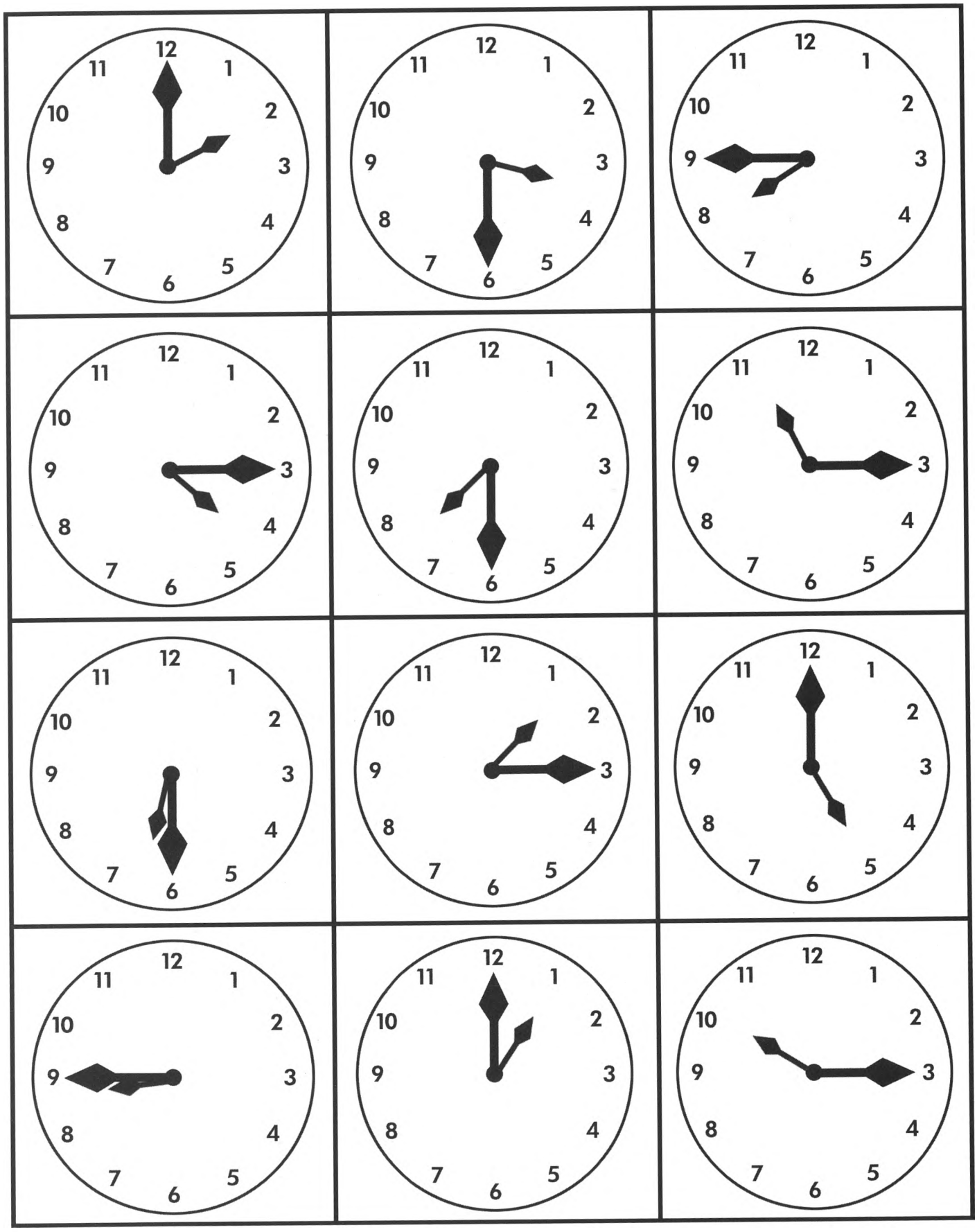
T	I	M	E
2:00	4:15	6:30	8:45
3:30	7:30	1:15	1:00
7:45	11:15	5:00	10:15

T	I	M	E
3:30	11:15	5:00	1:00
7:45	4:15	6:30	10:15
2:00	7:30	1:15	8:45

Times to be called out by the teacher: T-7:45, T-2:00, T-3:30 I-7:30, I-4:15, I-11:15 M-1:15, M-5:00, M-6:30 E-10:15, E-1:00, E-8:45

T	I	M	E
7:45	7:30	1:15	1:00
2:00	4:15	5:00	8:45
3:30	11:15	6:30	10:15

T	I	M	E
2:00	4:15	6:30	10:15
7:45	11:15	5:00	8:45
3:30	7:30	1:15	1:00



2:00	3:30	7:45
4:15	7:30	11:15
6:30	1:15	5:00
8:45	1:00	10:15

Simple Calculator Division Problems

1) $5\overline{)60}$

2) $4\overline{)40}$

3) $2\overline{)44}$

4) $8\overline{)72}$

5) $3\overline{)12}$

6) $2\overline{)10}$

7) $1\overline{)64}$

8) $6\overline{)54}$

9) $7\overline{)63}$

10) $2\overline{)24}$

11) $6\overline{)90}$

12) $2\overline{)28}$

Calculator Division Problems

1) $6\overline{)78}$

2) $5\overline{)305}$

3) $9\overline{)369}$

4) $7\overline{)630}$

5) $2\overline{)66}$

6) $1\overline{)498}$

7) $2\overline{)86}$

8) $8\overline{)480}$

9) $5\overline{)130}$

10) $2\overline{)622}$

11) $2\overline{)74}$

12) $1\overline{)225}$

Division Problems Checked by Multiplication

1) $4\overline{)16}$

2) $7\overline{)84}$

3) $3\overline{)24}$

4) $4\overline{)28}$

5) $5\overline{)20}$

6) $8\overline{)96}$

7) $3\overline{)12}$

8) $6\overline{)12}$

9) $3\overline{)99}$

10) $8\overline{)48}$

11) $5\overline{)25}$

12) $9\overline{)54}$

13) $7\overline{)84}$

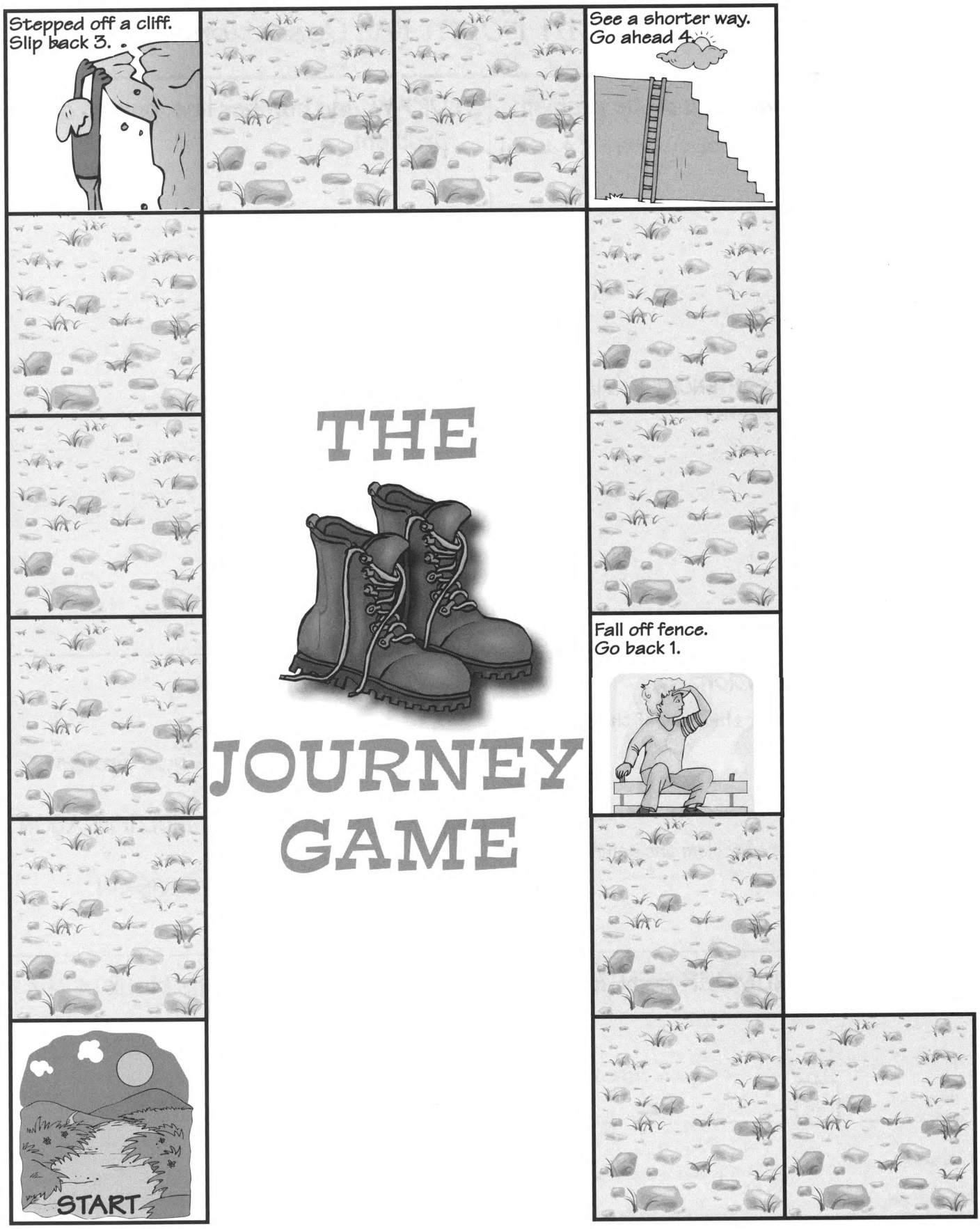
14) $3\overline{)21}$

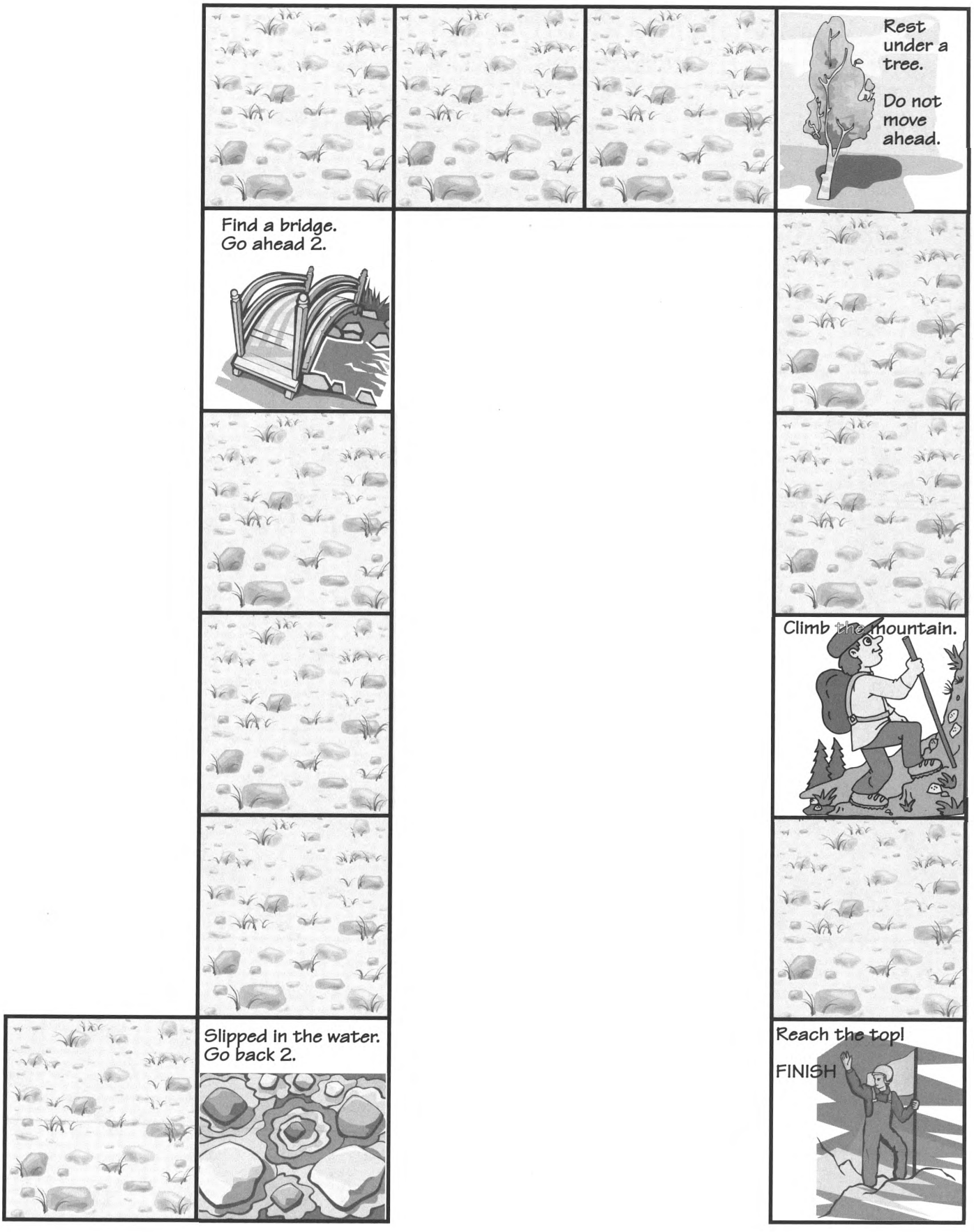
15) $9\overline{)63}$

16) $8\overline{)16}$

Word Problem Form (All Operations)

1) READ and understand the problem. (Write needed numbers with their labels; e.g., 4 desks) **WRITE THE ANSWER SENTENCE** with a blank. (e.g., Room has ____ desks.)	2) DRAW, use number line, or act out problem.
3) FIND the operation. Will the answer be larger than any of the numbers in the problem? Are the numbers in the problem the same or equal?	4) CIRCLE the operation. Equal Groups ↓ Larger → ADDITION MULTIPLICATION Smaller → SUBTRACTION DIVISION
5) WRITE THE NUMBER SENTENCE. _____ ☐ _____ = _____	6) WRITE ANSWER in the blank of the sentence in square 1.


Stepped off a cliff.
Slip back 3.
See a shorter way.
Go ahead 4.
THE
JOURNEY
GAME
Fall off fence.
Go back 1.
START


Rest under a tree.
Do not move ahead.
Find a bridge.
Go ahead 2.
Climb the mountain.
Slipped in the water.
Go back 2.
Reach the top!
FINISH

Journey Game Problem Cards

(Multiplication—Numbers, 1 and 2 digits)

YM1 $42 \times 6 =$ _____	YM2 $5 \times 54 =$ _____
YM3 $7 \times 22 =$ _____	YM4 $5 \times 70 =$ _____
YM5 $8 \times 9 =$ _____	YM6 $5 \times 7 =$ _____
YM7 $7 \times 43 =$ _____	YM8 $6 \times 22 =$ _____

Journey Game Problem Cards

(Division—Numbers, 1 and 2 digits)

ED1 $72 \div 8 =$ _____	ED2 $24 \div 6 =$ _____
ED3 $60 \div 6 =$ _____	ED4 $36 \div 6 =$ _____
ED5 $81 \div 9 =$ _____	ED6 $35 \div 7 =$ _____
ED7 $45 \div 5 =$ _____	ED8 $21 \div 7 =$ _____

Journey Game Problem Cards

(Subtraction—Numbers, 1 and 2 digits)

OS1 $42 - 6 = ____$	OS2 $85 - 54 = ____$
OS3 $27 - 22 = ____$	OS4 $95 - 70 = ____$
OS5 $8 - 3 = ____$	OS6 $25 - 7 = ____$
OS7 $7 - 4 = ____$	OS8 $36 - 22 = ____$

Journey Game Problem Cards

(Addition—Numbers, 1 and 2 digits)

TA1 $6 + 18 =$ _____	TA2 $17 + 9 =$ _____
TA3 $6 + 7 =$ _____	TA4 $8 + 7 =$ _____
TA5 $28 + 9 =$ _____	TA6 $5 + 7 =$ _____
TA7 $7 + 15 =$ _____	TA8 $41 + 7 =$ _____

Journey Game Problem Cards

(Addition—Word problems, 1-3 digits)

WA1 Michael read 27 books last year. This year he read 32 books. How many books has he read altogether?	**WA2** Brad has 25 stamps in his collection. He got 22 new stamps for his birthday. How many stamps does he have now?
WA3 Heather drives her daughter to school and picks up her friend Katie on the way. If she drives 2 miles to Katie's house and then 3 miles more to the school, how many miles has she driven to get the girls to school?	**WA4** Jeff and his family are eating spaghetti. Jeff has 3 meatballs on his spaghetti, Heidi has 2 meatballs, Katri has 3 meatballs, and Kyle has 5 meatballs. How many meatballs does the family have altogether?
WA5 Mark is on the football team that plays a 9 game season. There are 4 away games where the team is out of town. How many home (at their own stadium) games does he play?	**WA6** Jake went to the zoo. He saw 7 adult tigers and 2 baby tigers. How many tigers did he see at the zoo?
WA7 The cheerleading squad was collecting canned goods for the flood relief in the south of the state. Carol brought 9 cans, Marie 11 cans, Allison 23 cans, and Ellen 22 cans. How many cans did they get?	**WA8** McKenzie and her brother loved to play video games. Her brother's highest score was 203. She beat her brother's score by 10 points. What was her score?

Journey Game Problem Cards

(Subtraction—Word problems, 1-3 digits)

WS1 Veronica has a paper route. She usually delivers 37 papers each morning. She received notice that 4 families on her route were moving away. How many newspapers does she have to deliver now?	**WS2** Shelby is cooking the pasta for dinner. The recipe says to boil the pasta for 9 minutes. The pan has been boiling for 5 minutes already. How many minutes more does she have to let it boil?
WS3 Ryan is allowed to use the computer for 45 minutes to e-mail his friends. He has been on the computer for 15 minutes. How much more time does he have left to use the computer?	**WS4** Paul is saving up to buy a $200 guitar. He has saved $125 already. How much more does he have to save?
WS5 The school bus has 32 seats on it. The bus has picked up 20 children. How many empty seats are there on the bus?	**WS6** Kayli has a regular allowance of $5.00 a week. This week she has to repay her brother Brad the $1.25 that she borrowed. How much money will she have to spend this week?
WS7 The discount store is having a sale on DVD's. All DVD's are $16 each. Before the sale, all the DVD's were $20 each. How much money will Ann save if she buys one DVD?	**WS8** Jerry found 19 ants around the kitchen sink on Sunday. On Monday he saw 30 ants. How many more ants did he see on Monday than on Sunday? (He had better get the ant spray!)

Journey Game Problem Cards

(Multiplication—Word problems, 1 and 2 digits)

WM1 Mark and his 2 friends each bought eight bananas. How many bananas did they buy in all?	**WM2** Rowan put 6 stickers on each row of a sticker album. If there were 7 rows on a page, how many stickers are on the whole page?
WM3 Jake has 3 toy boats. His 2 friends also have 3 boats each. How many boats are there all together? *(Two steps)*	**WM4** Eric has 3 toy trains in the basement. If each train has 10 cars (including the engine and caboose) each, how many train cars does he have all together?
WM5 Season collects play frogs. She puts 3 frogs on each windowsill. If her house has 11 windows, how many frogs does she have in the windows all together?	**WM6** Ryan's schoolbus has 42 seats. How many students can ride on 4 schoolbuses?
WM7 How many eggs are in 4 cartons? Hint—each carton holds 12 eggs.	**WM8** Paul took 6 pictures of Hawaii every day he was on vacation. If he spent 5 days in Hawaii, how many pictures did he take of Hawaii?

Journey Game Problem Cards

(Division—Word problems, 1 and 2 digits)

WD1 Katri has 24 chocolate kisses. If she divides them equally between her 6 friends, how many will each friend get? (Do not include Katri; she already ate her share.)	**WD2** The stamp club ordered a pizza for their meeting. The pizza cost $12. If the 4 members of the club shared the cost equally, how much should each member pay?
WD3 Allison dealt out 40 cards for the Money game. If there were 4 card players, how many cards did each player get?	**WD4** Lindsay dug up 18 plants from her garden. She wanted to give her 3 friends an equal number of plants. How many did she give to each friend?
WD5 Brian went caroling at Christmas time. He sang a total of 24 songs. If he sang the same number of carols at each of 6 houses, how many carols did he sing at each house?	**WD6** Scott liked tee shirts with wild animal pictures on them; he had 12 of them. If his mom and dad each got him the same number of tee shirts, how many did each one of them get?
WD7 Steve made scrambled eggs for the whole family (4 members) and used 12 eggs. If they got the same amount of eggs, how many eggs did each get?	**WD8** Scott's friends asked him to reset their watches to Daylight Savings Time. Each friend had him fix 2 watches and he fixed eight watches total. How many friends had Scott fix their watches?

Journey Game Problem Card Answers

NUMBER PROBLEMS

YM1 = 252
YM2 = 270
YM3 = 154
YM4 = 350
YM5 = 72
YM6 = 35
YM7 = 301
YM8 = 132

ED1 = 9
ED2 = 4
ED3 = 10
ED4 = 6
ED5 = 9
ED6 = 5
ED7 = 9
ED8 = 3

OS1 = 36
OS2 = 31
OS3 = 5
OS4 = 25
OS5 = 5
OS6 = 18
OS7 = 3
OS8 = 14

TA1 = 24
TA2 = 26
TA3 = 13
TA4 = 15
TA5 = 37
TA6 = 12
TA7 = 22
TA8 = 48

WORD PROBLEMS

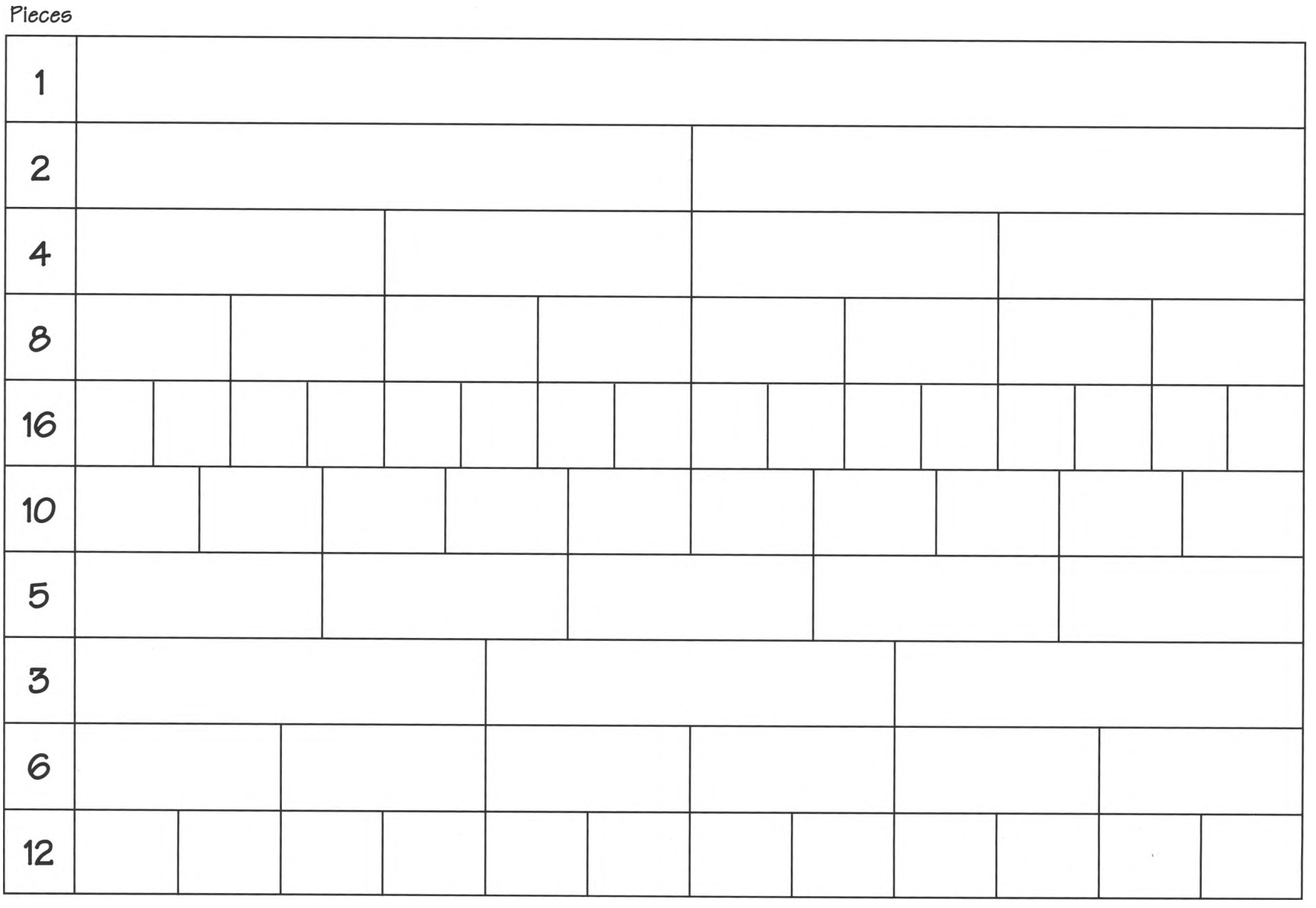
WA1 = 59 books
WA2 = 47 stamps
WA3 = 5 miles
WA4 = 13 meatballs
WA5 = 5 games
WA6 = 9 tigers
WA7 = 65 cans
WA8 = 213 points

WS1 = 33 newspapers
WS2 = 4 minutes
WS3 = 30 minutes
WS4 = $75
WS5 = 12 seats
WS6 = $3.75
WS7 = $4.00
WS8 = 11 ants

WM1 = 24 bananas
WM2 = 42 stickers
WM3 = 9 boats
WM4 = 30 cars
WM5 = 33 frogs
WM6 = 168 students
WM7 = 48 eggs
WM8 = 30 pictures

WD1 = 4 chocolate kisses each
WD2 = $3.00
WD3 = 10 cards
WD4 = 6 plants
WD5 = 4 carols
WD6 = 6 tee shirts each
WD7 = 3 eggs
WD8 = 4 people

Fraction Strips



Domino Fraction Cards

Domino Fraction Cards

Domino Fraction Board—1

$\frac{}{6}$s	$\frac{}{2}$s & $\frac{}{3}$s	$\frac{}{4}$s	$\frac{}{5}$s
$\frac{5}{6}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
$\frac{2}{6}$	$\frac{2}{3}$	1	$\frac{2}{5}$
$\frac{3}{6}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{5}$
$\frac{4}{6}$	1	$\frac{2}{4}$	$\frac{4}{5}$

Domino Fraction Board—2

$\frac{}{6}$s	$\frac{}{2}$s & $\frac{}{3}$s	$\frac{}{4}$s	$\frac{}{5}$s
$\frac{3}{6}$	$\frac{2}{3}$	1	$\frac{4}{5}$
$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{4}$	1
$\frac{2}{6}$	1	$\frac{2}{4}$	$\frac{2}{5}$
$\frac{5}{6}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{5}$

Domino Fraction Board—3

$\frac{}{6}$s	$\frac{}{2}$s & $\frac{}{3}$s	$\frac{}{4}$s	$\frac{}{5}$s
$\frac{4}{6}$	$\frac{2}{3}$	$\frac{2}{4}$	$\frac{3}{5}$
$\frac{5}{6}$	1	$\frac{1}{4}$	$\frac{1}{5}$
$\frac{3}{6}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{2}{5}$
$\frac{2}{6}$	$\frac{1}{3}$	1	$\frac{4}{5}$

Fraction Strips Worksheet

Figure what the shaded part of the fraction strip equals as a fraction. Put one of the following fractions on the right hand margin next to the fraction:

$1/2$ $1/4$ 1 $3/4$ $6/8$ $2/4$ $8/16$ $4/16$ $4/8$

Equivalent Fractions Worksheet

Use the Fraction Strips Worksheet (from Appendix B) to work these problems.

1) $\frac{6}{8} = \frac{\quad}{4}$

2) $\frac{2}{10} = \frac{\quad}{5}$

3) $\frac{4}{6} = \frac{\quad}{3}$

4) $\frac{3}{6} = \frac{\quad}{10}$

5) $\frac{2}{4} = \frac{\quad}{6}$

6) $\frac{4}{6} = \frac{\quad}{9}$

7) $\frac{4}{8} = \frac{\quad}{2}$

8) $\frac{3}{4} = \frac{\quad}{8}$

9) $\frac{1}{5} = \frac{\quad}{10}$

10) $\frac{1}{4} = \frac{\quad}{8}$

11) $\frac{4}{5} = \frac{\quad}{10}$

12) $\frac{2}{5} = \frac{\quad}{10}$

PLAY CURRENCY $1

PLAY CURRENCY $5

PLAY CURRENCY $10

PLAY CURRENCY $20

PLAY CURRENCY $50

$1.11	$17.62
$9.08	$6.10
$12.10	$18.12
$4.77	$8.23
$1.75	$6.22
$4.03	$1.04

$9.06	$6.10
$11.09	$7.05
$ .25	$ .05
$ .10	$ .50
$ .01	$ .90
$ .75	$ .30

Exact Dollar Amount	Next-Highest Dollar Amount
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.
9.	9.
10.	10.
11.	11.
12.	12.

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About the author:

DeAnna Horstmeier, Ph.D., is a retired educator, former instructor of special education and speech, language, and communication at the Ohio State University, and author of books and professional publications on math and reading.

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